The Impact of Electronic Trading and Exchange Traded Funds on the Effectiveness of Minimum Variance Hedging

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Abstract

This empirical study examines the impact of both advanced electronic trading platforms and index exchange traded funds (ETFs) on the minimum variance hedging of stock indices with futures. Our findings show that minimum variance hedging may provide an out-of-sample hedging performance that is superior to that of the naïve futures hedge, but only in markets without active trading of ETFs and advanced development of electronic communications networks. However there is no evidence to suggest that complex econometric models that include, for instance, time varying conditional covariances and error correction can improve on the simple ordinary least squares hedge ratio. Furthermore, in markets with actively traded index ETFs and where electronic trading has become established, no significant efficiency gains are apparent from any minimum variance hedge.

JEL Classification: C32, G10, G15
Keywords: Minimum Variance, Futures Hedging, Stock Indices, Exchange Traded Funds, Electronic Trading, Conditional Effectiveness Measure

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I  INTRODUCTION

The debate on econometric models for estimating the minimum variance futures hedge ratio has run for many years. Hedging commodities is an interesting econometric problem because carry costs are difficult to predict and the basis can be large and variable, but stock indices generally have much lower basis risk. Nevertheless econometric models for minimum variance (MV) hedging of stock indices continue to be the focus of a huge amount of empirical research.

The theme for our paper stems from a recent strand of literature on market microstructure relating to the impact of electronic trading on bid-ask spreads. Electronic trading reduces both human errors and, since smaller lot sizes become economically feasible, market impact. Also, given the discipline required to commit trading rules to execution algorithms, it increases objectivity. Moreover, electronic trading increases liquidity because transactions costs are reduced.

A recurrent proxy for transactions costs is the bid-ask spread and several academic papers have tested the impact of electronic trading on reducing this spread. Early studies in New Zealand (Blennerhasset and Bowman [1998]), Germany (Frino, McNish and Toner [1998]) and the cross listing of bund contracts in Germany and England (Pirrong [1996]) reported lower bid-ask spreads in electronic trading systems. Yet Massib and Phelps [1994] argue that open outcry can offer higher liquidity than electronic systems, and Shyy and Lee [1995] found that spreads are actually greater on electronic systems than on floor trading of the Bund futures contracts. More recently, Chordia, Roll and Subrahmanyam [2001] analyse the characteristics of US equity market spreads, depths and trading activities. They observe a downward trend in spreads and the opposite trend in depth and volume. They also find that bid-ask spreads respond asymmetrically to market movements, increasing significantly in downward markets but decreasing only marginally in up markets and they also argue that excessive market volatility can reduce rather than increase the bid-ask spread. Copeland, Lam and Jones [2004] conclude that screen trading does not improve efficiency of FTSE, CAC, DAX and KOSPI futures trading. Yet Fung, Lien, Tse and Tse [2005] argue that electronic trading on the Hang Seng Composite Index attracts more informed traders to the futures market and increases information flow, in addition to reducing the bid-ask spread. Gilbert and Rijken [2006], Tse and Zabotina [2001] and Aitken et al. [2004] analyze the FTSE 100 futures contracts as their trading migrated from open outcry to electronic trading in May 1999. However Aitken et al. [2004] find wider spreads while Gilbert and Rijken [2006] and Tse and Zabotina [2004] find lower spreads.

Contradicting arguments on the advantages of electronic systems for reducing transactions costs and providing effective liquidity, especially during highly volatile periods, are frequent in academic research. However a recent survey by Burghart [2006] on global trends in trading volumes and average bid-ask spreads provides a convincing argument that the growth in electronic trading systems since 1999 has been the driving force behind dramatically increased trading volumes and huge reductions in bid-ask spreads during the last six years. He shows that average, volatility adjusted, bid-ask spreads on several
electronically traded futures have steadily decreased to between 60% and 90% of their level in 1999, yet those on pit-traded contracts such as Soya beans and Crude oil have not reduced at all.

Whist the effect of electronic trading on market efficiency remains a point of academic debate, there is a consensus view that cash market efficiency increases following the introduction of an exchange traded fund (ETF) or iShare on the index. Switzer, Varson, and Zghidi [2000], Ackert and Tian [2000, 2001], Chu and Hsieh [2002] and Kurov and Lasser [2002] argue that an ETF or iShare contract facilitates spot-futures arbitrage, thus increasing cash market efficiency and reducing the no-arbitrage range for the future about its fair value.

In the presence of high trading costs and costly information, transitory deviations from the equilibrium relation between spot and futures prices might be observed. But as trading costs decrease and spot-futures arbitrage is facilitated by an ETF, the correlation between spot and futures returns increases and basis risk decreases. Hence the development of both index ETFs and advanced electronic trading network may reduce the efficiency of a MV hedge ratio relative to that of the naïve hedge, in which the short position in the future is matched exactly to the spot position. We shall show that no significant gains can now made from MV futures hedging of some major stock indices. However, MV hedging may still improve on a one-to-one hedge on less efficient exchanges. Furthermore we show that on those exchanges where MV hedging may still be more effective than a one-to-one hedge, it is not possible to distinguish which econometric model most efficiently reduces the variance. Finally, our results also support the growing evidence (e.g. from Copeland and Zhu [2006]) that more sophisticated econometric models, e.g. GARCH, incorporate too much noise to provide cost effective hedges.

The theoretical and empirical methodology in this paper extends previous research in two significant ways:

- Lien [2005] proves that it is not appropriate to evaluate the hedging performance of conditional variance minimisation using the unconditional effectiveness measure of Ederington [1979], even though it is used in most articles on this subject. Since performance is sample specific (it depends on both the estimation and the evaluation samples), we introduce a conditional measure of hedging effectiveness;

- We use this conditional effectiveness measure to examine the evolution of hedging efficiency over many years, before and after the introduction of electronic trading platforms. Our results are based on an out-of-sample performance test on nearly 4000 observations of most of the seven indexes, and this is considerably larger than in any other published research.

II LITERATURE ON MINIMUM VARIANCE HEDGING

Johnson [1960] was the first to derive the quantity of futures contracts necessary to hedge a certain spot position based on minimising the variance of the hedged portfolio. Much of the debate that followed concerned whether the minimum variance (MV) criterion is appropriate since it is based on a quadratic utility function, which is only one of many possible objective functions. Other utility functions (as in
Cecchetti, Cumby and Figlewski (1988) or alternative hedging objectives may be applied. For instance: Howard and D'Antonio (1984) design the hedge to maximize the Sharpe ratio; Cheung, Kwan and Yip (1990), Lien and Luo (1993) and Lien and Shaffer (1999) minimize the mean-Gini coefficient; and Eftekhar (1998), Lien and Tse (1998, 2000) and Mattos, Garcia and Nelson (2006) employ objectives that include minimization of the generalised semi-variance or higher lower partial moments.

The papers by Lien and Tse (2002) and Chen, Lee and Shrestha (2003) are dedicated exclusively to review the huge literature on futures hedging. Many papers consider MV hedge ratio estimation based on an advanced econometric model with time-varying hedge ratio given by the ratio of the conditional covariance of spot and futures returns to the conditional variance of the futures returns. The seminal paper by Baillie and Myers (1991) concluded that the generalised autoregressive conditional heteroscedastic (GARCH) model provides a superior performance to other dynamic or constant hedges, given the time-varying nature of the conditional distributions of commodities returns and their futures contracts. Moschini and Myers (2002) reject the hypothesis of a constant hedge for the corn weekly series under the assumption that spot and futures prices have GARCH effects. They also reject the hypothesis that seasonality and time-to-maturity account for the total variation in the optimal hedge ratios. Chan and Young (2006) incorporate a jump component to the bivariate GARCH model to hedge the copper market and find that the jump GARCH hedge performs better than the constant hedge for both daily and weekly frequencies. Other more advanced models, such as the Markov switching GARCH of Lee and Yoder (2005), also appear useful for hedging commodity prices.

It is not only in commodity markets that advanced econometric models may produce more efficient MV hedge ratios. Bhattacharya, Sekhar and Fabozzi (2006) show that the cointegration GARCH model provides a powerful means of pricing and hedging mortgage backed securities (MBS), which is more efficient than standard regression because it captures the dynamics between MBS and Treasury note futures in a low interest rate environment. Their hedging results show that the cointegration GARCH model is substantially better than the regression-based model at hedging various MBS with a 10-year Treasury note futures contract and another MBS, with the cross hedge effectively accounting for negative convexity. This extends the study by Koutmos and Pertieli (1999) who tested the effectiveness of the cointegration GARCH hedge ratios without a cross hedge.

In fixed income markets the underlying and hedging contracts often differ, indeed a portfolio of hedging instrument may be used. Also the underlying may be less liquid than the hedging instruments, hence their correlation will rarely be close to unity. And in commodity markets the basis may be extremely volatile and prices may not follow a random walk. In these circumstances advanced econometric models can be very useful for computing the most efficient hedge ratio. However, many econometricians apply similar MV hedge ratios to hedging stock indices – and it is the usefulness of this research that will be questioned in this article. The case for MV hedging of stock indices is much less sound than it is for commodities, fixed
income securities and indeed any asset where it is likely that the correlation between the underlying and the hedging instrument(s) is high, but far from perfect. Stock trading is now highly efficient on many exchanges and the basis risk on stock indices is usually very small indeed.

A seminal paper on minimum variance hedging of stock indices is by Figlewski [1984], who analysed the futures cross hedging and hedging with the S&P 500 index between June 1982 and September 1984. Subsequent papers investigate the effect of dividend yield (Graham and Jennings [1987]), futures mispricing (Merrick [1988]), duration and expiration effects (Lindahl [1992]) and investment horizon (Geppert [1995]). Sutcliffe [2005] contains a comprehensive review of the literature on hedging stock indices with futures.

Hedgers of stock indices include stock market makers, equity hedge funds and indeed any investor aiming to neutralise the market risk factor derived from a mandatory exposure in his portfolio. But by hedging an exposure to a stock index the investor is also giving up potential returns; in other words he pays a price for the hedge. A direct cost of rebalancing the hedged portfolio arises when MV hedge ratios are employed. Whilst the direct cost per transaction is likely to be small, the cumulative cost of MV hedging large positions over a long period may be significant. Advanced econometric models can produce hedge ratios that vary excessively over time, as shown by Lien, Tse and Tsui [2002], Poomimars, Cadle and Theobald [2003], Harris and Shen [2003], Choudhry [2003, 2004], Miffre [2004], Alizadeh and Nomikos [2004], and Yang and Allen [2005]. Thus increased transactions costs could offset any potential gain in efficiency. So, even if basis risk is high enough to warrant the use of MV hedge ratios, their costs may well be greater than their benefits.

Index spot and futures prices typically have a unit root and error correction hedging models will take account of the basis convergence. The papers by Garbade and Silber [1983], Myers and Thompson [1989] and Ghosh [1993] take cointegration and the lead-lag relationship between cash and futures prices into account. Kroner and Sultan[1993] and Miffre [2004] incorporate conditionality in the available information with error correction models. However Lien [2004] has proved that the omission of the cointegration relationship should have minimal impact on hedging effectiveness.

Several other papers aim to demonstrate the superiority of sophisticated dynamic hedge ratios for hedging stock indices with futures. Using daily prices from June 1988 to December 1991, Park and Switzer [1995] show that a symmetric bivariate GARCH hedge ratio outperforms the constant hedge ratio for the S&P500, MMI and Toronto 35 stock indices. Tong [1996] and Brooks, Henry and Persand [2002] support this general result, but the latter paper, using daily prices on the FTSE 100 index and futures contract from January 1985 to April 1999, finds no improvement in hedging efficiency when asymmetric volatility responses are added to the GARCH model. Choudhry [2003] compares naïve, OLS, and GARCH hedge ratios for stock indices in Australia, Germany, Hong Kong, Japan, South Africa and the United Kingdom.
with a two-year out of sample period between January 1998 and December 1999 to conclude that GARCH models perform the best. Floros and Vougas [2004] also find that GARCH hedge ratios perform better than OLS and vector error correction models (ECM) for hedging the Greek stock index market between 1999 and 2001. However, Laws and Thompson [2005] apply OLS, GARCH and exponentially weighted moving average (EWMA) hedge ratios to index tracking portfolios from January 1995 to December 2001 and conclude that the EWMA method provides the best performance.

More recent papers investigate hedging efficiency using even more advanced econometric techniques for computing minimum variance hedge ratios. Alizadeh and Nomikos [2004] compare Markov switching GARCH models with traditional GARCH, ECM and OLS methods using weekly prices on the S&P500 and FTSE-100 markets from 1984 to 2001, using a one-year out-of-sample period. They conclude that the Markov switching GARCH outperforms all other models in the FTSE market and both GARCH models are superior for the S&P500. Dark [2004] examines the bivariate error correction GARCH and fractionally integrated GARCH models applied to the Australian All Ordinaries Index, finding that these produce ratios that are superior to the OLS and naïve hedge ratios, a result that is supported by Yang and Allen [2005]. However, there is no evidence that fractional integration improves the effectiveness of the GARCH model. The out-of-sample period runs over three months only, ending in October 1999. Finally, Lai, Chen and Gerlach [2006] develop a copula threshold GARCH model to estimate optimal hedge ratios in the Hong Kong, Japan, Korea, Singapore and Taiwan indices, finding their model to improve on traditional OLS in three of the five markets. However, a recent discussion paper by Copeland and Zhu [2006] compares various dynamic ratios with the standard OLS hedge ratios for six equity markets (Australia, Germany, Japan, Korea, UK and US). They argue that there are no clear benefits from utilizing more sophisticated hedging models. Our study will add further weight to this argument.

An important critique of all this research is presented by Lien [2005]. The recurrent measure to evaluate the hedging performance is the unconditional effectiveness measure proposed by Ederington [1979], but Lien proves that this measure is inappropriate when spot and futures prices are cointegrated. In this paper, we shall therefore extend Ederington’s methodology by computing a conditional effectiveness measure that allows one to evaluate the dynamic characteristics of the effectiveness of different hedging strategies.

### III A SURVEY OF TRADING CHARACTERISTICS IN INDEX MARKETS

We shall investigate futures hedging effectiveness in seven stock indices that have different trading characteristics: the Nasdaq 100 and S&P 500 indices from North America, the FTSE 100 and CAC 40 indices from Europe, the Hang Seng Composite and Kospi 200 indices from Asia and the Ibovespa index from South America. The Nasdaq Exchange (Nasdaq) is arguably one of the most efficient and advanced electronic exchanges and it will be contrasted with the Hang Seng Composite, the Ibovespa and the Kospi 200 where the electronic platforms are at an earlier stage of development. We also include the S&P 500 and the FTSE 100 because these have been the focus of much previous academic research.
The Us Exchanges

Even though transactions costs on the technology stocks in the Nasdaq 100 index are relatively high, the Nasdaq exchange has a very highly evolved electronic communications network (ECN). The Nasdaq was originally a network of dealers but brokers introduced an ECN during 1996-1997 and, by 2002, even super-montage consolidated quotes had been introduced. In this sense the Nasdaq is more efficient than both the London and New York stock exchanges. Total trading volume on the Nasdaq 100 stocks during 2005 was very high (432,504 million US dollars) and since April 1999 there has also been a very liquid ETF (the ‘Cubes’) on the Nasdaq 100 index. In terms of assets under management the Cubes is the second largest ETF in the US, but trading volume during 2005 actually exceeded that on the Spider, averaging over 90 million US dollars per day.

The S&P 500 stocks are traded on both the New York and Nasdaq exchanges and total trading volume amounted to 483,815 million US dollars during 2005. This index also has one of the most actively traded index ETFs in the world: the so-called Spider (i.e. the ‘Standard and Poor’s Depositary Receipt’). The American Stock Exchange released the Spider in 1993 and it also started trading on the New York Stock Exchange (NYSE) in 2001. During 2005 the average daily trading volume on the Spider was over 60 million US dollars and by December 2005 the fund had a colossal 59 billion US dollars under management. This represents nearly one-quarter of the entire US market in passive ETFs. Another factor contributing to the efficiency of both the NYSE and the Nasdaq Exchange is that in April 1999 the US Securities Exchange Commission introduced new regulations governing the trading and execution on electronic order books, requiring market makers to compete fairly with limit orders.

The European Exchanges

FTSE 100 stocks trade on the London Stock Exchange (LSE) and some trading is still carried by dealers even though their electronic communications network, which is called the stock exchange trading service (SETS), was introduced in October 1997. The move towards electronic order book trading was slow initially, because the LSE lacked confidence in the system, but by the year 2000 a large proportion of trading was over SETS. During 2005 total trading volume on FTSE stocks was 395,070 million US dollars on bid ask spreads of around 25 basis points, depending on the share. Efficiency on the FTSE market is further enhanced by trading on the iFTSE 100 index share, although this contract is not nearly as liquid as many of the US index ETFs.

The CAC 40 futures contract has its spot trading on Euronext, a totally integrated European cross-border market that now encompasses the Liffe, Paris, Belfox, Amsterdam and Lisbon exchanges. Since 1st June 1998 Euronext has operated only electronic trading, and the CAC 40 future is one of the most actively traded contracts on this exchange. An ETF for the CAC 40 index (the Lyxor ETF CAC 40) was launched in January 2001 and by 30th December 2005 it had over 3 billion Euros of assets under management.
The Asian and South American Exchanges

For examples of markets where electronic trading is less developed than the US and European exchanges and index ETFs are not actively traded, we consider: the Hang Seng Composite Index (HSCI) in Hong Kong, the Kospi 200 from Korea and the Ibovespa index (IBOV) index in Brazil.

The HSCI stocks are listed on the Hong Kong Stock Exchange where brokers have operated an electronic auto-matching system since 1993. Futures trading migrated from the pit to an electronic platform in June 2000. The Hong Kong auto-matching system has recently been enhanced to upgrade the limit order system but it still has fewer advanced features than SETS and less regulation than the ECNs in the US. Moreover, total trading volume on the Hang Seng stocks during 2005 was only about 232,808 million US dollars, which is about 60% of the trading volume on the FTSE 100 and an even smaller fraction of the trading volume on either S&P 500 or Nasdaq 100. An ETF on HSCI, the Tracker Fund of Hong Kong was launched in November 1999. However even by 2005 the daily average trading volume on this fund was a mere 2.66 million US dollars, a small faction of the volume traded on the Cubes or the Spider.

The Kospi 200 futures contracts began trading at the Korean Exchange on May 3, 1996. The trading of the contract reached a volume of nearly 34 million contracts in 2005 with a trading value nearly 18 billion US dollars. An ETF on the Kospi 200 index (the Kodex 200) started trading in October 2002. Although it was introduced much later than the ETF on the HSCI, there is a much higher trading volume on the Kodex 200 and by the end of 2005 it had 800 million US dollars in assets under management. The Korea Exchange also trades all contracts through an electronic system. According to the 2005 Annual Report of the World Federation of Exchanges, it was the fifth largest exchange for trading of index futures contracts in 2005, after CME, Eurex, Euronext and the National Stock Exchange, India.

The Ibovespa stocks are listed in the Sao Paulo Stock Exchange and the futures contract is traded at the BM&F Futures Exchange. The Brazilian futures Ibovespa contract is traded in a hybrid market with both open outcry and electronic systems. The number of contracts traded on Ibovespa reached 6,065,361 in 2005. The BM&F is still in the process of migrating the futures contract to be traded solely on an electronic platform and it has specified limited maturities to be traded on the floor. The Ibovespa does not have a tracker fund with shares traded on the exchange. The only ETF in the Brazilian market is benchmarked to the IBX index, which is, however, highly correlated with the Ibovespa.

V THE DATA AND ITS CHARACTERISTICS

We use daily closing prices on S&P 500, FTSE 100, HSCI and their corresponding futures contracts from 19th April 1994. The Ibovespa dataset starts in 2nd August 1995, the Kospi 200 data starts on 6th May 1996, the Nasdaq 100 starts on 15th April 1996 and the CAC 40 data starts on 8th January 1999. The last trading day is 19th April 2006 in all datasets. We divide each sample into two periods, before and after 1st
April 1999. We choose this date because it marked the introduction of SEC regulation on ECNs and because index ETFs became established in the US and Europe during this period. If the US and European market trading efficiency was lower in the pre 1999 sub-sample we may find that MV hedging was more efficient then, compared with the post 1999 sub-sample.

Table 1 summarises the descriptive statistics of the two sub-samples. For the first sub-sample we observe high average returns as a result of the bull market of the 90’s. As the technology bubble burst in the second sub-sample, average returns were considerably lower with negative values for the Nasdaq 100 and FTSE 100. The highest volatility was observed in Ibovespa during the first sub-sample, as this period includes the period immediately after the Brazilian stabilisation plan. Volatility of the HSCI and Kospi 200 was also extremely high in the first sub-sample due to the Asian Crisis in 1997.

All spot market returns are highly correlated with the corresponding futures returns. As expected, this correlation is highest on the Nasdaq 100 and CAC 40 and lowest on the HSCI and Kospi 200. For all the indices except the Nasdaq 100 and Ibovespa, spot-futures returns correlation is notably higher during the second sub-sample. Finally, since all the spot and futures prices are cointegrated according to the Johansen maximum eigenvalue test (which has 1% critical value of 6.65) error correction models for MV hedge ratios should apply.

Before proceeding to the empirical results we point out that there is a discrepancy between closing times in most cash and futures markets. For instance, in the S&P 500, Nasdaq 100, and Kospi 200 the cash market closes 15 minutes before the futures market. In fact only the Ibovespa regular trading sessions of the futures and cash markets have synchronous closing times. Hence the use of daily closing prices on the spot index and the index future is likely to produce a downward bias on returns correlation and an upward bias on basis risk. Clearly this could result in the conclusion that MV hedge ratios are more effective than they really are.

The lack of synchronous daily data must have affected results in numerous previous empirical studies that use daily closing prices to demonstrate the superiority of econometric models for estimating MV hedge ratios. However in our study we aim to support the hypothesis that MV hedge ratios cannot improve on the naïve hedge once electronic trading has been fully developed and an ETF has become established. Hence non-synchronous data is not so much of an issue because it will tend to bias results in favour of the alternative hypothesis.

VI ECONOMETRIC MODELS FOR SHORT TERM FUTURES HEDGING

Two questions arise with futures hedging: the first is how to estimate the optimal number of futures contracts and the second is how to measure the efficiency of the hedging strategy. These two questions are deeply related and should be tackled in conjunction. The MV hedge ratio is defined as the number of
futures per unit of the spot asset that will minimize the variance of the hedged portfolio returns. In this study four different MV hedge ratios have been calculated as follows:

1. **OLS**: this is the estimated slope coefficient in the simple OLS regression:
   \[
   s_t = \alpha + \beta f_t + \varepsilon_t,
   \]
   where \( s_t \) and \( f_t \) denote the daily log returns on the spot index and the index future;

2. **ECM**: here a lagged disequilibrium term and lagged dependent variables are added in a bivariate vector error correction mechanism. The equation for the spot returns is:
   \[
   s_t = \alpha + \beta_1 f_t + \beta_2 s_{t-1} + \beta_3 f_{t-1} + \lambda \varepsilon_{t-1} + \varepsilon_t
   \]
   and \( \varepsilon_t \) is the difference between the log of the futures price and the log of the stock price. Here the OLS estimate \( \hat{\beta}_1 \) is the minimum variance hedge ratio;

3. **EWMA**: this is similar to the ordinary OLS ratio but it uses exponentially weighted average estimates of the futures returns variance (\( \hat{\sigma}^2_{f,t} \)) and of the spot and futures returns covariance (\( \hat{\sigma}_{sf,t} \)). That is, we put:
   \[
   \hat{\sigma}^2_{f,t} = \lambda \hat{\sigma}^2_{f,t-1} + (1 - \lambda) f_{t-1}^2
   \]
   and these give a time-varying estimate of the hedge ratio as:
   \[
   \hat{\beta}_t = \frac{\hat{\sigma}_{sf,t}}{\hat{\sigma}^2_{f,t}}.
   \]

4. **GARCH**: this calculated is in (3) but the variance and covariance estimates are obtained from a bivariate GARCH model. Here the two conditional mean equations are given by a bivariate vector error correction mechanism, following the general specification for cointegrated processes given by Engle and Granger [1987].

The models 1, 2, and 4 were estimated using the last 6 months of daily data, we chose one lag for the ECM and model 3 was based on a smoothing constant of \( \lambda = 0.95 \). These decisions carry a certain element of model risk. So we also examined results for different lags in (2), and based on different in-sample periods (of 3 months, 1 year and 2 years) and we also varied the value of \( \lambda \) within reasonable limits. Moreover in 4 we estimated a variety of different bivariate GARCH models. Whilst numerical results varied the qualitative nature of our conclusions remained unchanged.

A traditional measure of hedging effectiveness, derived by Ederington (1979) and since applied in numerous empirical studies, is the percentage reduction in variance:

\[
E = \frac{\sigma^2_s - \sigma^2_h}{\sigma^2_s}
\]

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where $\sigma_u^2$ and $\sigma_h^2$ denote the sample variance of the un-hedged and hedged portfolio returns respectively. Since the MV criterion is applied in-sample and the hedging effectiveness is tested out-of-sample there is no guarantee that MV hedging will produce more effective hedges, in terms of variance reduction, than the naive hedge.

Lien [2005] has emphasised the inadequacy of the regression $R^2$ to evaluate minimum variance hedge ratios other than OLS. He has also proved that (4) will favour the OLS hedge ratio when spot and future prices are cointegrated. For this reason, and also to provide a time-varying measure of out-of-sample hedging effectiveness, we propose the following conditional effectiveness measure:

$$E_t = \frac{\sigma_{u,t}^2 - \sigma_{h,t}^2}{\sigma_{u,t}^2}$$

where $\sigma_{u,t}^2$ and $\sigma_{h,t}^2$ denote conditional variances of the un-hedged and hedged portfolio out-of-sample returns respectively. For simplicity the EWMA variance with a smoothing constant of 0.95 is used in (5).

VII  **Empirical Results**

For each hedging model we perform an out-of-sample analysis of hedging performance with daily re-balancing. Each day we estimate the MV hedge ratio to determine the futures position to be taken at the end of that day until the following day. The sample is then rolled one day, the hedge ratios re-estimated, and the hedge re-balanced and held until the end of the next day. Each of the models is estimated using a 6-month in-sample period staring on 19th April 1994 (or 15th April 1996 for the Nasdaq 100, 8th January 1999 for the CAC 40, 2nd August 1995 for Ibovespa and 3rd May 1996 for the Kospi 200) and rolling the estimation period forward one day at a time until we reach the end of the entire sample.

Figure 1 shows how different hedge ratios have evolved over the sample period for the FTSE 100. For brevity the hedge ratios for the other indices are not shown, but in each case the ECM hedge ratio displays similar time-varying characteristics to the OLS ratio except that it is generally closer to one, and the EWMA and GARCH ratios are much more variable because they are based on conditional covariances. Hence both EWMA and GARCH models would require considerable re-balancing at the daily frequency, a feature that has also been observed by Lien, Tse and Tsui [2002], Poomimars, Cadle and Theobald [2003], Choudhry [2003], Miiffre [2004], Alizadeh and Nomikos [2004] and Yang and Allen [2005]. In both the FTSE 100 and the S&P 500 the hedge ratios increase towards one over time. In the CAC 40 and NasdaqAQ 100 the hedge ratios are very close to one over the entire period. As expected the HSCI has the lowest average hedge ratio and, as in the Ibovespa, we cannot distinguish any trend. The Kospi 200 has lower hedge ratios in the first sample period, and they are particularly low during the Asian Crisis; thereafter they increase towards unity.
Figures 2 to 4 illustrate the differences between the conditional effectiveness measures given by the four different MV hedge ratios and the naïve hedge, a positive value thus indicating that the MV ratio performs better than the one-to-one ratio. The time series in Figure 2 for the FTSE 100 index shows very clearly that, when effectiveness is measured in a time-varying framework, no significant variance reduction from MV hedging beyond the variance reduction offered by the naïve hedge has been possible since 2000. Before then the MV hedge ratios offered greater variance reduction than the naïve hedge. Yet it is not possible to decide whether the ordinary OLS, the ECM, the EWMA or the GARCH hedge was the better strategy. The equivalent figures for the S&P 500 and Kospi 200 indices (not shown) indicate very similar characteristics: MV hedging appears more efficient than one-to-one hedging prior to 2000, but since then no significant variance reduction from MV hedging is apparent. It should also be noted that an extremely high volatility on the Kospi 200 index may have contributed to higher spreads and thus increased the efficiency of MV hedges relative to the naïve hedge during the Asian crisis in 1997.

Figures 3 and 4 are in stark contrast to each other. From Figure 3 we see that the Nasdaq exchange is so efficient that MV hedging has never been able to reduce variance significantly compared with the naïve hedge, except for a few short and isolated periods in the sample. These are periods of high volatility on the Nasdaq 100 index, during the Asian Crisis in 1997, the Russian Crisis in 1998 and during the burst of the technology bubble in 2000. At these times there is a small increase in the conditional efficiency of the MV hedge ratios relative to the naïve hedge. However it is apparent that the efficiency of trading on the Nasdaq is sufficient to ensure that transactions costs and therefore basis risk remains low. The equivalent figure for the CAC 40 index (not shown) also indicates zero improvement from MV hedging, although only the second sub-sample is available. For the Ibovespa (also not shown) we find a few isolated periods when a MV hedge can be more marginally effective than the naïve hedge, and these are also associated with excessively high volatility in the market. By contrast, figure 4 shows that for the HSCI all MV hedges can dramatically improve on the naïve hedge, even during the latter part of the sample.

Table 2 reports the volatilities and the Ederington measure (4) for all of the out-of-sample hedged portfolio returns over both sub-samples. Except in the Nasdaq 100 and Ibovespa, where there is very little difference in the efficiency of different hedge ratios, the naïve hedge is clearly less efficient than the MV hedges during the first sub-sample. Hedging effectiveness in the Kospi 200 and HSCI is lower than for the other indices, although it improves during the second sub-sample, and for these indices the naïve hedge remains less effective than the other hedges even during the second sub-sample. During the second sub-sample and in the other indices there is no significant difference between the hedge portfolio returns distributions, irrespective of the hedge ratio used.

To support these observations we report in Table 2 the probability values of Kolmogorov-Smirnoff (KS) tests for the hypothesis that the MV hedge portfolio returns are drawn from the same distribution as the
naïve hedged portfolio returns. During the first sub-sample the KS statistics for FTSE 100 and HSCI indices are highly significant, and those for the Kospi 200 index also have low probability values.

The probability values of the KS statistics are uniformly greater in the second sub-sample, indicating that the distributions of MV hedged portfolio returns are moving closer to the distribution of naïve hedged portfolio returns. During the second sub-sample the probability values of the KS statistics for HSCI, Ibovespa and Kospi 200 are much lower than they are for the US and European indices. Compare, for instance the probability value of 0.9997 for the GARCH hedge of the S&P 500, with the probability value of 0.1263 for the equivalent hedge of the HSCI.

We have also employed KS statistics to test for any significant difference between hedged portfolio’s returns based on different MV methods. These results have been omitted, for brevity, but all excluded results are available from the authors by request. No significant results were found so we must conclude that there is no discernable difference between any of the MV hedging strategies for any of the indices in either sub-sample.

VIII SUMMARY AND CONCLUSIONS

Our study adds weight to the argument against MV hedge ratios for short-term futures hedging of stock indices in markets where trading mechanisms are highly efficient. The advanced electronic trading platforms in the US and European markets have increased trading volume and reduced transactions costs. Actively traded ETFs on these indices have further increased trading efficiency. We have shown that since the turn of the century, MV ratios in these markets have offered no discernable improvement on the naïve futures hedge. However, in markets where trading is less efficient, such as in the Hang Seng composite index, econometric models may still provide hedge ratios with more efficient variance reduction than the naïve hedge. Even so, we found no evidence to suggest that complex econometric models such as GARCH can improve on a simple OLS regression for estimating this hedge ratio.

This last finding accords with those of Poomimars, Cadle and Theobald [2003], who compare the empirical performance of several dynamic and static models in seven markets: S&P 500, Nikkei-225, FTSE100, JPY, GBP, Gold and Silver. They conclude that hedging performance is similar for most models. Moosa [2003] also concludes that ‘although the theoretical arguments for why model specification does matter are elegant, the difference model specification makes for hedging performance seems to be negligible. What matters for the success or failure of a hedge is the correlation between the prices of the un-hedged position and the hedging instrument. Low correlation invariably produces insignificant results and ineffective hedges, whereas high correlation produces effective hedges irrespective of how the hedge ratio is measured.’

Our results are also consistent with previous empirical research that demonstrates that some MV models incorporate too much noise to be effective for hedging purposes. The benefits of an active hedging
strategy should be economically justifiable yet these models do not account for transactions costs, such as margins and commissions. When the costs of hedging are considered the case against MV hedge ratios based on conditional covariances is strengthened even further. We find that the more advanced the econometric model used, the greater the variability in the hedge ratio and the more frequently the hedged portfolio would be rebalanced in practice. In this respect we agree with Lence [1995], who argues that sophisticated econometric models for estimating MV hedge ratios provide negligible economic benefits and suggests that the effort dedicated to estimate better MV hedges 'has been a waste of resources'.
References


Table 1: Summary Statistics

The table reports the first four moments of the daily returns distributions over two samples, the correlation between spot and futures returns, the OLS hedge ratio measured over the entire sample and the R squared from this regression. We also report the Johansen maximum eigenvalue cointegration test (which has 1% critical value of 6.65). When spot and future prices are cointegrated, error correction models for MV hedge ratios should apply.

<table>
<thead>
<tr>
<th>Sample I</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
<th>Nasdaq 100</th>
<th>CAC 40</th>
<th>HSCI</th>
<th>Ibovespa</th>
<th>Kospi 200</th>
</tr>
</thead>
<tbody>
<tr>
<td>19/04/91* to 31/03/99</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
</tr>
<tr>
<td>Average annual return</td>
<td>11.04%</td>
<td>10.96%</td>
<td>14.57%</td>
<td>14.57%</td>
<td>40.34%</td>
<td>40.32%</td>
<td>N/A</td>
</tr>
<tr>
<td>Volatility</td>
<td>13.90%</td>
<td>16.03%</td>
<td>13.14%</td>
<td>14.43%</td>
<td>28.25%</td>
<td>29.63%</td>
<td>N/A</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.1049</td>
<td>0.0668</td>
<td>-0.5167</td>
<td>-0.4131</td>
<td>-0.3321</td>
<td>-0.3382</td>
<td>N/A</td>
</tr>
<tr>
<td>Unconditional Correlation</td>
<td>0.9564</td>
<td>0.9626</td>
<td>0.9714</td>
<td>N/A</td>
<td>0.9369</td>
<td>0.9740</td>
<td>0.8447</td>
</tr>
<tr>
<td>Beta coefficient (OLS)</td>
<td>0.82937</td>
<td>0.87639</td>
<td>0.92615</td>
<td>N/A</td>
<td>0.81771</td>
<td>0.9367</td>
<td>0.6432</td>
</tr>
<tr>
<td>R-square</td>
<td>0.95475</td>
<td>0.92658</td>
<td>0.94354</td>
<td>N/A</td>
<td>0.8778</td>
<td>0.9487</td>
<td>0.7135</td>
</tr>
<tr>
<td>Johansen Cointegration</td>
<td>0.02</td>
<td>2.87</td>
<td>0.01</td>
<td>N/A</td>
<td>4.86</td>
<td>2.37</td>
<td>5.52</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample II</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
<th>Nasdaq 100</th>
<th>CAC 40</th>
<th>HSCI</th>
<th>Ibovespa</th>
<th>Kospi 200</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/04/99 to 19/04/06</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
<td>Future</td>
<td>Spot</td>
</tr>
<tr>
<td>Average annual return</td>
<td>-0.45%</td>
<td>-0.48%</td>
<td>0.25%</td>
<td>0.23%</td>
<td>-2.64%</td>
<td>-2.68%</td>
<td>2.82%</td>
</tr>
<tr>
<td>Volatility</td>
<td>18.08%</td>
<td>18.31%</td>
<td>18.05%</td>
<td>18.28%</td>
<td>37.11%</td>
<td>36.53%</td>
<td>22.77%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.2099</td>
<td>-0.1623</td>
<td>0.1049</td>
<td>0.058</td>
<td>0.2735</td>
<td>0.118</td>
<td>-0.0751</td>
</tr>
<tr>
<td>Xs Kurtosis</td>
<td>3.168</td>
<td>3.1499</td>
<td>2.3848</td>
<td>2.5263</td>
<td>3.6612</td>
<td>3.5287</td>
<td>2.9769</td>
</tr>
<tr>
<td>Unconditional Correlation</td>
<td>0.97945</td>
<td>0.9719</td>
<td>0.97116</td>
<td>0.99016</td>
<td>0.9530</td>
<td>0.97254</td>
<td>0.94206</td>
</tr>
<tr>
<td>Beta coefficient (OLS)</td>
<td>0.96763</td>
<td>0.95952</td>
<td>0.98659</td>
<td>0.98252</td>
<td>0.8497</td>
<td>0.91924</td>
<td>0.88349</td>
</tr>
<tr>
<td>R-square</td>
<td>0.95933</td>
<td>0.94458</td>
<td>0.94315</td>
<td>0.98042</td>
<td>0.9082</td>
<td>0.94583</td>
<td>0.88748</td>
</tr>
<tr>
<td>Johansen Cointegration</td>
<td>1.63</td>
<td>2.19</td>
<td>1.36</td>
<td>1.01</td>
<td>1.95</td>
<td>2.10</td>
<td>0.55</td>
</tr>
</tbody>
</table>

* For Nasdaq sample I starts in 15/04/96, for Ibovespa sample I starts in 02/08/1995 and for Kospi 200 samples I starts in 06/05/1996.

The CAC sample I data are omitted since they start only on 08/01/1999.
Table 2: Volatilities, Ederington Effectiveness (E) and Kolmogorov-Smirnoff (KS) Tests

The table reports the unconditional volatility of un-hedged positions on the spot index and the index future. Below this we report the unconditional volatility of the hedged portfolio and the Ederington measure (E) for the naïve hedge and for each of the four MV hedges. This is followed by the average hedge ratio taken over the sample period, and in every case this average is greater for sample II than it is for sample I. For the MV hedges the KS statistic tests whether the distribution of the hedged portfolio returns is significantly different from the distribution of the naïve hedged portfolio returns. The probability values of the KS statistic are reported in bold type. Sample I is before 1st April 1999 (and it excludes the first estimation sample) and sample II is from 1st April 1999 to 19th April 2006.

<table>
<thead>
<tr>
<th>Sample</th>
<th>FTSE 100</th>
<th>S&amp;P 500</th>
<th>Nasdaq 100</th>
<th>CAC 40</th>
<th>HSCI</th>
<th>Ibovespa</th>
<th>Kospi 200</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>I</td>
<td>II</td>
<td>I</td>
<td>II</td>
<td>I</td>
</tr>
<tr>
<td>Spot</td>
<td>14.06%</td>
<td>18.08%</td>
<td>13.21%</td>
<td>18.05%</td>
<td>29.44%</td>
<td>37.11%</td>
<td>N/A</td>
</tr>
<tr>
<td>Futures</td>
<td>16.16%</td>
<td>18.31%</td>
<td>14.57%</td>
<td>18.28%</td>
<td>31.08%</td>
<td>36.53%</td>
<td>N/A</td>
</tr>
<tr>
<td>Naïve</td>
<td>4.90%</td>
<td>3.69%</td>
<td>4.03%</td>
<td>4.31%</td>
<td>7.35%</td>
<td>8.86%</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>87.83%</td>
<td>95.83%</td>
<td>90.69%</td>
<td>94.29%</td>
<td>93.77%</td>
<td>94.30%</td>
<td>N/A</td>
</tr>
<tr>
<td>OLS</td>
<td>4.08%</td>
<td>3.60%</td>
<td>3.57%</td>
<td>4.26%</td>
<td>7.05%</td>
<td>8.91%</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>91.60%</td>
<td>96.04%</td>
<td>92.69%</td>
<td>94.43%</td>
<td>94.26%</td>
<td>94.23%</td>
<td>N/A</td>
</tr>
<tr>
<td>Average Hedge Ratio</td>
<td>0.8200</td>
<td>0.9660</td>
<td>0.8870</td>
<td>0.9630</td>
<td>0.9310</td>
<td>1.0004</td>
<td>N/A</td>
</tr>
<tr>
<td>KS</td>
<td>0.0477</td>
<td>0.9094</td>
<td>0.3192</td>
<td>0.9661</td>
<td>0.7532</td>
<td>0.9266</td>
<td>N/A</td>
</tr>
<tr>
<td>ECM</td>
<td>4.07%</td>
<td>3.61%</td>
<td>3.61%</td>
<td>4.24%</td>
<td>7.11%</td>
<td>8.87%</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>91.62%</td>
<td>96.02%</td>
<td>92.54%</td>
<td>94.49%</td>
<td>94.18%</td>
<td>94.29%</td>
<td>N/A</td>
</tr>
<tr>
<td>Average Hedge Ratio</td>
<td>0.8450</td>
<td>0.9720</td>
<td>0.9210</td>
<td>0.9780</td>
<td>0.9580</td>
<td>0.9930</td>
<td>N/A</td>
</tr>
<tr>
<td>KS</td>
<td>0.0797</td>
<td>0.9418</td>
<td>0.3687</td>
<td>0.9955</td>
<td>0.9764</td>
<td>0.9987</td>
<td>N/A</td>
</tr>
<tr>
<td>EWMA</td>
<td>4.10%</td>
<td>3.65%</td>
<td>3.64%</td>
<td>4.33%</td>
<td>7.10%</td>
<td>8.93%</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>91.49%</td>
<td>95.93%</td>
<td>92.42%</td>
<td>94.25%</td>
<td>94.18%</td>
<td>94.22%</td>
<td>N/A</td>
</tr>
<tr>
<td>Average Hedge Ratio</td>
<td>0.8240</td>
<td>0.9670</td>
<td>0.8890</td>
<td>0.9640</td>
<td>0.9340</td>
<td>1.0010</td>
<td>N/A</td>
</tr>
<tr>
<td>KS</td>
<td>0.0530</td>
<td>0.5804</td>
<td>0.2745</td>
<td>0.9955</td>
<td>0.9875</td>
<td>0.9975</td>
<td>N/A</td>
</tr>
<tr>
<td>GARCH</td>
<td>4.17%</td>
<td>3.64%</td>
<td>3.59%</td>
<td>4.25%</td>
<td>6.99%</td>
<td>9.02%</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>91.22%</td>
<td>95.95%</td>
<td>92.63%</td>
<td>94.46%</td>
<td>94.36%</td>
<td>94.09%</td>
<td>N/A</td>
</tr>
<tr>
<td>Average Hedge Ratio</td>
<td>0.8160</td>
<td>0.9900</td>
<td>0.8940</td>
<td>0.9620</td>
<td>0.9380</td>
<td>1.0010</td>
<td>N/A</td>
</tr>
<tr>
<td>KS</td>
<td>0.0344</td>
<td>0.9418</td>
<td>0.3192</td>
<td>0.9977</td>
<td>0.9601</td>
<td>0.9975</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure 1 – Hedge Ratios, FTSE 100

![Graph showing Hedge Ratios](image1)

Figure 2 – Difference in Conditional Efficiency, FTSE 100

![Graph showing Difference in Conditional Efficiency](image2)
Figure 3 – Difference in Conditional Efficiency, Nasdaq 100

Figure 4 – Difference in Conditional Efficiency, HSCI