On the Aggregation of Market and Credit Risks

ISMA Centre Discussion Papers in Finance 2003-13
1st October 2003

Carol Alexander and Jacques Pézier
ISMA Centre, University of Reading, UK

Paper Abstracted from:
9th Annual Round Table of the International Financial Research Institute
IFCI
March 2003

© IFCI Foundation-International Financial Risk Institute,
Professor Carol Alexander & Professor Jacques Pezier, ISMA Centre, University of Reading
(reprinting or quotations citing source permitted).
Abstract
This paper presents a new approach to aggregating market and credit risks in large complex financial firms, banks in particular. By identifying risk factors that are common to many business activities, dependencies between different risk types across various lines of business can be properly accounted for in the aggregate risk estimate. The risk factor aggregation model is illustrated using historical data on market and credit risk factors that are common to many business units, including interest rates, credit spreads, equity indices and implied volatilities. Economic capital estimates obtained using the model are compared with the economic capital data from several major banks. Applications to optimal risk diversification shows that, whilst the independent control of economic capital by business unit can be sub-optimal, the risk factor aggregation approach has the great advantage of allowing both risks and returns in different business activities to be modeled in the same framework. We show that this greatly facilitates the constrained optimization of a risk/return objective.

JEL Classification: C32, C61, G18, G21
Keywords: Credit Risk; Economic Capital; Market Risk; Risk Aggregation; Risk Diversification; Value-at-Risk; Factor Model; Risk Adjust Return on Capital

Acknowledgements
This is the first paper based on the research that we performed for the 9th Annual Round Table of International Financial Risk Institute (IFCI). We would like to thank the members of the IFCI foundation that made this work possible. Several preliminary discussions, including useful information about economic capital calculations and economic capital data, were obtained from Richard Evans, Michael Brockmann and Mark Laycock of Deutsche Bank, London; Tobias Guldimann and Christian Walter of Credit Swiss Group; and Andrew Threadgold and John Wilmot of JP Morgan Chase. Particularly useful comments on the first draft were given by Richard Evans of Deutsche Bank and Mattia Rattaggi of UBS, and we extend our thanks to Professor Elias Dinensis, the discussant, and to all the participants in the 9th Annual IFCI Roundtable for their excellent contributions to the discussion.

Contacting Authors:
Prof. Carol Alexander
ISMA Centre, University of Reading, Whiteknights, Box 242, Reading, RG6 6BA,
Tel: +44 1189 316134; fax: +44 1188 314741.
E-mail address: c.alexander@ismacentre.rdg.ac.uk

Dr. Jacques Pezier
ISMA Centre, University of Reading, Whiteknights, Box 242, Reading, RG6 6BA,
Tel: +44 1189 316675; fax: +44 1188 314741.
E-mail address: j.pezier@ismacentre.rdg.ac.uk
I Introduction
Capital adequacy and the efficient use of capital resources in banking have not always been primary concerns for risk managers. In the past, in most countries, the banking sector has been heavily regulated and protected from competitive pressures. Indeed banking plays a crucial economic and social role. Especially during the reconstruction period following WWII, governments have used the banking industry as a means of economic control. The collapse of the Bretton Woods agreement in 1972 marked the breakdown of exchange rate controls and was followed by easing of capital transfers and relaxation of credit controls. It signaled the beginning of a new, freer regime for the banking industry, although governments, as lenders of last resort, remain ultimately accountable for crises in banking and will always want to ensure the soundness of the banking system.

Deregulation opened the gates to expansion and greater risks. Western banks started to lend excessively to Latin America whilst Japanese banks rushed to acquire market share in the West. Competition for domestic lending led to a major Savings and Loans crisis in the USA. Banks in London paid over the odds to acquire brokers and jobbers after the deregulation of the Stock Exchange and to develop large investment banking divisions. New risks were acquired through expanded trading in capital markets. OTC derivatives overcame all other markets in notional size. But capitalisation of the banking system did not follow apace with these developments – indeed, as a whole, it decreased during the 70s and the early 80s - and some individual banks, if not national banking industries, became highly vulnerable.

It is against this background that the first Basel Accord (1988) was forged, setting minimum regulatory capital (MRC) requirements as a buffer against credit risk which was then regarded as the overwhelming source of banking risks. The Accord was crude but simple. Reputational pressures led to its rapid acceptance in all major economies and now in more than 100 countries. It forced many banks to raise capital and to become more risk conscious; in this respect, it has been a success.

In the 1988 Accord, credit risk was assessed as the sum of risk weighted assets (RWA), that is, the sum of notional exposures weighted by a coefficient reflecting the credit worthiness of the relevant obligors. The implementation was simplistic to the extreme.¹ Nonetheless, the seed was planted; minimum regulatory capital (MRC) was to be regarded as a key regulatory tool. In the 1996 Amendment to the Accord, market risks were recognized as another major contributor to banking risks and additional capital requirements were introduced correspondingly. More recently, a major revision of the Accord has been undertaken to

---
¹ Credit worthiness was cast roughly into three categories: governments, regulated banks and others. There was little attention given to the type of exposure (instrument, maturity, etc…) or to diversification (although concentration limits are set) or hedging (simple substitution of credit weights for qualifying guarantees, strict qualifications for eligible collateral, etc.); individual RWAs were simply added-up.

Copyright © 2003 IFCI Institute, Carol Alexander and Jacques Pezier
refine and make more risk sensitive the evaluation of capital requirements for credit risks as well as to introduce new capital requirements for operational risks.

But the intention of the Basel committee remains purely prudential: primarily, to protect banks’ depositors and other creditors by ensuring a certain level of financial soundness and, secondarily, to provide a basis for fair competition among banks and to contain systemic risks; it is neither to define an optimal capital ratio nor to provide guidance to bank managers on how to run their business. Naturally, in as much as the capital constraints are or could become biting, banks have a definite interest in refining the MRC calculation method. However, beyond regulations, banks saw in risk assessment and the corresponding linkage to capital, a tool for efficient management.

Indeed, bank managers must also satisfy the wider and complementary interests of shareholders. To the latter, insolvency is just a worst case to consider when balancing firm-wide risks and returns. In this respect, risk managers must look well beyond regulatory requirements to assess risks and returns of various activities and determine the most attractive mix for shareholders. Thus, as far back as the late 1970s, some banks were designing their own, internal risk assessment, aggregation and risk adjusted performance measurement methodologies. Because the goals are broader than those of the regulator and the application specific to a bank, these internal methodologies differ from regulatory rules in many respects. However some key features - and consequently aggregation problems - are common to most internal methodologies as well as to regulatory rules.

In this paper we present a new approach to risk aggregation that is applicable to internal firm-wide risk assessment. Aggregate risk estimates are based on the correlation between risk factors that are common to the different business activities in the firm. Thus our approach has its roots in the arbitrage pricing theory introduced by Ross (1976), where the returns to business units take the place of stock or bond returns. In addition to risk aggregation, applications to optimal risk diversification and optimizing the mix of risks and returns are given.

The outline of the paper is as follows: section 2 describes the need for a new approach to risk aggregation. Section 3 introduces a common risk factor model as the underlying framework for risk aggregation. It separates explainable risks that can be logically aggregated, from residuals risks that should be largely independent. It also focuses on the modeling the tail dependencies between marginal distributions of risk.

---

2 For example, Bankers Trust formally developed the concept of Risk-Adjusted Return on Capital (RAROC™) in the late 1970s and claimed to have been measuring the required risk capital for all activities in the entire bank by 1983.

3 In some respects, such as the use of credit portfolio models, they are much more refined; often they take into consideration subjective views (e.g., on business and reputation risks) that are pertinent but would be difficult to validate by banking supervisors and therefore inappropriate to include in statutory reports.
factors that is key to the aggregation of tail losses under exceptional circumstances, for the purpose of regulatory or economic capital estimation. A specific implementation of the risk factor model is described in section 4. First, historical data are used to estimate the joint and marginal distributions of selected common risk factors, and we relate our findings to recent academic research on the relationship between market and credit risk factors. Comparisons are made (a) of the total risk evaluated by the factor model under different distributional assumptions for the risk factors, and (b) between the total risk from the factor model and the economic capital data from three major banks. Section 5 describes further application of the common risk factor model, to optimal risk diversification and to the optimization of a risk vs return objective when risk factor sensitivities may be constrained to lie within reasonable bounds. We summarize our results and conclude in section 6.

II Application Based Risk Aggregation

We describe a bank as a collection of activities and positions in financial instruments generating a net operating income and, in the case of positions, changes in position values (realized and unrealized). We are interested in the ‘net value change’, that is, the sum of net operating income and net changes in position values over the risk horizon. For simplicity, we shall call this change, which may be positive or negative, profit and loss (P&L). But it is important to note that our definition of P&L is wider than and should be distinguished from the accounting definition which (in the absence of fair valuation) does not take into account unrealized profits or losses except for general and specific provisions.

In this paper we take banking risks to mean uncertainties in P&L, as we have defined it, and stop short of a wider discussion of shareholder value. P&L impacts shareholder value in complex ways because of tax effects, provisioning and distribution policies. Shareholder value would also be affected by the loss of a major client or the hiring of a star manager but, whilst these may be extremely important when taking a strategic view of the business, the aggregation model in this paper is aimed at the medium-term efficient allocation of resources, so we shall ignore these wider issues.

More specifically, we shall translate banking risks into a single metric and call it, as many banks have done, ‘economic capital’ (EC) because capital in such amount should be available to absorb the relevant risk. By extension we also refer to the ‘allocation of economic capital’ to describe a set of risk limits or capital charges imposed on various business units. But economic capital is not real capital; it has nothing to do with, say, funding or distribution policies, it is only a risk metric. As such, economic capital is

---

4 In this context, cost of capital is a shadow cost in an optimization problem. The underlying capital measure is not linked to either a funding need or actual capital allocation. It is a measure of marginal risk contribution to the total risk of the firm.
Banking risks may be assessed using a variety of methodologies. However among internal methods and also with new regulatory proposals that follow best practices there are remarkable similarities. One common feature is that most methods proceed from the ‘bottom up’. That is, risks and returns are first assessed at the most elemental level (e.g., instrument by instrument) and by risk type (separately for credit, market and other risks). Then, individual assessments are progressively aggregated into portfolios of similar instruments or activities, then by business units and, usually only at the very end, across major classes of risks, to obtain a global representation of firm-wide risk.

A second common feature is that risks are assessed, initially, over different time horizons according to liquidity. Since liquid risks tend to evolve rapidly and it would be difficult to represent the dynamics of these risks over the long term, the most common ‘bottom-up’ risk assessment paradigm assumes that current positions will remain static over a certain time period, that we know precisely the value of these positions now and that all we want to assess is the uncertainty about the value of these positions at the end of the stated time period.

A third common feature is that risk assessments are summarized by a single number at an early stage. Probabilities provide a universal language to describe uncertainties, but full probability distributions may be too difficult to communicate. Instead, summary statistics are used. First, the expected value of a return distribution is assessed; it is the figure that is usually reported when discussing returns. Then, a second statistic summarizes potential deviations from the expected value, which can be a standard deviation or the difference between some quantile and the expected value of the distribution. In this paper, we call this second statistic the risk metric. Because the main role of capital in banks is to absorb risks, the terms ‘risk capital’ or ‘economic capital’ are now very widely used to describe the chosen risk metric and to distinguish this internal assessment of capital needs from the externally imposed MRC. We define economic capital (EC) as an internal assessment of the difference between some upper quantile of the P&L distribution and its expected value – a quantity that is also often referred to as the ‘unexpected loss’. These three features make it very difficult to generate a single, global risk assessment. The high-level risk aggregation methodologies currently in use are very primitive and do not account for the purpose of the aggregation exercise. On the regulatory side, simple addition of capital requirements, which, in practice,
amounts to the addition of quantiles from separate distributions, is recommended – or even required, because it is supposedly safe. But it ignores the first rule of risk management, which is to look for diversification and mitigation, and by ignoring such possibilities it encourages bad practices. Thus internally, some banks are already using or are testing more sophisticated aggregation techniques but without any best practice emerging yet. Many risk managers attempt to aggregate risks by ascribing a correlation matrix to their risk metric for different business units and risk types. These correlations are subjective, infrequently updated and not reflective of changes in exposures or policies. Many risk managers also choose for their risk metric a quantile that is compatible with the desired credit rating, even though rating agencies view capitalization as only one of the important determinants of credit quality and experience shows that there is no statistically significant relationship between a bank’s core capital and its credit rating.

But why should banks seek a single, global risk assessment? The choice of descriptive statistics, the level of detail and accuracy required, the relevant time scale, all depends very much on the purpose of the exercise. Is it for risk management to make efficient use of capital and other resources over a relatively short term horizon? Or, at the other extreme, is it to guard against the risk of insolvency over the long term? Or could it be to address one of a number of intermediate problems? In particular, could it be to help risk managers forecast the impact of extreme risks over a medium term, such as one year, in the absence of management intervention; artificial as it may seem, that is indeed a question raised by regulators, and which therefore must be answered.

The aggregation methodology that is developed in this paper is based on a common risk factor model for P&L by business unit. Thus uncertainties about each P&L are related through sensitivities to uncertainties about explanatory factors. They may be market factors, such as an interest rate or equity index, or more general economic factors, such as inflation. Management decisions cannot affect these factors but can alter the exposure of the business to these factors. Line managers ought to know the main sources of uncertainty affecting their P&L but they may be less aware of how the same factors may affect other business units. A degree of independence between the P&L of different business units would be expected, but if some correlation is apparent (either positive or negative), it points to the possibility of one or more common factors.

---

6 For example, AA rated banks have historical default probabilities of about 0.03% per year, therefore banks aiming at a AA rating will choose a 0.03% quantile on one year P&L for the risk metric.

7 These two purposes require very different approaches, to risk assessment and aggregation, to the choice of time horizon and even to the recognition of important risks. For the efficient management of a bank the main risks are short-term market and credit risks. But for solvency considerations the important risks include business, reputational and systemic risks, over the medium to long-term.
These common factors provide a logical answer to the aggregation of risks. If common factors are found that explain most of the P&L uncertainty (by business unit, and consequently in aggregate), the analysis will have been very useful. If they explain only a small fraction of total P&L uncertainty, the residuals will have to be examined to see whether they exhibit some correlation. They would either (i) make the case for independence of the residuals or (ii) point to the existence of more common factors that should be worth investigating and including in the analysis.

Most interesting for top management are the risk factors that affect several business units and to which exposures can be altered. Aggregation of the risk metric across business units and risk types is determined by sensitivities to these common risk factors, and the risk factor dependencies. Knowledge of the marginal and joint distributions of the risk factors is thus an important determinant of the aggregate risk estimated by the model. In a properly specified factor model the unexplained P&L will be largely independent across business units, so the residual contribution to the aggregate risk may be calculated via simple rules.

Risk factor models are not new in asset management. Following Ross (1976) the literature on developments of the arbitrage pricing theory is huge. Research includes: the identification of factors affecting stock returns (Beckers, Conner and Curds, 1996; Connors and Korajczyk, 1993; Roll and Ross, 1980; Solnik, 1983; and many others); the analysis of investment risks (Chan, Karceski, and Lakonishok, 1998; Sweeney and Warga, 1986; and many others); the analysis of different investment styles (Connor and Korajczyk, 1991; Roll and Ross, 1984; Sharpe, 1992; and many others); the effect of distributional assumptions for the risk factors (following Elton and Gruber, 1992); and the identification of common factors affecting returns on stocks and bonds (following Fama and French, 1993). However, to the authors’ knowledge, this paper is the first application of a common risk factor model to the analysis of risks and returns of the different business activities in a bank.

III The Risk Factor Approach to Aggregation

In this section the model is first defined for a ‘normal case’ that is able to provide a framework for efficient risk management. Given a certain amount of capital, management should try to increase profits and reduce the uncertainty of P&L. Then the key purpose of the risk assessment model would be to describe the aggregate P&L uncertainty over the short to medium term for a going concern. In this case, extreme rare events are not of much concern, and neither are the uncertainties about the current values (fair prices) of positions, if the uncertainties remain similar at the beginning and the end of the reference period; it is uncertainties about changes in value that matter. Positive deviations from the expected P&L are of equal importance to negative deviations of the same size, and the P&L contribution over the chosen time horizon is not expected to be a large fraction of current capital (say –20% to +20% at most over a
Thus the ‘normal case’ simplifies the treatment of probability distributions describing various sources of P&L uncertainty under the following assumptions:

- **Common time scale** – results from all activities should be described over a common, relatively short, time period.\(^8\)
- **Simple statistics** – multidimensional probability distributions describing sources of P&L uncertainty will be summarized by a few statistics: expected values, standard deviations, and linear correlation coefficients.

However, the normal case is not designed to produce extreme quantiles of the P&L distribution over longer periods with accuracy. Nevertheless, these extreme quantiles are required for a firm-wide risk assessment based on economic capital. For this purpose the following assumptions are necessary:

- **Longer time scale** – although it may not be realistic to assess market and credit risks over the same time horizon, and under a ‘business as usual’ policy, this is not a decision theoretic model that takes into account management reactions; therefore we assume the same (one year) time horizon for all risks.
- **Tail statistics** – the assessment of extreme P&L variations requires attention to the tail behavior of risk factors, the tail dependencies among risk factors, and non-linear effects of risk factors on P&L.

**III.1 Model Outline**

**Notation:**

\(n\) = number of business units, indexed by \(i = 1, 2, \ldots n\).

\(P_i\) = P&L generated by business unit \(i\) over relevant time horizon

\(V = n \times n\) matrix of variances and covariances between \(P_i\) and \(P_j\)

\(x = m \times 1\) vector of risk factor changes over relevant time horizon

\(V_x = m \times m\) risk factor covariance matrix

\(\beta_i = m \times 1\) vector of risk factor sensitivities for business unit \(i\)

\(B = m \times n\) matrix of risk factor sensitivities with \(j^{th}\) column equal to \(\beta_j\)

\(e_i\) = residual P&L of business unit \(i\) that is not explained by the risk factors

\(\alpha_i\) = expected P&L that is not explained by the risk factors

---

\(^8\) The choice of the period may vary from institution to institution depending on the dynamics of the business. For example, banks mostly involved in market activities such as trading and sales or brokerage services may want to choose a period as short as a few weeks, whereas institutions involved in retail and commercial banking may choose a few months. When a particular risk is assessed over a different period, it will have to be appropriately scaled up or down to the correct horizon.
\( \mathbf{v} = (v_{ij}) = n \times n \) residual covariance matrix

The choice of common factors should be firm specific. The betas are nominal risk factor sensitivities representing the gain or loss in P&L per unit change in the relevant risk factors. P&L sensitivities of each business unit to the risk factors should be determined in the same way as exposures that are routinely used in the estimation of MRC or EC. The risk factor covariance matrix, which is verifiable in the same way as the covariance matrices used in Value-at-Risk calculations, has a time horizon equal to the risk horizon of the model. Typically it will be a period during which business can be considered as running in a normal steady state. In this case, for each of the \( n \) business units, a linear factor model is prescribed as follows:

\[
P_i = \alpha_i + \beta_i' \mathbf{x} + \epsilon_i \quad [i = 1, 2, \ldots, n]
\]

Total P&L is then

\[
P = P_1 + \ldots + P_n
\]

The variance of \( P \) is now described in terms of the factor model, assuming each \( \epsilon_i \) is independent of \( \mathbf{x} \):

\[
Var(P_i) = \beta_i' \mathbf{V_x} \beta_i + v_{ii} \quad (3)
\]

\[
Cov(P_i, P_j) = \beta_i' \mathbf{V_x} \beta_j + v_{ij} \quad (4)
\]

In matrix form the above two equations may be written together as:

\[
\mathbf{V} = \mathbf{B}' \mathbf{V_x} \mathbf{B} + \mathbf{v} \quad (5)
\]

Now \( Var(P) \), which is the sum of all the elements of \( \mathbf{V} \), can be written as:

\[
Var(P) = \beta' \mathbf{V_x} \beta + u' \mathbf{v} \quad u
\]

where \( \beta \) is the \( m \times 1 \) vector with terms equal to the sum of the terms in the corresponding rows of matrix \( \mathbf{B} \), that is the total sensitivities to the risk factors, and \( u \) is a \( n \times 1 \) unit vector. Thus, if we choose standard deviation as the metric to express risk we obtain the total risk accounted for by the factor model, by setting all \( v_{ij} \) to zero, as:

i. Total risk over all business units and risk types is the square root of the sum of all the elements in \( \mathbf{V} \);

ii. Total risk generated by a subset of risk factors aggregated over all business activities is the square root of the sum of all the elements in \( \mathbf{V} \), where the betas with respect to risk factors not in the subset are set to zero;
iii. Total risk within the $i$th business activity over all risk types is the square root of the $i$th diagonal element of $V$.

Note that the above decomposition of risks depends on the independence of the residual P&L and the included risk factors. This will not be the case if important risk factors have been omitted, and these omitted factors are correlated with the included risk factors. Also, the total risk accounted for by the factor model does not include the specific risks in each business unit. And if, in addition, the risk factors are assumed to be normally distributed with their dependencies summarized by linear correlations, on translating from the standard deviation to a percentile metric (see section IV.2), the total risk is likely to be substantially smaller than the aggregate market and credit economic capital figures. Exceptionally, they might be larger because one risk factor might affect several risk types other than market and credit risk. For example, an equity price variation might affect all major risk types: market risk obviously, but also credit spread risks and consequently default risk, business risk (e.g., decline in brokerage activity) and operational risk (e.g., increased problems in collateral management).

**III.2 Assumptions for the Estimation of Extreme Risks**

The above definition of the risk factor model may be used to assist bank managers in evaluating the degree of risk diversification in their activities under normal business circumstances, providing a basis for risk adjusted performance measurements and more efficient planning and budgeting. However, many risk managers are also asked to provide a description of extreme risks. Indeed, internal EC estimates in many banks are based on a confidence level of 99.95% or better per year, that is on a probability of losing more than the EC figure over a year of less than 0.05%. As already remarked, these high levels of confidence are deemed compatible with the stated objective of some banks to maintain their credit rating at a certain level such as ‘AA’.

The risk factor model estimates of such extreme risks will be sensitive to the form of the P&L profile as a function of the risk factors. It may be inappropriate to assume that P&L is linearly related to the risk factors over wide ranges, as in (1). In the most general form we have instead a set of non-linear factor models for each of the $n$ business units:

$$P_i = f_i(x) + \epsilon_i \quad [i = 1, 2, \ldots, n]$$ (8)

and the equivalent model for the total P&L given by (2) is:

$$P = f(x) + \epsilon \quad [i = 1, 2, \ldots, n]$$ (9)
However, unless P&L is highly irregular, extreme variations in total P&L will be associated with extreme
variations in at least one of the risk factors. Only when a net risk factor sensitivity is very small would an
extreme variation in that risk factor not induce an extreme P&L variation. In this case a second order
approximation to (8) is given by:

\[ P_i = \alpha_i + \beta_i' \gamma_i \gamma + \epsilon_i \quad [i = 1, 2, \ldots, n] \] (10)

Similarly for the total P&L (9) we have the approximation

\[ P = \alpha + \beta' \gamma + \epsilon \quad [i = 1, 2, \ldots, n] \] (11)

where \( \alpha \) is the sum of the intercept terms in (9) and \( \beta \) is the \((m \times 1)\) ‘net beta’ vector with terms equal to
the sum of the terms in the corresponding rows of matrix \( B \) and \( \gamma \) is an \((m \times m)\) matrix equal to the sum
of matrices \( \gamma_i \). Note that in this case

\[ E[P_i] = \alpha_i + \text{sum}(1/2 \gamma_i \otimes V_x) \quad [i = 1, 2, \ldots, n] \] (12)

where \( \otimes \) stands for the matrix where each element is the product of the corresponding elements in the two
matrices \( \gamma_i \) and \( V_x \) and \( \text{sum(.)} \) denotes the sum of all the elements in that matrix.

Thus equations (5) and (6) still hold. However, because the P&L function is non-linear, the translation of
the risk factor model covariance estimates to estimates of extreme quantiles of each P&L will depend on
the functional form of the P&L distribution. We must consider appropriate marginal distributions for the
risk factors and take care to specify the type of dependency that is important for the extreme movements
in P&L. Certainly it would be dangerous to assume that the joint distribution of risk factors is multivariate
normal with covariance matrix estimated as an equally weighted average of all covariances over a long
historical period. More appropriate would be to admit non-normal marginal distributions for each risk
factor, and to employ a risk factor covariance matrix in (5) that relates only to their extreme covariances.
In addition, as we shall see in section IV.2, the scaling factor that translates the standard deviation metric
of the risk factor model to the percentile metric for economic capital also depends on the distributional
assumptions.

IV Model Implementation

IV.1 Definition and Properties of Risk Factors

It is up to each organization to find out which are the most useful and convenient explanatory factors
depending on its mix of activities. Table 1 defines the six common risk factors we have selected for our
illustration. Other candidate factors could be, for example, trade weighted currency indices, real estate
indices and commodity indices. Factors should be defined for each major currency or currency group. All
these factors relate here to the US market. Similar factors for currencies other than the US dollar and for
key exchange rate could be introduced as applicable. We take the 1-year interest rate as a useful common
factor for parallel shifts in the yield curve, assuming that the average maturity of interest rate exposures in
our sample bank is close to one year. To capture the changes in value of long interest rate positions, the
difference between the 10-year rate and the 1-year rate is taken as a second risk factor indexing tilts in the
yield curve. The interest rate volatility is also included to capture changes in value of interest rate options
(a 3 month volatility on the 1 year interest rate has been used). For equity positions a market index, and
the index volatility at either 3 months (assuming the average maturity of option portfolios is around 3
months) or 1 month (for the VIX index) are used. To reflect credit risks we take the credit spread of a 10-
year Baa bond index over matching treasuries as an indicator.

Table 1: Definition of Risk Factors

<table>
<thead>
<tr>
<th>Main Risk</th>
<th>Factor</th>
<th>Variable</th>
<th>Notation</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rates</td>
<td>Parallel shift</td>
<td>1 yr tsy rate</td>
<td>‘tsy’ or ‘r’</td>
<td>1 bp</td>
</tr>
<tr>
<td></td>
<td>Slope</td>
<td>10yr tsy – 1yr tsy</td>
<td>‘slope’ or ‘s’</td>
<td>1 bp</td>
</tr>
<tr>
<td></td>
<td>Volatility</td>
<td>Implied volatility index</td>
<td>‘move’ or ‘σ,’</td>
<td>1% abs.</td>
</tr>
<tr>
<td>Equities</td>
<td>Overall level</td>
<td>Market index</td>
<td>‘spx’ or ‘e’</td>
<td>1% rel.</td>
</tr>
<tr>
<td></td>
<td>Volatility</td>
<td>Implied volatility index</td>
<td>‘vix’ or ‘σ_e’</td>
<td>1% abs.</td>
</tr>
<tr>
<td>Spread</td>
<td>Credit Spread</td>
<td>10yr credit spread</td>
<td>‘spread’ or ‘c’</td>
<td>1 bp</td>
</tr>
</tbody>
</table>

Daily historical data on these risk factors, covering the period 4th January 1999 to 31st December 2002 (a
total of 1000 observations) we obtained as follows: *Interest Rates and Implied Volatility*. Data were
obtained from www.federalreserve.gov/releases/h15/data.htm, for Treasury 1 year and 10 year closing
mid value interest rates. The implied volatility index is the “Merrill option volatility estimate” (MOVE)
obtained from 30-day ATM options on 2, 5 10 and 30 year US treasury bonds, weighted by the volume
traded.9 *Equity Index and Implied Volatility*: Daily closing prices on the S&P 500 equity index was
obtained from www.yahoo.com. The equity implied volatility was the VIX CBOE volatility index,
available from www.cboe.com is calculated by taking a weighted average of the implied volatilities of
eight OEX calls and puts. The chosen options have an average time to maturity of 30 days. *Credit Spread:*
The Goldman Sachs par asset 10 year US BBB swap spread, which is based on constant maturity bonds.10

---

9 Many thanks to Harley Bassman of Merrill Lynch, New York for kindly providing these data.
10 Many thanks to Francesco Garzarelli, Senior Economist at Goldman Sachs in London, for kindly providing these data.
Table 2 states the sample moments of the marginal distributions of the risk factor changes based on the daily data described above. The standard deviations are expressed in basis points for the interest rate \( r \), the interest rate slope \( s \), and the credit spread \( c \), in percentage points for the equity index \( e \) and the equity volatility \( \sigma_e \). There is, as expected, strong evidence that risk factor changes are not normally distributed. Approximate standard errors of the skewness and excess kurtosis estimates are 0.19 and 0.76 respectively, so all risk factors (except the equity implied volatility factor) have much heavier tails than normal variables. There is less evidence of skewness in the risk factor distributions: some positive skew in the treasury curve slope and the ‘move’ implied volatility, and a significant negative skew in the 1 year treasury rate.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Risk Factor</th>
<th>tsy</th>
<th>slope</th>
<th>move</th>
<th>spx</th>
<th>vix</th>
<th>credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td>-0.3267</td>
<td>0.2405</td>
<td>0.0057</td>
<td>-0.0806</td>
<td>0.0063</td>
<td>0.1743</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td></td>
<td>5.1446</td>
<td>4.5212</td>
<td>4.4334</td>
<td>3.7013</td>
<td>1.7868</td>
<td>3.6011</td>
</tr>
<tr>
<td>Skewness</td>
<td></td>
<td>-1.1031</td>
<td>0.6521</td>
<td>0.9545</td>
<td>0.1714</td>
<td>0.2882</td>
<td>-0.3627</td>
</tr>
<tr>
<td>XS Kurtosis</td>
<td></td>
<td>10.1488</td>
<td>5.1759</td>
<td>6.7786</td>
<td>1.1805</td>
<td>2.6830</td>
<td>11.1527</td>
</tr>
</tbody>
</table>

In view of these findings, each risk factor \( i \) is assumed to have a density of the form:

\[
\eta_i(x_i) = w_i \phi(x_i | \mu_i, \sigma_i^2) + (1 - w_i) \phi(x_i | \mu_i, \sigma_i^2)
\]

where \( \phi(x_i | \mu, \sigma^2) \) denotes the normal density function with mean \( \mu \) and variance \( \sigma^2 \). That is, we assume a mixture of two normal distributions to describe each risk factor distribution. Since both densities in the above have the same mean, the normal mixture density has positive excess kurtosis; thus by fitting such a density to historical data on risk factor changes, we match the leptokurtosis but ignore the skewness.

Table 3: Estimated Parameters for Normal and Normal Mixture Distributions

<table>
<thead>
<tr>
<th>(a) Model Parameters</th>
<th>Normal</th>
<th>Normal Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma )</td>
<td>( w )</td>
</tr>
<tr>
<td>tsy</td>
<td>5.14</td>
<td>0.111</td>
</tr>
<tr>
<td>slope</td>
<td>4.52</td>
<td>0.076</td>
</tr>
<tr>
<td>move</td>
<td>4.43</td>
<td>0.277</td>
</tr>
<tr>
<td>spx</td>
<td>1.40</td>
<td>0.120</td>
</tr>
<tr>
<td>vix</td>
<td>1.79</td>
<td>0.100</td>
</tr>
<tr>
<td>credit spread</td>
<td>3.60</td>
<td>0.079</td>
</tr>
</tbody>
</table>
Table 3 shows the results of fitting mixtures of two normal densities to the historical data on risk factor changes. The four parameters \{w, \mu, \sigma_i^2 \text{ and } \sigma_i^2\} were optimized by matching the mean, variance, kurtosis and lower 1%-ile of the empirical density with the same parameters of the theoretical density. In the table, the mixing weight \(w\) refers to the high volatility component, \(\sigma_1\).

Although the variance mixture of two normal densities representation is unlikely to provide the best possible fit to historical data, it has the advantage of an intuitive interpretation. Two components are used in the mixture model to provide more flexibility to model a population that is not adequately captured by a single normal distribution. In this case the density with the higher variance is associated with the observations in the tail determined by the weight \(w_1\). For example, if \(w_1 = 10\%\) the observations in the 5% tails of the density determine the high volatility component (McLachlan and Peel, 2000).

The relationships between risk factors may be significantly different in the tails than for smaller variations. We investigate this by evaluating pair-wise linear correlations after separating from our data sample a ‘core’ of observations consisting of all data points where each variable is observed in its middle \((1 - w_i)\%\) bracket. The pair-wise tail correlations are based on observations where either one or both of the two factors fall into the \((w_i/2)\%\) tails of their histograms. Equally weighted correlation is estimated over the whole sample period, using (a) all data; (b) only data from the upper and lower tails of the empirical distributions; and (c) only data from the inner core of the distributions. 11

To be more precise, for case (b) we have excluded all data points that lie within the rectangular area \{X_L < X < X_U \text{ and } Y_L < Y < Y_U\} where \(X\) and \(Y\) are daily changes (relative for the equity index and absolute for the other variables) and the subscripts “L” and “U” refer to the lower and upper \((1 - w_i/2)\%\)-iles of the empirical density. The remaining points are used to estimate the tail correlations. For case (c) the points \{X_L < X < X_U \text{ and } Y_L < Y < Y_U\} are the only points used to estimate correlations. In this way we gain some idea of the overall correlation and how this is related to the points that lie in the tails of the distributions.

Table 4 shows the sample estimates of these correlations. Only a few ‘overall’ correlations between risk factors are significant. In particular negative correlations are evident between equity index and index implied volatility, and between the interest rate, credit spread and slope. The correlations are far stronger in the tails of the joint distribution: in part (b) of table 4, most of the correlations are significant at the 1% or 5% level. Correlations are weaker in case (c) which examines the core of the joint distribution: the only

11 We have also examined core and tail correlations for a 90% inner core (and 5% tails) for all risk factors. The empirical results are very similar: only tail correlations are significant, and their estimates only differ from those in table 4(b) by less than 0.01.
highly significant relationships found are a negative correlation between interest rates and credit spread, and a negative correlation between equity index and index implied volatility.\textsuperscript{12}

**Table 4: Risk Factor Correlations**

**a) Overall Correlation**

<table>
<thead>
<tr>
<th></th>
<th>tsy</th>
<th>slope</th>
<th>move</th>
<th>spx</th>
<th>vix</th>
<th>spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>tsy</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>-0.162</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>move</td>
<td>-0.025</td>
<td>0.039</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>spx</td>
<td>0.190</td>
<td>0.107</td>
<td>-0.028</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vix</td>
<td>-0.162</td>
<td>-0.102</td>
<td>-0.010</td>
<td>-0.834</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>spread</td>
<td>-0.395</td>
<td>-0.225</td>
<td>0.014</td>
<td>-0.141</td>
<td>0.108</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**b) Tail Correlation**

<table>
<thead>
<tr>
<th></th>
<th>tsy</th>
<th>slope</th>
<th>move</th>
<th>spx</th>
<th>vix</th>
<th>spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>tsy</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>-0.260</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>move</td>
<td>-0.092</td>
<td>0.191</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>spx</td>
<td>0.254</td>
<td>0.179</td>
<td>-0.158</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vix</td>
<td>-0.214</td>
<td>-0.175</td>
<td>0.229</td>
<td>-0.906</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>spread</td>
<td>-0.402</td>
<td>-0.258</td>
<td>0.057</td>
<td>-0.211</td>
<td>0.139</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**c) Core Correlation**

<table>
<thead>
<tr>
<th></th>
<th>tsy</th>
<th>slope</th>
<th>move</th>
<th>spx</th>
<th>vix</th>
<th>spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>tsy</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>-0.007</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>move</td>
<td>-0.047</td>
<td>0.114</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>spx</td>
<td>0.097</td>
<td>0.011</td>
<td>-0.188</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vix</td>
<td>-0.077</td>
<td>0.015</td>
<td>0.139</td>
<td>-0.734</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>spread</td>
<td>-0.376</td>
<td>-0.180</td>
<td>-0.029</td>
<td>-0.052</td>
<td>0.053</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Bold figures denote statistical significance at 1%; grey figures indicate significance at 5%.

\textsuperscript{12} Note that the multivariate normal mixture density based on the marginal densities in table 3 may be represented as a weighted sum of multivariate normal densities, with weights given by products of the weights in each marginal normal mixture density. The multivariate normal with the greatest weight has the ‘core’ covariance matrix given by $DCD$, where $D$ is the diagonal matrix of the six low volatilities and $C$ is the ‘core’ correlation matrix. The ‘tail’ correlation matrix as we have defined this relates to all the remaining eleven terms.
Interest rates, credit spreads and equity values are factors that are fundamental to the contingent claim analysis of equity and debt. Examples include Merton (1974), Longstaff and Schwartz (1995), Madan and Unal (1999) and Duffie and Singleton (1999). Consequently, much research has been focused on the relationship between these factors. Blume, Keim, and Patel (1991) and Cornell and Green (1991) find that low-grade bonds returns are more sensitive to equity returns than high-grade bond returns. Even at the aggregate level, Fama and French (1993) and Campbell and Ammer (1993) and others have found a positive relationship between market indices of bonds and equities. Our results in table 4 indicate that treasury rates have a significant positive correlation with the equity index returns, but only in the tails of the distribution.

More recently, with research focusing on the relationship between credit spreads, rather than corporate bond returns, and interest rates, the relationships again appear stronger for lower credit quality. A negative correlation between credit spreads and short term interest rates, which increases as credit quality deteriorates, has been found by many authors, including Longstaff and Schwartz (1995), Duffee (1998), Collin-Dufresne, Goldstein, and Martin (2001) and Kiesal, Perraudin and Taylor (2002).13 These results concur with our finding in table 4 of significant negative correlation between the BBB credit spread and the 1 year treasury rate in the US.

As expected, table 4 shows a very strong negative correlation between the equity index and the equity implied volatility, due to the leverage effect where equity volatility increases more for price falls than for price increases of the same size. However there is little evidence of any significant correlation for the 1 year treasury rate, or the treasury slope, with the treasury bond implied volatility index. Finally, there is a strong positive tail correlation between the equity and interest rate implied volatilities, probably a result of rises in interest rates during equity market falls.

The yield curve slope factor encompasses two effects, liquidity and expected future short rates, both of which should have a negative relationship with the credit spread. Table 4 indicates a negative correlation between the credit spread and the slope of the treasury curve. This can occur, for example, if a decrease (increase) in the slope arising from a fall (rise) in 10 year treasury rates relative to the 1 year rate is not immediately matched by a commensurate fall (rise) in 10 year corporate bonds. Huang and Nefciti (2003) also find a negative relationship between swap spreads and the yield curve slope of the treasury curve. However, other empirical results are mixed: Duffie and Singleton (1997) find a weakly positive liquidity

---

13 Duffee (1998) attributes part of this negative correlation to call features embedded on corporate bonds, although he still finds a weaker negative correlation between interest rates and credit spreads at the monthly maturity on straight bonds.
effect for swap spreads; Collin-Dufresne, Goldstein, and Martin, (2001) find no significant link between slope and credit spreads in any rating or maturity.

Contingent claim models predict a negative relationship between credit spread changes and equity returns, and this is confirmed by the results of Collin-Dufresne, Goldstein, and Martin (2001). However, they find little evidence of a relationship with equity volatility (a positive relationship is predicted by structural models). In table 4 we find only a weak negative ‘tail’ correlation between equity returns and changes in the credit spread and, whilst the correlation with equity volatility is positive, it is not significant.

IV.2 Translation of Risk Metrics
In section IV.5 we shall compare the results of an implementation of the risk factor model with some real EC data from major banks given in section IV.3. The EC data are defined by extreme quantiles of the P&L distribution, but the risk metric for the factor model is standard deviation. Therefore, for each risk type \( j \), a ‘scaling factor’ \( f_j \) is defined so that, given the economic capital estimate \( C_{ij} \) for business unit \( i \) and risk type \( j \), the corresponding standard deviation is \( C_{ij} / f_j \). That is, the scaling factor is the ratio of the ‘unexpected loss’ to the standard deviation of the P&L distribution, where ‘unexpected loss’ is the EC capital estimate defined as the difference between the upper percentile and the expected P&L.

The calculation of the scaling factor should be straightforward for banks, since it is common practice to assess the standard deviation of the aggregate P&L per risk type in the process of generating EC estimates within each business unit. It increases as the quantile becomes more extreme (for example, it is approximately 50% larger for the 0.03% quantile than it is for the 1% quantile) and it also increases with the skewness and leptokurtosis in the P&L distribution, as shown in table 5.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Normal Mixture (Tail Correlations)</th>
<th>Normal Mixture (Zero Correlations)</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1%</td>
<td>4.46</td>
<td>3.86</td>
<td>3.09</td>
</tr>
<tr>
<td>0.05%</td>
<td>4.91</td>
<td>4.22</td>
<td>3.29</td>
</tr>
<tr>
<td>0.03%</td>
<td>5.24</td>
<td>4.47</td>
<td>3.43</td>
</tr>
</tbody>
</table>

The first row of table 5 uses the Cholesky decomposition of the tail correlation matrix in table 4(b) to obtain correlated standard normal variables, which are then transformed into normal mixture simulations with the appropriate volatilities from table 3. Each set of correlated realizations is weighted, using the net betas for each risk factor, and summed to obtain a realization for the total P&L. The middle row replaces
the empirical tail correlations with zero correlations and the last row uses a normal distribution for each risk factor with the same volatility. Each total P&L distribution is based on one set of 1.8 million P&L simulations. The standard error of the scaling factors varies from 0.04 for the 0.1% scaling factor to 0.07 for the 0.03% scaling factor. The scaling factors for normal risk factors in the last row of the table were obtained from standard normal tables.

The standard deviation of P&L depends on the assumptions made about risk factor correlations: it is 4.445 when the empirical tail correlations are used in the simulations and 6.451 (45% larger) when a zero correlation between risk factors is assumed. However, table 5 shows that de-correlation has the opposite effect on the scaling factor. Multiplying the scaling factor by the standard deviation of P&L gives the total EC estimate at the chosen percentile, and with the assumed (zero or tail) correlations. Thus, for the 0.03%-ile the EC estimate is 28.84 with zero correlation between risk factors, but only 23.20 when the empirical tail correlations are used. This is when risk factors have normal mixture distributions. If the risk factors were normal, the corresponding EC estimates would be 22.19 (zero correlation) and 15.29 (tail correlation).

Most banks use similar scaling factors but perhaps without strong justification and with a degree of uneasiness. The method we suggest here should provide a more systematic and less uncertain basis for the evaluation of extreme quantiles for EC estimation. It yields the entire distribution for P&L by business unit (and in total, as illustrated by the example above) under different assumptions about risk factor distributions and different assumptions about their dependencies.

IV.3 Economic Capital Data

For the purposes of this study we have obtained sample data on economic capital from three major banks, each corresponding to the lowest 0.03% quantile of the P&L distribution over one year. To preserve anonymity of the detailed data, we have taken averages of these, to construct the economic capital for a fictitious ‘sample’ bank and these economic capital estimates are given – as a percentage of total economic capital – in Table 6. The figures for our ‘sample’ bank could refer to a universal bank with an emphasis on commercial banking, with some corporate finance, retail banking and asset management (which includes private equity business).

The data in table 6 refer only to market and credit risks: ‘other’ risks, including operational and business risks are not included. Total EC is calculated in table 6 as the sum of each EC estimate. The total EC

---

14 The estimated standard errors were obtained by dividing the 1.8 million simulations into many ‘blocks’ of 30,000 simulations and taking the standard error of the mean in each block. Computation time should not be a problem: it is around 20 seconds on a standard desktop PC without using any special acceleration technique.
under a zero correlation assumption is also reported, where the total EC is the square root of the sum of
the squares of the EC estimates. With zero correlation, the reduction in total risk is significant as
expected. But it is not so much the new total of 37.22 that is significant, it is more that a few single risks
would appear to be dominant - the percentage contribution of each risk to the total is in proportion to its
square. Thus in particular, credit risks in commercial banking appear relatively more important under the
zero correlation assumption.

Table 6: Economic Capital for the Sample Bank

<table>
<thead>
<tr>
<th>Economic Capital</th>
<th>Market</th>
<th>Credit</th>
<th>Total EC</th>
<th>Total (Zero Correlation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>8.21</td>
<td>5.47</td>
<td>13.68</td>
<td>9.87</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>15.17</td>
<td>8.32</td>
<td>23.49</td>
<td>17.30</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>3.42</td>
<td>11.74</td>
<td>15.17</td>
<td>12.23</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>2.39</td>
<td>27.48</td>
<td>29.87</td>
<td>27.58</td>
</tr>
<tr>
<td>Payment &amp; Settlement</td>
<td>0.34</td>
<td>1.82</td>
<td>2.17</td>
<td>1.86</td>
</tr>
<tr>
<td>Agency &amp; Custody</td>
<td>0.80</td>
<td>1.48</td>
<td>2.28</td>
<td>1.68</td>
</tr>
<tr>
<td>Asset Management</td>
<td>7.41</td>
<td>3.88</td>
<td>11.29</td>
<td>8.36</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>1.03</td>
<td>1.03</td>
<td>2.05</td>
<td>1.45</td>
</tr>
<tr>
<td><strong>Total EC</strong></td>
<td><strong>38.77</strong></td>
<td><strong>61.23</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Total (Zero Correlation)</strong></td>
<td><strong>19.28</strong></td>
<td><strong>31.84</strong></td>
<td></td>
<td><strong>37.22</strong></td>
</tr>
</tbody>
</table>

IV.4 Estimation of Risk Factor Sensitivities

Regression based estimates of the model betas in equation (1) should be obtained using historical data on
$P_i$ and $\mathbf{x}$, based on estimation techniques that account for the correlations between risk factors.
Regressions will also provide estimates of the residuals, which are important for completing the
specification of the risk factor model with equation (6). In many cases, however, sensitivities may also be
estimated using known pricing models, for example sensitivities of bond prices to parallel shifts and tilts
of the yield curve.

For our ‘sample’ bank the assumed risk factor sensitivities by business unit, and summed over all activities,
are shown in Table 7. Thus in corporate finance, participating in new issues for bonds, equities, options
on these and various hybrid instruments such as convertible bonds, the exposures to bond prices, equities
and volatilities are all positive. Being purely corporate bonds, the credit spread and interest rate
sensitivities are approximately equal. Trading and sales activities are relatively neutral in the sample
bank, seeking to balance books with delta hedging. Positions are in government as well as corporate
bonds (hence the interest rate sensitivity is greater than the sensitivity to the credit spread) and volatility sensitivities are positive because of the dominance of short options positions. As market makers in equities, they often prefer to trade from a long equity position. For reasons of space, interpretation of other risk factor sensitivities is left to the reader.

Table 7: Risk Factor Betas

<table>
<thead>
<tr>
<th></th>
<th>tsy</th>
<th>slope</th>
<th>move</th>
<th>spx</th>
<th>vix</th>
<th>spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>-0.0086</td>
<td>-0.0024</td>
<td>0.0050</td>
<td>0.0235</td>
<td>0.0123</td>
<td>-0.0122</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>-0.0073</td>
<td>-0.0017</td>
<td>-0.0128</td>
<td>0.0135</td>
<td>-0.0316</td>
<td>-0.0052</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>-0.0095</td>
<td>-0.0011</td>
<td>-0.0027</td>
<td>0.0087</td>
<td>-0.0068</td>
<td>-0.0101</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>-0.0233</td>
<td>-0.0040</td>
<td>-0.0054</td>
<td>0.0171</td>
<td>-0.0268</td>
<td>-0.0333</td>
</tr>
<tr>
<td>Payment &amp; Settlement</td>
<td>0</td>
<td>0</td>
<td>0.0004</td>
<td>0.0012</td>
<td>0.0010</td>
<td>-0.0010</td>
</tr>
<tr>
<td>Agency &amp; Custody</td>
<td>0</td>
<td>0</td>
<td>0.0004</td>
<td>0.0013</td>
<td>0.0010</td>
<td>-0.0010</td>
</tr>
<tr>
<td>Asset Management</td>
<td>-0.0053</td>
<td>-0.0010</td>
<td>-0.0020</td>
<td>0.0324</td>
<td>0.0101</td>
<td>-0.0050</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>0</td>
<td>0</td>
<td>0.0015</td>
<td>0.0059</td>
<td>0.0037</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Global Beta</td>
<td>-0.0540</td>
<td>-0.0102</td>
<td>-0.0157</td>
<td>0.1037</td>
<td>-0.0371</td>
<td>-0.0689</td>
</tr>
</tbody>
</table>

IV.5 Application of the Factor Model to EC Estimation

A proper use of the risk factor model will take into account the best estimates of risk factor distributions that can be obtained from historical data. We have used the normal mixture risk factor distributions estimated in table 3 and correlations based only on the tails of the risk factor distributions as in table 4(b).

For comparison with table 6, the 0.03% quantile of the P&L for each business activity is obtained by translating the P&L standard deviation estimated by the factor model using the scaling factor 5.24 calculated in table 5.15

Table 8 reports economic capital estimates obtained from the factor model, disaggregated according to three factor groups: the interest rate risk factors (i.e. the 1 year interest rate, the slope of the yield curve and interest rate implied volatility), the equity risk factors (i.e. the equity index and the index implied volatility), and the single credit risk factor (the credit spread). The total EC explained by the factor model is 27.42, a little more than one quarter of the total EC for the sample bank of 100. Of course, an important difference between the two EC estimates is that the sample bank EC data are summed, whereas the factor

---

15 Extrapolation to longer horizons should be done with caution; while the ‘square root of time’ rule may be applied safely to some factors, it becomes questionable when applied to mean reverting series such as the credit spread and the implied equity volatility. However, since the purpose of this section is to compare the ability of the factor model to forecast extreme quantiles, we shall not delve into these difficulties here and, for the purposes of comparison with the EC data above, the square root of time rule has been applied.

Copyright © 2003 IFCI Institute, Carol Alexander and Jacques Pezier
model aggregates EC using empirical correlations. Note that the total risk explained by risk factors in corporate finance is 4.41 (on the first row and fourth column of table 8) and this is less than half the sum of the three component risks: $4.03 + 1.33 + 3.64 = 9.00$. In fact the total risk is not much greater than the interest rate risk alone, not just in corporate finance but also in retail banking. In all cases the correlated total EC in column four is much less than the sum of the risks over the three factor groups in column five. Clearly the simplistic addition of risks can give a very distorted picture of the net risk that is taken by different activities.

Table 8: Factor Model EC Estimates

<table>
<thead>
<tr>
<th>Interest Rate Factors</th>
<th>Equity Factors</th>
<th>Credit Spread</th>
<th>Correlated Total</th>
<th>Sum over Factors</th>
<th>Sample Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>4.03</td>
<td>1.33</td>
<td>3.64</td>
<td>4.41</td>
<td>9.00</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>5.44</td>
<td>6.14</td>
<td>1.56</td>
<td>8.35</td>
<td>13.14</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>4.00</td>
<td>1.97</td>
<td>3.03</td>
<td>4.14</td>
<td>9.00</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>9.73</td>
<td>5.84</td>
<td>9.94</td>
<td>11.51</td>
<td>25.51</td>
</tr>
<tr>
<td>Payment &amp; Settlement</td>
<td>0.14</td>
<td>0.06</td>
<td>0.29</td>
<td>0.35</td>
<td>0.49</td>
</tr>
<tr>
<td>Agency &amp; Custody</td>
<td>0.15</td>
<td>0.07</td>
<td>0.30</td>
<td>0.37</td>
<td>0.52</td>
</tr>
<tr>
<td>Asset Management</td>
<td>2.27</td>
<td>2.48</td>
<td>1.50</td>
<td>3.06</td>
<td>6.25</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>0.55</td>
<td>0.30</td>
<td>0.27</td>
<td>0.73</td>
<td>1.12</td>
</tr>
<tr>
<td><strong>Total EC</strong></td>
<td><strong>22.70</strong></td>
<td><strong>17.18</strong></td>
<td><strong>20.54</strong></td>
<td><strong>27.42</strong></td>
<td><strong>60.41</strong></td>
</tr>
</tbody>
</table>

The factor model can also use a zero correlation assumption, but this time between risk factors, not between risk types and business activities as assumed in the last row and column of table 6. Retaining the assumption that risk factors have normal mixture distributions defined in table 3, but now assuming zero correlations and the corresponding scaling factor of 4.47 for the 0.03%-ile (see table 5) we obtain the EC estimates given in table 9. The total risk for each business unit is the square root of the sum of the squared risks by risk factor type. It is thus directly comparable with the last column of table 9 – which reports again the original EC data, but now under the zero correlation (between risk types) assumption. The square of the ratio of column four to column five gives the percentage of variation explained by the factor model. The smaller this percentage, the greater the specific risks that are not captured by the factor model. We see that specific risks are largest for payment & settlement, and agency & custody businesses. In these business units the market and credit factors that we have chosen are not as important as other risk factors relating to ‘other’ risks such as operational risks. However, payment & settlement, and agency & custody businesses make only a small contribution to the total risk in the bank. Of the business activities that make a more important contribution to global risks, the factor model has less ability to explain risks.
in both trading & sales and retail banking. For example, in trading & sales the total EC from the factor model is 6.55 compared with the total EC of 17.30, and this implies that the total variance explained by the factor model is only 14%. In corporate finance, commercial banking and asset management the specific risks are smaller.

Table 9: Factor Model EC Estimates (Zero Correlation)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Interest Rate Factors</th>
<th>Equity Factors</th>
<th>Credit Spread</th>
<th>Total EC</th>
<th>Sample Variance Explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>3.56</td>
<td>2.80</td>
<td>3.11</td>
<td>5.49</td>
<td>9.87</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>4.84</td>
<td>4.22</td>
<td>1.33</td>
<td>6.55</td>
<td>17.30</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>3.57</td>
<td>1.22</td>
<td>2.58</td>
<td>4.57</td>
<td>12.23</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>8.74</td>
<td>3.79</td>
<td>8.48</td>
<td>12.75</td>
<td>27.58</td>
</tr>
<tr>
<td>Payment &amp; Settlement</td>
<td>0.12</td>
<td>0.17</td>
<td>0.25</td>
<td>0.33</td>
<td>1.86</td>
</tr>
<tr>
<td>Agency &amp; Custody</td>
<td>0.13</td>
<td>0.18</td>
<td>0.26</td>
<td>0.34</td>
<td>1.68</td>
</tr>
<tr>
<td>Asset Management</td>
<td>2.05</td>
<td>3.45</td>
<td>1.28</td>
<td>4.21</td>
<td>8.36</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>0.47</td>
<td>0.75</td>
<td>0.23</td>
<td>0.91</td>
<td>1.45</td>
</tr>
<tr>
<td><strong>Total EC</strong></td>
<td><strong>20.49</strong></td>
<td><strong>11.28</strong></td>
<td><strong>17.52</strong></td>
<td><strong>29.22</strong></td>
<td><strong>37.22</strong></td>
</tr>
</tbody>
</table>

In table 9 the zero risk factor correlation assumption implies the total EC for each business unit is the square root of the sum of the squared EC estimates by risk factor. However, the column totals (the total risk by risk factor type) in table 9 are not estimated under a zero correlation assumption, and neither is the total EC estimate of 29.22, because business activities are still very likely to be correlated, unless one activity is affect only by risk factor which do not affect the other (which is unlikely). Note that the total EC estimate from the factor model is larger than in table 8, because of the many negative risk factor correlations in table 4(b). Also equity risks in general appear less important now, as do the risks from trading and sales activities, whilst corporate finance and commercial banking risks have a proportionally larger contribution to the total risk.

V Optimal Risk Diversification and Constrained Optimization of the Sharpe Ratio

Armed with the common risk factor model for banking activities, it becomes easy to explore whether more diversification can be created by altering some sensitivities. Some sensitivities would be difficult to alter (e.g., sensitivities to credit spreads) but some others could be changed by acquiring liquid market instruments (e.g., sensitivities to interest rate and equity exposures). For example, we could ask what
alteration of interest rate and equity exposure in trading & sales activities would bring the maximum reduction in the total EC for the bank?

From (6) the portfolio variance explained by the factor model has the representation $\beta' V_x \beta$ where $\beta$ is the $m \times 1$ vector having elements $\beta_1, \beta_2, \ldots, \beta_m$ being the sums of the betas for each risk factor, over all the $n$ business units. Thus, to answer the question just posed, the first step is to solve the following optimization problem:

$$\text{Choose } \beta_1, \beta_2, \beta_4 \text{ to minimize } \beta' V_x \beta \text{ keeping the other elements of } \beta \text{ fixed.}$$

We simply have to solve a set of three linear equations with three unknowns. Then, given the optimal elements for $\beta$, the second step is to adjust the betas in trading & sales to achieve these sums. Table 10 shows how global sensitivities to interest rate, interest rate slope and equity indices would have to be changed to achieve the maximum reduction in total risk, whilst holding the other sensitivities constant. Interest rate and equity global betas have been significantly reduced and the negative sign on the equity beta indicates a net short position in equities.

Table 10: Modification of Exposures to Minimize Risks

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Global Betas Before Optimization</th>
<th>Global Betas After Optimization</th>
<th>Trading &amp; Sales Betas Before Optimization</th>
<th>Trading &amp; Sales Betas After Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate</td>
<td>-0.0540</td>
<td>-0.0236</td>
<td>-0.00734</td>
<td>0.02311</td>
</tr>
<tr>
<td>Slope</td>
<td>-0.0102</td>
<td>-0.0176</td>
<td>-0.00167</td>
<td>-0.00914</td>
</tr>
<tr>
<td>Equity</td>
<td>0.1037</td>
<td>-0.0561</td>
<td>0.01347</td>
<td>-0.14630</td>
</tr>
<tr>
<td>EC</td>
<td>27.42</td>
<td>17.81</td>
<td>8.35</td>
<td>16.29</td>
</tr>
</tbody>
</table>

The implementation of the desired change in risk factor exposures could be achieved in a number of ways. In table 10 we might assume that proprietary trading within trading & sales will be asked to make the appropriate changes. An alternative, but more radical, plan would be to place risk management in charge of a new hedging unit within trading and sales. As a result, the EC for all other business units remain unchanged, but the EC for trading & sales has been increased from 8.35 to 16.29 even though the overall EC has been substantially reduced, from 27.42 to 17.81. In the last column of table 10 we see that the optimal interest rate beta in trading and sales is now positive and the optimal equity beta is now negative. The implication is that, in order to offset the risks from other activities where global exposures to bonds and equities are long, the total activities in trading and sales activities should have large, net
short exposures to both bonds and equities. But of course, such trades are likely to produce unattractive returns.

To include the impact of risk diversification on expected returns, $ER$, we now consider the optimization of a Sharpe ratio, $SR$ corresponding to a risk-adjusted return on capital. The objective function to maximize is defined as follows:

$$SR = \frac{ER - rEC}{EC}$$

where $r$ is the risk free rate of return. The expected return is the sum over all risk factors of the product of the net factor beta with the expected return on the risk factor.

For the purpose of this illustration we have taken $r$ to be 1.32%, being the 1 year treasury bond rate at the end of the data sample (31st December 2002). The expected return was based on the following values for expected risk factor returns (net of funding costs): -30bp for the interest rate; -150bp for the slope; 7% for equity; for simplicity all other risk factors were assumed to have zero net return. With these expected returns the result of the optimal risk diversification exercise reported in table 10 is to reduce the risk adjusted return on capital quite significantly. Before optimization the expected annual return explained by the factor model was 3.87%. But after changing the betas to achieve the optimal reduction in EC, the total expected return falls to 2.95% p.a. (and the expected return from trading and sales changes from +0.56% to −0.34%)! so the reduction in EC from 27.42 to 17.81 would not be justified from a Sharpe ratio perspective.

We now consider the optimization of the Sharpe ratio (14) where the expected returns ($ER$) and aggregate risk ($EC$) are both estimated by the same risk factor model. For the expected risk factor returns defined above, the unconstrained maximization of Sharpe ratio gives optimal risk factor sensitivities which are not possible to implement by the alteration of trading and sales activities alone. This time not because of the implied short positions in bonds and equities – in fact it is quite the contrary – some very large long bond positions, particularly in long maturity bonds, are implied. More realistic would be to maximize the Sharpe ratio subject to constraints on the values of betas. To begin with a simple example, suppose again that all the necessary adjustments to exposures are to be made within the hedging activities in trading and sales, but now suppose that the bank wishes to constrain the trading and sales interest rate and slope betas to be less than 0.01 in absolute value, and the equity beta to be less than 0.02 in absolute value.

The maximization of (14) subject to these constraints is a straightforward optimization problem having the solution shown in table 11. A significant increase in Sharpe ratio, from 13.8% to 19.7%, can be achieved if the additional hedging activities in the trading and sales business unit take some large positions on long maturity treasury bonds and a short equity position.
Table 11: Constrained Maximization of SR within Trading & Sales

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Global Betas</th>
<th></th>
<th>Trading &amp; Sales Betas</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td></td>
<td>Optimization</td>
<td>Optimization:</td>
<td>Optimization</td>
<td>Optimization:</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-0.0540</td>
<td>-0.0459</td>
<td>-0.00734</td>
<td>-0.0059</td>
</tr>
<tr>
<td>Slope</td>
<td>-0.0102</td>
<td>-0.0185</td>
<td>-0.00167</td>
<td>-0.0100</td>
</tr>
<tr>
<td>Equity</td>
<td>0.1037</td>
<td>0.0185</td>
<td>0.01347</td>
<td>-0.0200</td>
</tr>
<tr>
<td>EC</td>
<td>27.42</td>
<td>24.27</td>
<td>8.35</td>
<td>6.86</td>
</tr>
<tr>
<td>ER</td>
<td>3.87</td>
<td>4.84</td>
<td>0.56</td>
<td>1.54</td>
</tr>
<tr>
<td>SR</td>
<td>13.8%</td>
<td>19.7%</td>
<td>3.8%</td>
<td>19.4%</td>
</tr>
</tbody>
</table>

The last example of the application of the risk factor model is to the case where all activities in corporate finance, trading and sales, and asset management are encouraged to make small changes in a direction that will increase the overall Sharpe ratio for the bank. Thus (14) is optimized by changing all betas in rows one, two and six of table 7, but with the constraint that each of these changes must be relatively small.\(^{16}\) The results of the constrained optimization are summarized in table 12. This shows that the Sharpe ratio has increased in each of the three business units, mainly through a reduction in risk – note that the increase in expected returns are small – and the overall Sharpe ratio increases from 13.8% to 17.3% due to the reduction in total EC from 27.42 to 22.26. The modification of activities in each of the business units has been constrained to small adjustments in the direction of increasing Sharpe ratio. The lower part of table 12 reports the risk factor sensitivities for the three activities, before and after optimization.

At the global level the largest changes in exposures are to interest rate and equity options positions. Both the implied volatility sensitivities are approximately one half of their pre-optimized values and this is a result of additional long option positions in all activities. At the same time, exposures to short term corporate bonds and equities are reduced as positions are shifted towards longer maturity government debt.

The results presented in this and the previous section apply to the risk profile of our ‘sample’ banks. The optimal mix of risks and returns shown in table 12 also depends on the risk factor covariance matrix and the expected returns to the risk factors. Whilst we have employed 1000 observations of historical data to estimate appropriate ‘tail’ risk factor correlations, it is beyond the scope of this paper to provide more

\(^{16}\) ‘Relatively small’ means that changes in any beta were usually of the order of 50% or less. For reasons of space the exact specification of the constraints imposed in this example, which depends on the asset and activity types, is not detailed here but is available from the authors on request.
exact values for expected returns to the risk factors. These are known to depend on a number of international market and macroeconomic factors (see Korajczyk and Viallet, 1989; Priestly, 1996; Solnik, 1983 and many others). Nevertheless, the examples given here are representative of the power of the risk factor approach to go beyond the problem of a proper aggregation of risks, towards the optimization of activities to achieve the optimal mix of risks and returns.

### Table 12: Constrained Maximization of the Sharpe Ratio

(a) Economic Capital and Expected Return

<table>
<thead>
<tr>
<th></th>
<th>Before Optimization</th>
<th>After Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EC</td>
<td>ER</td>
</tr>
<tr>
<td>Corporate Finance</td>
<td>4.41</td>
<td>0.786</td>
</tr>
<tr>
<td>Trading and Sales</td>
<td>8.35</td>
<td>0.565</td>
</tr>
<tr>
<td>Asset Management</td>
<td>3.06</td>
<td>0.536</td>
</tr>
<tr>
<td>Global</td>
<td><strong>27.42</strong></td>
<td><strong>3.87</strong></td>
</tr>
</tbody>
</table>

(b) Risk Factor Sensitivities

<table>
<thead>
<tr>
<th></th>
<th>Interest Rate</th>
<th>Slope</th>
<th>IR Implied Volatility</th>
<th>Equity Implied Volatility</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before Optimization</td>
<td>After Optimization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate Finance</td>
<td>-0.0086</td>
<td>-0.0024</td>
<td>0.0050</td>
<td>0.0235</td>
<td>-0.0122</td>
</tr>
<tr>
<td>Trading and Sales</td>
<td>-0.0073</td>
<td>-0.0017</td>
<td>-0.0128</td>
<td>0.0135</td>
<td>-0.0316</td>
</tr>
<tr>
<td>Asset Management</td>
<td>-0.0053</td>
<td>-0.0010</td>
<td>-0.0020</td>
<td>0.0324</td>
<td>-0.0101</td>
</tr>
<tr>
<td>Global Beta</td>
<td>-0.0540</td>
<td>-0.0102</td>
<td>-0.0157</td>
<td>0.1037</td>
<td>-0.0371</td>
</tr>
</tbody>
</table>

VI Summary and Conclusion

The methods that banks currently use to aggregate market and credit risk across business units are often simplistic in the extreme and so are the rules prescribed by current and proposed regulations. Because of the bottom up approach to risk assessment that has been established during the last few years, banks are left with no option but to assume correlations between risk types in different business units that are difficult to justify – often a unit correlation between all risks as taken as an approximate upper bound for the total risk, while a zero correlation is assumed to be close to a lower bound. Neither of these assumptions is appropriate, leading to very crude and improper aggregate risk assessments.
This paper has introduced a new method for risk aggregation that is nevertheless based on a well established framework in the theory of finance – that of a risk factor model. Having its roots in the arbitrage pricing theory developed by Ross (1976) and many others, the risk factor approach is shown to have some far reaching and new applications to economic capital calculations for banks. By identifying market and credit risk factors that can be common to many business units, aggregation through the risk factor model takes proper account of correlations between the risks in different business units.

We have used historical data on six common risk factors, including interest rate, equity, volatility and credit spread factors to describe a particular implementation of the model. Risk factor returns are assumed to have leptokurtic (normal mixture) distributions, and only tail correlations are used to aggregate economic capital estimates. In comparison with the case when every risk factor is assumed to be normally distributed, the normal mixture economic capital estimates at the 0.03%-ile are approximately 30% larger. Also, because of the many negative tail correlations between risk factors the effect of using empirical correlations in place of a zero correlation assumption is to reduce the economic capital estimate, by approximately 20%.

The economic capital estimates by business line that are obtained from the factor model are compared with the actual economic capital data from three major banks. This comparison shows that simple summation of individual EC estimates is considerably over-stating the total risk, whilst aggregation under the assumption of zero correlation is clearly not a lower bound. The total EC estimated by the factor model is a little more than one quarter of the total EC of the bank. This large difference is due, primarily, to the use of empirical risk factor correlations to aggregate risks using the factor model, whereas the total EC in the sample banks is simply added. Since many of the empirical correlations between risk factors are negative, the aggregate EC under the factor model is lower than the EC aggregated under the assumption that all activities and risk types have zero correlation. The factor model can also assume a zero correlation, but only between risk factors and not between business activities. Comparison between the two types of ‘zero correlation’ EC estimates indicates that the factor model explains more of the risks in some activities (such as corporate finance and asset management) than others (such as payments and settlements, or agency and custody).

We have also applied the factor model to some important optimization problems for banks. The first of these is the reduction of overall economic capital, to be achieved by changing sensitivities to certain risk factors. In our first example we suppose that optimal changes to interest rate and equity risk factors are implemented by trading and sales, and we find that this business unit should actually increase its risk in order to reduce the overall level of risk for the bank. However, the practical implementation of such a
solution is not feasible because it would imply a negative expected return for trading and sales, and this is unrealistic.

One of the main strengths of the risk factor approach to aggregating risks is that it allows expected returns for each business unit to be estimated within the same framework as the risks. Thus the mix of risks and returns within each business unit can be optimized, provided that there is some flexibility to re-direct activities in the appropriate direction. Our last example concerned the optimization of a firm-wide Sharpe ratio, subject to many constraints on the alteration of existing positions. Our example applications show that the independent minimization of risks can be a sub-optimal target for risk management, but with the unified risk and return aggregation framework provided by the risk factor model, feasible solutions to the constrained optimization of risk vs return objectives can be derived.

References

Basel, 2001b, “Potential Modifications to the Committee’s Proposals”, Basel Committee on Banking Supervision
Elton, E. J. and M. J. Gruber, 1992 “Portfolio Analysis with a Nonnormal Multi-Index Return-Generating Process.”  
*Review of Quantitative Finance and Accounting* **2** 5-16


