Application-Based Financial Risk Aggregation Methods

ISMA Centre Discussion Papers in Finance 2003-11
10 September 2003

Jacques Pézier
ISMA Centre, University of Reading, UK

Paper presented at the 1st International Derivatives and Financial Market Conference organized by BM&F
Campos de Jordao, Brazil, August 20th – 23rd, 2003

Copyright 2003 Jacques Pézier. All rights reserved.
Abstract

Financial risks are usually analysed by type and by activity using different assumptions and methodologies as may seem appropriate in each case. This approach makes it very difficult to ascertain the degree of diversification between various activities and to obtain a proper estimate of global risk. We show that different risk aggregation methodologies should be used depending on the purpose of the exercise. In particular, if it is to promote an efficient allocation of resources, a short term, normal circumstances view should be adopted, but if it is to ensure a high degree of financial soundness over the long term, then extreme circumstances and contingency plans should be explored. We propose a simple linear risk factor model in the first case but suggest that a full business model is required for the second. Finally, financial regulators raise an intermediate question that is almost impossible to answer, namely, what is the minimum level of capital consistent with a probability of default of the firm of 0.1% over one year, that is consistent with a single ‘A’ rating. We suggest that an extension of our normal risk factor model to estimate ‘tail’ effects could give a better approximation than the current regulatory rules.

Contacting Author Details:
Jacques Pezier
ISMA Centre, University of Reading, Whiteknights, Box 242, Reading, RG6 6BA,
E-mail address: j.pezier@ismacentre.rdg.ac.uk

Acknowledgements

I wish to thank the IFCI Foundation\(^1\) for sponsoring research on this subject and the invaluable support received from representatives of their member firms, in particular, the Credit Swiss Group, Deutsche Bank, JP Morgan Chase and UBS. I also wish to thank my wife, Carol Alexander, who collaborated with me on this research and carried out most data modeling and analysis.

This discussion paper is a preliminary version designed to generate ideas and constructive comment. Please do not circulate or quote without permission. The contents of the paper are presented to the reader in good faith, and neither the author, the ISMA Centre, nor the University, will be held responsible for any losses, financial or otherwise, resulting from actions taken on the basis of its content. Any persons reading the paper are deemed to have accepted this.

\(^1\) IFCI Foundation-International Financial Risk Institute, Geneva, Switzerland
Is it Possible to Aggregate Financial Risks Intelligently?

That is the question that has exercised the minds of a great many risk managers over the last few years. Financial regulators are pushing for ever more sophisticated methods to estimate ‘risk sensitive’ capital requirements, whilst still asking under Basel II proposals for requirements calculated separately for various types of risks and businesses to be simply added together to yield a global requirement.

The commonly heard excuse for this blunt approach is that regulators must take a prudential view and that addition is safe because it corresponds to a supposedly worst-case scenario. That is simply not true, at least the way regulatory capital requirements are being calculated; and even if it were true, it would not be desirable since ignorance of diversification or hedging of risks can only provide wrong incentives for capital allocation.

The question was formally put to us recently by IFCI, an association of major financial institutions and consulting firms, that is trying to promote best practice in risk management. This topic has also been picked up by the Joint Forum’s Working Group on Risk Assessment and Capital that has recently reported on the state of the art in risk integration and aggregation in the financial sector (Basel 2003). Interestingly, the Working Group distinguishes integration, meaning the development of a common infrastructure for risk management (including policies, personnel, systems, databases etc.) from aggregation, meaning the development of “quantitative risk measures that incorporate multiple types or sources of risks”. Here we address the aggregation problem only, leaving aside the integration issue and, indeed, most matters of risk management policies.

Risk Aggregation for a Purpose

But why should a firm want to estimate its global financial risk in the first place? Estimating a global risk is a description problem and as for all such problems, it is impossible to say, except from an aesthetic point of view, whether a description is good or bad or simply satisfactory without having a purpose in mind, a problem that the description should help resolve. Indeed, without a purpose, it would be questionable whether the description of complex risks faced by modern financial institutions could ever be reduced meaningfully to a single probability distribution on an appropriate scale, not to say to a single number.

In view of the uncritical enthusiasm with which new risk assessment methods, value-at-risk figures and other economic capital numbers, have been embraced by the financial industry and its suppliers and the vast sums that have been invested in the
development of risk integration systems, it may be worth recalling the characteristics of what Sir Peter Medawar called ‘unnatural science’. To quote:

(a) The belief that measurement and numeration are intrinsically praiseworthy activities (the worship, indeed, of what Ernst Gombrich calls idola quantitatis)
(b) The whole discredited farrago of inductivism – especially the belief that facts are prior to ideas and that a sufficiently voluminous compilation of facts can be processed by a calculus of discovery in such a way as to yield general principles and natural-seeming laws;
(c) (…) faith in the efficacy of statistical formulae, particularly when processed by a computer

Medawar was making these remarks about the pseudo-science of IQ measurement, but those who might want to develop a universal science of risk measurement could equally reflect on them.

For a start, we should ban the term measurement and say risk ‘assessment’. Risks are about the future, an appreciation of potential difficulties or disasters. Measurements can only be about things that exist now or were recorded in the past. To jump to the future we need to make assumptions and to use models that are logical constructs build on these assumptions. Assumptions and models can never be validated in an absolute sense but they may be deemed acceptable in certain limited circumstances in order to support certain decisions.

Decisions that depend on risk descriptions are many. For example, on a short time scale, assuming markets behave normally, risk assessments should help make efficient use of resources, that is, achieve the most desirable risk/return trade-offs. Over the long term, managers as well as regulators are concerned about the viability of financial institutions. But a firm’s survival depends not only on capital now but also on a host of other factors such as a good reputation, a viable franchise, an astute and cautious management, and a well-developed risk management function. On some intermediate time scale, regulators would like to ensure that financial firms, given their current activities and policies, have enough capital to absorb extreme shocks to their P&L such as might happen with a probability as low as 0.1% over one year.

The assumptions, the time scale, the range of factors to be taken into consideration and the desired degree of accuracy in the assessment of global financial risks, all depend on the application. We shall propose three risk aggregation methodologies to address the three types of problems outlined above. But before we do so, for the sake of comparison and to understand the component risks, we need to review the state of the art in risk assessment and aggregation, both as advanced by the regulators and as promoted internally within firms.

From Minimum Regulatory Capital to Economic Capital

Governments are well aware of the crucial role played by financial industries and financial markets in free market economies. They are particularly concerned about ensuring the twin goals of fair competition and financial soundness among financial institutions as well as about ensuring fair business practices. The corollary of freer markets is the reinforcement of supervision and regulation of the financial industries and in particular the development of so-called risk sensitive capital requirements to ensure that financial firms are sufficiently capitalized with regard to the risks they take.

We have now seen considerable progress since the first Basel Accord (1988) setting minimum regulatory capital (MRC) requirements for banks as a buffer against credit risks, which were then regarded as the overwhelming source of banking risks. Successive amendments and now the new Basel II proposals have introduced detailed rules to take into account market and other risks and have greatly refined the assessment procedures. Basel has no statutory powers, only moral suasion, but competitive pressures and the willingness of many national authorities to follow suit has led to a near universal acceptance of these principles in major economies.

Basel’s intention is prudential: primarily, to protect banks’ depositors and other creditors by requiring a certain level of financial soundness and, secondarily, to provide a basis for fair competition among banks and to contain systemic risks. Basel’s intention is neither to define an optimal capital ratio nor to provide guidance to bank managers on how to run their business. Naturally, in as much as the MRC constraint is or could become biting, banks have a definite interest in refining its calculation method. But beyond regulations, banks and other financial institutions saw in risk assessment and the corresponding linkage to capital, a tool for efficient management.

Indeed, company directors must also satisfy the wider and complementary interests of shareholders. To the latter, insolvency is just a worst case to consider when balancing risks and returns. Interestingly, the ratio of average excess return over risk free return to volatility of returns, the well-known Sharpe ratio often used in investment selection, is not so much dependent on the ratio of equity to total liabilities as it is on an appropriate selection of assets, liabilities and flow businesses. It is easy to verify that if a bank funds itself at near the risk free rate, then the Sharpe ratio for shareholders is about equal to the Sharpe ratio of the assets, whatever the leverage. Firms must therefore assess risks not so much to satisfy regulatory capital requirements but to evaluate risk/return ratios and select an appropriate business mix.

Thus, as far back as the late 1970s, some firms were designing their own, internal risk assessment and risk adjusted performance measurement (RAPM) methodologies. For example, Bankers Trust formally developed the concept of Risk-Adjusted Return on Capital (RAROC™) in the late 1970s and claimed to have been measuring the required risk capital for all activities in the entire bank by 1983.

---

3 It is easy to verify that if a bank funds itself at near the risk free rate, then the Sharpe ratio for shareholders is about equal to the Sharpe ratio of the assets, whatever the leverage.
4 For example, Bankers Trust formally developed the concept of Risk-Adjusted Return on Capital (RAROC™) in the late 1970s and claimed to have been measuring the required risk capital for all activities in the entire bank by 1983.
Because the goals are broader than those of the regulator and the applications are firm specific, these internal methodologies may differ in some respects from regulatory rules. For example, they may use assumptions and internal models adapted to their business (e.g., credit portfolio models), subjective views about business and reputational risks, and higher standards of minimum capitalization.

But in many respect there are remarkable similarities among internal methods and statutory rules. Typically:

- Risks are assessed from the bottom up starting at the most elemental level (e.g., instrument by instrument) and by risk type (i.e., separately for credit, market and other risks). Risk aggregation rules are primitive: often down to a simple addition of standard deviations, sometimes, a square root of the sum of the squares if there is no evidence of dependency.

- Risks are assessed under static portfolios or business status quo assumptions and therefore over different time horizons according to the realism of such assumptions. Obviously, some risks will naturally evolve faster than others and indeed some exposures can be reduced, if need be, much faster than others. But it would be difficult to describe the dynamics of these risks or the effect of management policies over the long term. Thus, typically, liquid market risks are assessed over a couple of weeks, operational risks over a year and the less liquid credit risks perhaps until maturity of the relevant exposures.

- Risk assessments are summarized into a single number at an early stage. Commonly, the difference between the expected value and some low quantile of a P&L distribution is defined as the risk ‘metric’. Because the main role of capital in financial firms is to absorb risks, the terms risk capital or economic capital (EC) are now widely used to describe the chosen risk metric and to distinguish this internal assessment from the externally imposed MRC.

These three features and a few others – such as the uneven attention given to various types of risks – make it difficult to generate a reliable, comprehensive picture of all risks in a firm. The recent survey of internal methods of risk aggregation (Basel, 2003) confirms a wide interest in this problem but also the current lack of satisfactory methodologies. It concludes that these efforts should be strongly encouraged but that regulators should remain ‘cautious’, read, be wary of any method that could lead to a reduction in MRC. Hence the current primitive but supposedly conservative view of aggregation across risk types and among operational risks adopted in the Basel II proposals.

**Risks in Risk Models**

A philosopher wrote:⁵ “If a man will begin with certainties, he will end up with doubts, but if he will be content to begin with doubts, he may end with certainties”. It

---

⁵ Francis Bacon (1561-1626), *Great Instauration*
is indeed salutary to realize how much uncertainty there is in current financial risk assessments before putting forward new ideas for better risk aggregation.

(i) Uncertainty within each risk class

Even at the most elemental level, there are uncertainties about risk parameters. These are smaller for market risks, which are continuously observed, than they are for relatively rare credit events and, a fortiori, for extremely rare but significant operational risks. We argue (Alexander and Pézier, 2003) that the figures in Table 1 give a fair order of magnitude of relative errors in the determination of risk estimates according to risks class and the risk metric used. These are calculated as the ratio of the standard deviation of the risk estimate over the expected value of the risk estimate (a ratio commonly known as the coefficient of variation, or CV). The second column of the table shows the CV of the standard deviation risk metric, and the other two columns estimate the CV of two quantile based risk metrics.\(^6\)

<table>
<thead>
<tr>
<th>% Relative error for</th>
<th>Standard Dev.</th>
<th>1% quantile</th>
<th>0.1% quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market risk</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Credit risk</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Operational risk</td>
<td>100</td>
<td>130</td>
<td>160?</td>
</tr>
</tbody>
</table>

(ii) Risk in Aggregation across Risk Classes/ Business Units

We encounter three main difficulties in aggregating individual risk estimates: (a) difficulties due to the choice of metric, in particular, the extreme quantiles often used in the definition of EC, (b) difficulties in the description of co-variations and co-effects, particularly relevant with extreme risks and multi-risk products, and (c) impact of errors in individual estimates.

The use of extreme quantiles for risk metrics, as recommended by the regulator and as used in many EC definitions, increases the difficulty of devising a sensible aggregation model. Quantiles may be super-additive\(^8\) as well as sub-additive. In the

---

\(^6\) Quantile risk metrics are defined as the return levels that will have a probability equal to the quantile of not being exceeded. When making risk assessments, many banks use loss distributions rather than return distributions. We prefer to keep to returns as the underlying variable; a low quantile on returns such as 1%, corresponds to a 99% quantile on losses.

\(^7\) It is the author’s view that it is nigh impossible to determine such number. How can one ascertain with any accuracy the level of operational losses over the forthcoming year that would not be exceeded with a probability of more than 0.1%?

\(^8\) If \(x_\alpha\) represents the \(\alpha\)-quantile of a probability distribution on \(X\), and \(Y\) is another random variable, then it may well be that the total risk at the same quantile level is greater than the sum of the quantiles of each risk: \((x + y)_\alpha > x_\alpha + y_\alpha\). This is called super-additivity and is not a mere mathematical curiosity, it does happen with skewed, uncorrelated distributions that one would encounter, for example, in bond portfolios. Some authors (Artzner et alii 1999) have argued for the use of ‘coherent’
first case addition does not provide a ‘safe’ answer. In the second (and more common case) it ignores the benefits of diversification.

The use of linear correlations to describe co-variations has severe limitations. Correlation estimates are famously unstable. Often co-dependencies are different in extreme circumstances from what they are under normal market conditions. Copulas may offer a more fitting description of dependencies under both normal and extreme circumstances, using still a limited number of parameters. But, whatever the description, there will only be scant relevant historical evidence on extreme co-variations to support it.

In some instances, there are known functional relationships between risk types at the instrument level, but these are overlooked when doing separate evaluations for each risk type. For example, in a forward contract, counterparty exposure exists only when the underlying price moves in a favourable direction; it is impossible to incur both market and credit losses. That relationship is lost if the two risks are estimated separately. But for any sizable portfolio it appears unfeasible to analyse all risks simultaneously.

There is however one saving grace in the midst of all these difficulties: only the most important risks and dependencies matter. The smaller risks, by themselves, are negligible if they are not significantly correlated with major risks. This is typical of the aggregation of operational risks with other risks. Operational risks by themselves are assumed by Basel to be only a fraction, perhaps 10%, of market and credit risks. Our argument is that the marginal effect of operational risks on total risk should be negligible unless operational risks are significantly correlated to market and/or credit risks; presently, for most operational risks, there is no strong argument in favor of either a positive or a negative correlation with other risks.

(iii) Errors due to inadequate accounting and overlooked risks

The inadequate recognition of expected gains and losses under accrual accounting has led the regulator to include some assessment of expected losses for credit and operational risks in the evaluation of MRC. Thus, in the regulator’s view, capital is

---

9 See for example Bouye et alii 2001
10 For this reason, as noted by Pezier (1997), the integration of credit risk with market risk for par value OTC derivatives produces a total standard deviation smaller than with market risks only. On a global scale, the effect may be considerable. Worldwide, the underlying notional of OTC derivatives exceeds the notional of loans by a factor of about twelve. Credit risks for OTC derivatives are based on mark to market value when positive plus add-ons, with an adhoc but not generous formula for the netting of add-ons. As a result, total OTC derivative credit risk, amounts to a significant fraction (25%?) of credit risks attributed to loans, when realistically the marginal effect above market risks may be very small.
11 Some type of operational risks may increase in a period of stress when profitability is squeezed; other types may be more frequent when business is good and the volume of transactions is high.
not strictly a buffer against risks; it is also a buffer to absorb unaccounted expected losses. This expedient solution to the accounting problem raises its own difficulties:

- Expected losses (or gains) should be estimated over the whole life of the corresponding instruments, whereas risks are usually estimated over a shorter, single time horizon like one year (although regulations are not always clear about that choice).
- Expected losses are always additive, whereas risks generally are not.
- Subtracting a loss from eligible capital (numerator) does not produce the same effect on the capital ratio as adding it to the risk weighted assets (denominator) – even after multiplication by an ad hoc factor.\(^{12}\)

Also linked to accounting limitations are a number of risks that are simply ignored because they are too difficult to assess. These risks include most market risks in banking books, but more importantly, most business, reputational and systemic risks. This is all the more disturbing that many bankruptcies can be traced to such risks.

Accounting standards are not designed for risk management, nor can they ever be, but they form a basis for the recognition of risks. The development of new regulatory risk assessment rules and capital standards should therefore be intimately linked to the development of new international accounting standards (IAS39). In the meantime, regulatory rules must be judiciously adapted to local accounting standards.

(i) Consequences of Risk Model Risks

The global risk picture that emerges is likely to be incomplete and distorted, probably drawing unwarranted attention to minor risks, and not reflecting diversification that could lead to an improvement in the balance of risks and returns.

To illustrate this point, Alexander and Pézier (2003) consider a bank with three business lines reporting the 0.1% confidence level EC figures shown in Table 2a. Total EC figures are assumed to be evaluated accurately and to be additive (correlation of +1) across business lines and risk classes. Total EC is 100 so that figures in each cell can be read as the marginal percentage contribution of each risk category to the total. For example, a 1% increase in retail banking credit risk would increase total risk by 0.5%, and a 1% increase in retail banking overall risk would increase total risk by 0.59%.

\(^{12}\) To illustrate, consider a bank has been set a minimum solvency ratio of 12%; its qualifying capital is 100 and risk weighted assets 600. Its solvency ratio is 16.7%, well above the 12% minimum. However, new operational risks estimates reveal expected losses (EL) of 8 and unexpected losses (UL) of 12 that must now be taken into account. There is a choice between (a) recognizing EL in the accounts and deducting it from capital, and (b) including (EL) in the operational risk capital charge. Note that capital charges for market and operational risks are multiplied by 12.5 and added to risk weighted assets in the denominator of the capital ratio. The new solvency ratios would be:

With (a): \((100 – 8)/(600 + 12.5*12) = 12.27\%\) still above the minimum of 12%
With (b): \((100)/(600 + 12.5*(8+12)) = 11.76\%\) below the minimum of 12%
Table 2a – Percentage contributions to total risk: 
Precise EC valuations, complete dependence

<table>
<thead>
<tr>
<th></th>
<th>Credit</th>
<th>Market</th>
<th>Operational &amp; Business</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Banking</td>
<td>50</td>
<td>5</td>
<td>4</td>
<td>59</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>10</td>
<td>25</td>
<td>4</td>
<td>39</td>
</tr>
<tr>
<td>Brokerage</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total per class</td>
<td>60</td>
<td>30</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

Suppose now that all risk categories are independent and that the 0.1% EC evaluations are subject to the measurement error shown in the last column of Table 1. The marginal percentage contribution of each risk category to the total would then be as shown in Table 2b (rounded to nearest per cent). Because of uncertainty in risk assessments, and independence of errors, relatively large credit risks are perceived as even larger, whereas small ones, like operational & business risks nearly disappear.13

Table 2b – Percentage contributions to total risk:
Uncertain EC valuations, independence of risks

<table>
<thead>
<tr>
<th></th>
<th>Credit</th>
<th>Market</th>
<th>Operational &amp; Business</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Banking</td>
<td>81.4</td>
<td>0.5</td>
<td>0.9</td>
<td>82.8</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>3.3</td>
<td>12.7</td>
<td>0.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Brokerage</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Total per class</td>
<td>84.7</td>
<td>13.2</td>
<td>2.1</td>
<td>100</td>
</tr>
</tbody>
</table>

Finally, revisit the results in Table 2b by introducing a perfect correlation in each business line between operational & business risks and credit risks (errors in estimates are still taken to be independent). The marginal percentage contribution of each risk category to total risk would be as shown in Table 2c.

Table 2c – Percentage contributions to total risk:
Independence except between Operational & Business Risks and Credit Risks

<table>
<thead>
<tr>
<th></th>
<th>Credit</th>
<th>Market</th>
<th>Operational &amp; Business</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Banking</td>
<td>79.4</td>
<td>0.5</td>
<td>2.4</td>
<td>82.3</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>3.7</td>
<td>12.2</td>
<td>1.5</td>
<td>17.4</td>
</tr>
<tr>
<td>Brokerage</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Total per class</td>
<td>83.2</td>
<td>12.7</td>
<td>4.2</td>
<td>100</td>
</tr>
</tbody>
</table>

13 Figures in Table 2b are obtained by squaring each central cell in Table 1, multiplying the result by \((1+CV^2)\) where CV is taken from the last column of Table 1 for the relevant risk class, adding to obtain marginal totals and scaling the grand total to 100\(^2\).
Suddenly operational & business risks have regained in importance to such an extent that they become much more significant than market risks in retail banking and a third of market risks in total.

Looking at risks with different dependency assumptions is like looking through a kaleidoscope. The image can change suddenly and reveal significant features that were hidden under different assumptions.

The use by banks of current MRC or EC evaluations for setting overall capital ratios compatible with a solvency target is even more problematic. The financial soundness of a bank depends on much more than its capital ratio. Factors such as the quality of its management and controls, the ability to generate profits, the buoyancy of the markets in which it operates, the capacity to raise capital and the likelihood to receive government support if difficulties surface, are all relevant and should be considered over the long term. All such factors are taken into account by international rating agencies. Their ratings have proved, on average over the years, to be closely related to historical frequencies of default. However the correspondence between credit ratings and capital ratios is weak.

**Figure 1. Relationship between credit rating and capital**

![Figure 1](image-url)  
Figure 1 presents data from Fitch IBCA comparing Tier 1 capital ratios to Moody’s ratings for a selection of major international banks over the late 1990s. Chris Matten (2000) who reproduces these figures gives various reasons for some outliers but concludes: “Even deleting the outliers, however, there is no statistical relationship between credit ratings and capital in this sample”.

Copyright © 2003 Jacques Pezier.
Three Adapted Risk Aggregation Models

Is it possible to improve on the current risk aggregation methodologies that are limited to simple addition of risk metrics or the use of unreliable correlation coefficients? One should note first that great progress have been made over the last ten years within two major risk classes: market and credit risks. Basel has recognized the use of value-at-risk (VAR) models (with precise specifications) and is even basing new credit risk capital requirements for the Internal Ratings Based (IRB) approach on its own portfolio model including various correlation assumptions for different markets and some measures of concentration (Basel 2001a and 2001b). The same cannot be said about operational risks where the diversity of circumstances and the rarity of large losses conspire to make it nearly impossible to arrive at reliable estimates of losses at the required confidence level of 99.9% over a year. In the circumstances, capital requirements for operational risks look rather premature.

Where the main challenge remains, however, is in risk aggregation across risk classes. The Joint Forum Work Group (Basel 2003) reports that many financial firms have started to use more refined methods than simple addition of economic capital across business units and risk types, but with mixed results. A promising approach, they note, is to identify core factors for each of the major risk types, to determine the sensitivities of the P&L of each business units to such factors and to conduct global VAR analyses on groups of business units up to the firm level. The results are global economic capital figures by risk types that must still be combined to reach a single total economic capital figure. As we remarked earlier, this last step is particularly difficult as the distributions for various risk types will have different shapes and the co-dependencies between these distributions are neither well understood nor easily verifiable empirically. Different correlation assumptions may lead to widely different results.

Alexander and Pezier (2003) take this approach further by recognising three main applications of global EC estimation that they call Normal, Tail and Strategic Cases and designing adapted assumptions and models for each case. The models are as follows:

**Normal Case: A Linear Common Risk Factor Model**

The Normal case refers to a short-term time horizon, assumes normal market circumstances and business as usual. The purpose of the exercise is to assess risks in order to achieve an efficient allocation of resources based on the firm’s risk appetite.

A number of common risk factors are identified that affect the performance of various business units. The choice depends on the main activities of the firm. The most interesting factors are obviously those that would affect a large number of activities and to which exposures can be adjusted. A given risk factor may influence several risk types; for example a credit spread index may affect simultaneously market, credit
and operational risks. In fact, when thinking in terms of common risk factors, the traditional categories of risks may appear somewhat artificial.

Using eight common risk factors and a sample bank reflecting an average of the distributions of economic capital in a few major banks, the following illustrative results are obtained. Table 3 shows the economic capital of the sample bank by business lines and risk types. Total EC is normalized at 100. Table 4 shows the economic capital explained by the common risk factors (i.e., residual risks excluded) under two assumptions: zero correlations among risk factors and empirical correlations over three different time periods. Finally, Table 5 shows how the evaluation of risks could be used to minimize the economic capital by adjusting three exposures in one business unit, Trading and Sales, namely, Interest rate, Interest rate slope and Equity, leaving all other exposures unchanged.

Table 3: Economic Capital for the Sample Bank

<table>
<thead>
<tr>
<th>Economic Capital</th>
<th>Market</th>
<th>Credit</th>
<th>Operational</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>7.2</td>
<td>4.8</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>Trading and Sales</td>
<td>13.3</td>
<td>7.3</td>
<td>3.8</td>
<td>24.4</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>3</td>
<td>10.3</td>
<td>2.2</td>
<td>15.5</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>2.1</td>
<td>24.1</td>
<td>1.3</td>
<td>27.5</td>
</tr>
<tr>
<td>Payment and Settlement</td>
<td>0.3</td>
<td>1.6</td>
<td>1.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Agency and Custody</td>
<td>0.7</td>
<td>1.3</td>
<td>0.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Asset Management</td>
<td>6.5</td>
<td>3.4</td>
<td>0.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>0.9</td>
<td>0.9</td>
<td>0.3</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>34</strong></td>
<td><strong>53.7</strong></td>
<td><strong>12.3</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Table 4: Risk Capital Estimates by Risk Factors

<table>
<thead>
<tr>
<th>Risk Factors</th>
<th>Jan 66 - Dec 85 Correlations</th>
<th>Jan 86 - Dec 98 Correlations</th>
<th>Jan 99 - Dec 02 Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rates</td>
<td>51.24</td>
<td>35.40</td>
<td>20.74</td>
</tr>
<tr>
<td>Equity</td>
<td>8.49</td>
<td>8.86</td>
<td>12.77</td>
</tr>
<tr>
<td>Credit Spread</td>
<td>12.49</td>
<td>5.33</td>
<td></td>
</tr>
<tr>
<td>All Market and Credit Operational</td>
<td>56.30</td>
<td>38.84</td>
<td>26.30</td>
</tr>
<tr>
<td>Total</td>
<td>57.73</td>
<td>39.91</td>
<td>27.16</td>
</tr>
</tbody>
</table>
Table 5: Modification of Exposures to Minimise Risks

<table>
<thead>
<tr>
<th>Market and Credit Risk Capital</th>
<th>Before Optimization</th>
<th>After Optimization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>5.65</td>
<td>5.65</td>
</tr>
<tr>
<td>Trading and Sales</td>
<td>10.98</td>
<td>14.34</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>5.27</td>
<td>5.27</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>11.92</td>
<td>11.92</td>
</tr>
<tr>
<td>Payment and Settlement</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Agency and Custody</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Asset Management</td>
<td>3.66</td>
<td>3.66</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Total EC:</td>
<td>26.69</td>
<td>22.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Sensitivities</th>
<th>Before Optimization</th>
<th>After Optimization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate, r</td>
<td>-0.0379</td>
<td>-0.0191</td>
</tr>
<tr>
<td>Slope, s</td>
<td>-0.0310</td>
<td>-0.0096</td>
</tr>
<tr>
<td>Equity, e</td>
<td>0.1048</td>
<td>-0.0545</td>
</tr>
</tbody>
</table>

A few comments are called for:

- The total of economic capital of 100 in Table 3 corresponds to the unrealistic assumption of perfect positive dependence among all risks. An equally unrealistic assumption of zero correlation among all risks and business units would yield a total EC of 33.31.

- In Table 4, the zero correlation assumption is now between risk factors; total EC varies markedly with the time period (between 57.73 and 27.16) demonstrating large changes in risk factor volatilities over time.

- Since the figures in Table 4 do not include the residual risks not explained by the risk factors – which residuals may be of the same order of magnitude as the explained risks – the figure of 30.84 for zero factor correlation during the last period seem relatively large compared to the 33.31 obtained with zero correlation among risk types and business units. This suggests that the assumption of zero correlation between business units is probably unwarranted. Several business units may indeed be exposed to the same risk factors in the same direction.

- Table 4 also shows that empirical correlations vary significantly over time. A detailed look at the risk factors over the corresponding periods would rapidly reveal why (e.g., effect of the technology bubble on credit spreads during the last period). Economic capital under empirical correlations may or may not be smaller than under zero correlation. The zero correlation case is not an extreme case. It all depends on the relative sensitivities to various factors.
Table 5 refers to only the last period (Jan 99 – Dec 02) and does not include operational risks, hence the lower total EC of 26.69 before optimization. The optimization shows that the three sensitivities that have been modified are all reduced, but not to zero because of correlations among risk factors. Total EC is reduced from 26.69 to 22.09 but the economic capital of the Trading and Sales unit that has modified its exposures jumps up from 10.98 to 14.34. Thus the unit implementing the optimization suffers from an increase in its stand-alone EC whilst effecting a reduction in total EC. Its performance measurement should therefore reflect its marginal risk contribution. More realistic optimization problems (e.g., taking into account expected returns as well as risks, limits on sensitivities, transaction costs, etc.) could be addressed in a similar way.

Tail Case: A Non-Linear Risk Factor Model with Tail Dependencies

Many risk managers are asked to consider extreme circumstances over periods extending to one year and to ensure that capital is adequate to cover risks with a confidence level of 99.9% or more. Some banks actually set their capital targets as high as the 99.98% confidence level over a year, to be consistent with a target rating of ‘AA’ (although, as we have seen earlier, there is no clear relationship between capital ratios and credit ratings).

Such questions are impossible to answer without making heroic assumptions about policies and the dynamics of a number of activities in addition to the dynamics of the markets. But because these questions are raised by the financial supervisors themselves, they must be answered. Alexander and Pézier (2003) indicate that a mere extrapolation of their linear factor model would be extremely unreliable. Fudge factors would have to be invented to scale risks over time and to translate standard deviations into extreme quantiles when the shapes of the underlying distributions are not well known.

As a palliative they suggest that at least two improvements be made to the ‘normal case’ model just described. First that possible non-linear effects for large variations of risk factors be taken into account in the P&L; there is no theoretical difficulty in that. Second, that the description of the tails of the risk factor distributions and of their dependencies be refined. They put forward some relatively simple suggestions for these improvements. They describe major risk factors with a mixture of two normal distributions designed to fit not only the mean and standard deviations of the empirical distributions but also their excess kurtosis and 1% quantiles. Tail dependencies are estimated by calculating correlations after eliminating the core observations where two factors are observed simultaneously in their mid 90% range. They find that tail dependencies are generally stronger and more significant than

---

14 For a discussion of normal mixture modeling, see Alexander, 2001
Overall dependencies (e.g., the daily tail correlation between the S&P500 index and the VIX volatility index is –0.91, whereas the overall correlation is –0.83). They also find that daily correlations are often different from correlations estimated on monthly variations (e.g., on a monthly basis there is often a positive correlation between the slope of the yield curve and the 10year Baa credit spread over 10year US-Treasuries, whereas the correlation is significantly negative on a daily basis).

Overall, the tail case model shows that the P&L distribution has lower standard deviation with empirical tail correlations among risk factors than with no correlations. However the tails of the distribution are much fatter with empirical correlations than with zero correlations. A simple extrapolation of the ‘normal case’ linear model could not account for these complex effects.

**Strategic Case: A Business Model**

Despite all the efforts put into refining a ‘tail case’ factor model, I do not believe that it will ever be able to produce a total economic capital estimate that – if met – would be a major determinant of solvency over the long term and therefore of credit rating. Capital adequacy at any point in time is indeed only one among many factors that will affect the long-term survivability of a firm, and probably not the most important factor.

If one is serious about exploring the long term, one should draw a dynamic picture where management reactions to possible extreme circumstances are taken into account. It takes a broad brush to paint this vast canvas. Only major factors should be described. Among these are the ability to generate fee based earnings as well as the impact of possible extreme market conditions. Management’s reactions should be focused on major strategic decisions from hedging some major exposures to realizing some assets and even closing down some activities.

A generic tool for depicting such a business model is an ‘influence diagram’. The diagram consists of three types of nodes representing risk factors, decisions and outcomes. With arrows, it describes the direction of causal relationships between external risk factors, management decisions and outcomes. The risk factors are the major sources of uncertainty that could affect the firm over the long-term. They include market factors but also competition, the strength of the management team and the quality of the risk management function – factors which are routinely taken into account by equity analysts and rating agents. Decisions are choices among key strategies. As we draw an influence diagram, each decision may be set at the status quo and the likely outcomes can be examined on this basis. But the main value of the analysis is to identify and evaluate alternative courses of action, especially under exceptional circumstances.
Business models have been used in many industries and government agencies to facilitate strategic planning. They are also used by corporate finance houses to explore the effects of mergers and acquisitions, but, to my knowledge, they are not routinely used by financial firms to explore their own future. Long-term strategic plans and forecasts are notoriously precarious, if only because it is impossible to predict true innovations, but a dynamic business model – as opposed far-fetched extrapolations from the status quo – will shed some light and get a firm better prepared for the future.

Summary and Conclusions

Basel II ‘risk sensitive’ capital requirements form the first pillar of regulatory control. A minimum regulatory capital is set at a level which, the Basel Committee believes, is consistent with a single ‘A’ credit rating or better. But both theoretical considerations and empirical evidence show that, given the regulatory assessment methods for capital ratios, there is no significant relationship between capital ratios and probabilities of default. The two main weaknesses of current practices lie in inadequacies in risk assessment over the medium term and primitive risk aggregation methods.

Financial firms are trying to palliate these weaknesses by developing their own risk assessment methodologies and translating the results into a measure of so-called ‘economic capital’ for comparison with ‘minimum regulatory capital’. Central to these internal methodologies is the issue of aggregation among various risk types in order to understand the contribution of each activity to overall risks and returns and to set a target level for overall capital adequacy. We have argued that these two problems – as well as intermediate questions – justify separate approaches, each with their own assumptions and analytical tools.

A large fraction of short-term P&L variations by business unit and overall can be explained by a linear, common risk factor model. Unexplained residual risks are likely to be independent or to point towards additional common risk factors. There is no need to explore extreme variations and calculate capital requirements at a very high level of confidence when the purpose is only to decide on an efficient allocation of resources. The linear model is sufficiently simple to address a variety of optimization problems. But we stopped short of discussing the twin issues of choice of risk adjusted performance measures and implementation of optimal allocation of resources because it would require too much space.

The use of a model designed for normal circumstances would give unreliable estimates of extreme losses, that is losses which might have a probability of occurrence as low as 0.1% per annum. So we have proposed several refinements to

15 See the recent study by the Committee of Sponsoring Organizations of the Treadway Commission (Coso 2003) for a comprehensive discussion of the methodologies, benefits and limitations of enterprise risk management.
the basic linear risk factor model including: a better description of extreme movements of risk factors, estimations of co-dependencies for extreme movements, and non-linearities in the P&L as a function of the risk factors. Although we did not have a full, realistic set of data to test these refinements, they appear to provide a useful basis for assessing extreme quantiles of the P&L distribution over the medium term. A major weakness of this approach, however, is that it does not account for any corrective action by management under extreme adverse conditions.

The nub of the problem, when exploring extreme circumstances, is indeed to create and evaluate contingency plans. A typical ‘A’ rated financial firm would not suddenly find itself in serious difficulties unless it was hit by a totally unforeseen event; in which case a large capital buffer might still be insufficient. More likely, the situation could deteriorate over months, or years, due to adverse circumstances, or poor strategy – but management, in the meantime, would take all possible corrective measures. It seems, therefore, eminently useful to develop – with the assistance of senior management – a business model to draw strategic plans for normal and adverse circumstances. Such models have proved useful in many other industries.

References


Basel, 2001b, “Potential Modifications to the Committee’s Proposals ”, cEng, Basel Committee on Banking Supervision, Nov. (www.bis.org)


