Long-term Information, Short-lived Securities

Dan Bernhardt
463 Commerce West, University of Illinois, Champaign, IL 61820, USA
Ryan J. Davies
ISMA Centre, University of Reading, Whiteknights, P.O. Box 242, Reading RG6 6BA, UK
John Spicer
European Economic Research Ltd., Chancery House, 53–64 Chancery Lane, London WC2A 1QU, UK

Copyright 2003 Bernhardt, Davies and Spicer. All rights reserved.
Abstract

We explore strategic trade in short-lived securities by agents who possess long-term information. Trading short-lived securities is profitable only if enough of the private information becomes public prior to contract expiration; otherwise the security will worthlessly expire. We highlight how this results in trading behavior fundamentally different from that observed in standard models of informed trading in equity. Specifically, we show that informed agents are more reluctant to trade shorter-term securities too far in advance of when their information will necessarily be made public, and that existing positions in a shorter-term security make future purchases more attractive. Because informed agents prefer longer-term securities, this can make trading shorter-term contracts more attractive for liquidity traders. We characterize the conditions under which liquidity traders choose to incur extra costs to roll over short-term positions rather than trade in distant contracts, providing an explanation for why most longer-term derivative security markets have little liquidity and large bid-ask spreads.

JEL Classification: G1; D82
Keywords: Private information; Derivative securities; Rolling the hedge; Fixed transaction costs

Contacting Author Details:

Ryan J. Davies
ISMA Centre, University of Reading, Whiteknights, Box 242, Reading, RG6 6BA,
E-mail address: r.davies@ismacentre.rdg.ac.uk (R.J. Davies).

The views expressed are those of the authors. We are grateful for comments received from Melanie Cao, David Brown, and seminar participants at: Queen’s University, University of Toronto, Dalhousie University, Simon Fraser University, Memorial University, Western Michigan University, University of Miami, Fordham University, University of Colorado, Federal Reserve Bank of New York; the 2000 Western Finance Association Meetings; the 2000 Canadian Economic Theory Meetings; and the 1999 Northern Finance Association Meetings.
1 Introduction

We explore strategic trade in *short-lived* securities by an “informed agent” who possesses long-term private information. From the informed agent’s perspective, the risk associated with trading short-lived securities is that his position will be profitable only if his private information is revealed publicly (and is reflected in prices) prior to contract expiration. As time passes, newly-issued contracts with later expiration dates become available for purchase. We model the informed agent’s decision regarding when, and how frequently, to purchase these contracts.

When an informed agent purchases a short-term security, he conveys information to the market *even if* the contract does not pay off, adversely affecting future contract prices. These strategic information costs are higher for short-term securities than for equity, and more generally for nearby contracts than distant contracts. This is because, in contrast to trading equity, where an informed agent will ultimately benefit from his trades, trading short-term securities is profitable only if the private information is impounded in the price before expiry. Ignoring leverage concerns, this makes an informed agent more reluctant to trade immediately in shorter-term securities. Indeed, even if it is profitable in expectation to trade now, an informed agent may prefer to defer in the hope of obtaining better prices in the future. The attendant risk with this strategy is that his information may be revealed publicly before he can exploit it.

In further contrast to equity, a risk neutral informed agent’s holding of a short-lived security affects his trading behavior: Higher past informed trading leads to greater future informed trading. As the informed agent’s holdings of the short-lived security increase, the less he minds if his subsequent trading reveals his information, since he will profit from his existing stake. In contrast, equity holdings do not affect his trading strategy in the same manner, since he will eventually profit on his existing stake, provided he does not close his position. So, too, the time-to-expiry of the short-lived affects trading intensities. We show that informed trading intensities rise as the time-to-expiry approaches, so that prices become more sensitive to order flow.

Our model also provides insights into a puzzling empirical feature of derivative security markets — despite the existence of a widespread desire to hedge against long-term risk, longer-term option and futures contracts have little liquidity and large spreads. For example, Fleming and Sarkar (1999) found that 90% of total trading volume in Treasury Futures in 1993 occurred in the *nearby* contract (the contract with expiration
month closest to the trading date), and that distant contracts generally had far lower trading volumes and larger realized spreads. Comparable rises in open interest and volume as time-to-maturity falls are found in commodity-based futures contracts and other derivative security contracts, (e.g. longer-term options such as LEAPs (Long-term Equity AnticiPation Securities)).

Heuristic industry evidence suggests that the strategy of sequentially rolling over nearby futures contracts is quite common. Rolling over shorter-term positions, often referred to as “rolling the hedge,” has an additional cost over buying longer-term contracts — each time a position is rolled over, transactions costs (commissions, bid-ask spread) must be incurred. This paper details conditions under which liquidity traders chose to incur extra costs to roll over their short-term positions, rather than trade distant option and futures contracts. The intuition is that informed agents place a greater value on distant contracts because their information is more likely to be revealed before the contract expires, while liquidity traders value only the reduced roll-over costs. Provided fixed trading costs are small, the greater adverse selection costs in distant contracts more than offset the increased fixed trading costs, so that long-term liquidity traders prefer to trade nearby contracts. If all liquidity traders prefer the nearby contract, informed agents must also trade it.

Our model captures a variety of situations in which informed agents, with trading horizons extending beyond the date when their information will become public, trade in markets where they have to realize their position before this date. For example, our model captures an agent attempting to leverage up his capital by trading in options. Alternatively, our model captures futures trading in commodity markets, where an informed agent cannot readily take a position by purchasing the commodity (e.g. soybeans) and where futures contracts with longer time horizons either do not exist or have poor liquidity. So, too, our analysis is relevant for informed agents who want to short-sell and face margin requirements. If an informed agent has limited resources, and the price moves against him, he may have to close his position before his information is incorporated into price.

**Related Literature:** Our model shares features of the overlapping generations model proposed by Dow and Gorton (1994). Dow and Gorton endogenize the choice to act on private information made by informed agents with short-horizons and one trading opportunity. In their model, information is only revealed through trade. Hence, an informed agent trades only if he believes it sufficiently likely that a future informed agent will trade before he has to realize his position, so that prices will reflect his information. Because Dow
and Gorton allow an informed agent only one opportunity to trade, their model does not capture the essence of repeatedly trading short-lived securities when informed agents have long-term intrinsic private information. In particular, their informed agents do not internalize the effects of current trade on future trading opportunities, so that immediate trade is more attractive and private information becomes public sooner.

In contrast to the Dow and Gorton model, we consider a single informed agent contemplating purchases of a series of short-lived securities. We show that the informed agent’s trading decisions and profit depend subtly on both the likelihood of liquidity trade and how far the market maker’s competitive price is expected to diverge from its ‘fundamental’ value. We then extend the model to consider how the informed agent’s strategy depends on his accumulated position and the availability of contracts of different durations.

In another model with short trading horizons, De Long et al. (1990) offer a rationale for why risk averse arbitrageurs may have a limited willingness to bet against noise traders: a concern that noise trader beliefs may not revert to the true value for a long time. Even if there is no ‘fundamental’ risk associated with purchasing ‘under-priced’ stocks, ‘arbitrage’ may be imperfect because of this ‘noise trader’ risk.

Back (1993) integrates a long-lived call option into a continuous time Kyle (1985) model. Back shows that if trades to the option and the underlying asset convey different information, then the option cannot be spanned by the underlying asset (i.e. it is not redundant) and thus cannot be priced by arbitrage arguments. Indeed, the empirical observations that long-dated contracts typically have little volume and large bid-ask spreads can only be reconciled by the presence of market microstructure effects.

Unfortunately, one cannot modify Back’s model of a long-lived option to explain these empirical regularities and capture the strategic impacts of trading short-lived securities. Incorporating short-lived securities immediately entails consideration of how asset holdings affect strategic trading behavior. Also, a significant portion of the strategic tradeoff between long- and short-lived securities for both liquidity and informed agents concerns the avoidance of fixed trading costs associated with rolling over positions. The requirement that trade be normally distributed precludes the possibility that a Kyle-style model such as Back’s can capture the strategic tradeoff between higher fixed trading costs of rolling over shorter-term securities, and their reduced adverse selection costs.

Other models of informed trade in option markets include Biais and Hillion (1994) and John et al. (2001). Biais and Hillion consider a static model in which a single trader (either informed or a rational, risk
averse liquidity trader) chooses whether to trade the stock or the option. Adding the opportunity to trade options may reduce informed profits because of the effects on liquidity trading strategies. John et al.’s static model explores how margin requirements affect the choice by informed agents of which asset to trade.

Our model contrasts with models of informed trading in equity (e.g., Kyle (1985), Back et al. (2000), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996)) where informed agents have multiple opportunities to trade a long-lived asset, but do not need to realize positions before their information becomes public. The informed agents care about both current and future prices, but because they do not have to unwind positions there is no penalty to investing early; eventually stock prices reflect private information.

The paper is organized as follows. The next section sets out the basic model. Section 3 characterizes the informed agent’s equilibrium trading strategy and details how outcomes vary with the parameters describing the economy. Section 4 introduces short-lived contracts that exist for longer periods. First, we explore how the informed agent’s accumulated position affects his trading strategy. An informed agent with an accumulated position takes into consideration that submitting an order this period increases the probability that his information will be revealed and that his previously purchased contract will then be exercised at a profit. Next, we extend the analysis to consider: (i) longer-lived liquidity traders; and (ii) the choice between trading securities with different holding periods. We characterize how the time until expiry affects trading behavior, and show that when agents can choose whether to trade long- or short-lived securities, then for reasonable parameterizations of the economy, only shorter-lived securities are traded. Section 5 concludes.

2 The Basic Model

Consider a multi-period economy with two primitive assets: a riskless asset that returns \( r = 0 \), and a risky asset. At some distant date \( T \), the value of the risky asset will be \( v \in \{ B, G \}, G > B \). Ex ante the good state \( (v = G) \) and the bad state \( (v = B) \) occur with equal probability. Let \( i = T - t \) be the number of periods remaining until date \( T \): Period \( i \) corresponds to the date with \( i \) periods remaining until date \( T \).

There are three types of market participants: (1) a risk neutral, competitive market maker; (2) short-lived liquidity traders who arrive randomly; and (3) an informed agent. With probability \( \delta \), an informed agent has private information about the risky asset: he knows whether the realization was \( G \) or \( B \). The true future
state may be revealed to the public in period \( i \) in two ways: either by the equilibrium trading order flow (in a manner detailed below); or by an information leak, which occurs with probability \( \lambda_i. \)\(^1\) As period 0 approaches, the asset’s value is more likely to be revealed to the public, \( \lambda_i \leq \lambda_{i-1}. \) Let \( \theta_i \in \{b, n, g\} \) reflect whether bad news (b), no news (n), or good news (g) leaked out at period \( i. \)

Market participants can trade a series of one-period European binary call options. The option is available for purchase at the beginning of each period \( i \) at price \( P_{o,i} \) and expires at the end of period \( i \) for a payoff of \( P_{c,i}. \) At the beginning of period \( i, \) given current and past order flows, the competitive market maker assigns a probability \( \beta_{o,i} \) to the good state. She then prices the option sold at the beginning of period \( i \) at

\[
P_{o,i} = \begin{cases} 
1 & \text{if } \beta_{o,i} = 1 \\
\beta_{o,i} \lambda_i & \text{if } \beta_{o,i} < 1. 
\end{cases}
\]

At the end of period \( i, \) the market maker updates her belief about the good state to \( \beta_{c,i} \) to reflect the possible leakage of private information during the period. The payoff at expiration to an option that matures at the end of period \( i \) is:

\[
P_{c,i} = \begin{cases} 
1 & \text{if } \theta_i = g \\
0 & \text{if } \theta_i = n \text{ or } \theta_i = b. 
\end{cases}
\]

We focus on binary options for the same reason that Dow and Gorton (1994) consider assets that pay off zero or one — they reduce algebra and the qualitative predictions extend to more general short-lived contracts (e.g., standard options, futures contracts). Specifically, the binary options are in the money if and only if the private information is revealed before they expire. In this way we capture the essence of trading short-lived securities written on longer-lived assets, while circumventing the need to model trade in multiple markets at each date. Section 4 extends the model to allow for multiple securities of different durations, and the choice of which security to trade.

All agents have a sufficient endowment that they are not wealth constrained in equilibrium. Agents incur a transaction cost of \( c \geq 0 \) when buying or selling the short-term security. This fixed trading cost represents brokerage fees plus time costs, etc. Without loss of generality, we assume that agents can only submit orders in round (integer) lots (this assumption never impacts on equilibrium outcomes).

\(^1\)An alternative interpretation of our economic environment is that the risky asset pays a dividend of \( B \) or \( G \) at a random time between dates \( t = 0 \) and \( t = T. \) The informed agent knows the dividend realization but not the timing of the dividend payment. Then \( \lambda_i \) corresponds to the probability the dividend is paid at period \( i, \) conditional on the dividend not previously being paid.
The basic model only considers the “buy” side of the market.\textsuperscript{2} As a result, the subgame equilibrium when the bad state occurs just has the informed agent declining to place an order. Since we want to consider situations where the informed agent may trade, without loss of generality, we focus on the case where the good state occurs.

It is important to emphasize that we can extend our model to incorporate the “sell” side of the market. This has a non-trivial effect on outcomes, but \textit{not} on any of our qualitative findings. If market participants could trade on both sides of the market, then for some parameterizations the informed agent might want to (probabilistically) trade against his information in order to manipulate market maker beliefs. In equilibrium, however, the market maker accounts for this possibility, revising her beliefs less dramatically in response to an order flow of two. Allowing for this possibility does not otherwise change the model’s qualitative predictions, but it does complicate informed agent strategies and reduce the clarity with which private information is revealed through trade. It is for these reasons that we restrict attention to the “buy” side.

\textbf{Short-lived liquidity traders:} Each period $i$, a short-lived liquidity trader enters the market with probability 0.5 to place a buy order of size one for the short-term security. The liquidity trader is uninformed and trades only once. With equal probability no short-lived liquidity trader trades in period $i$. Let $Z_i \in \{0, 1\}$ represent the short-lived liquidity trader’s period $i$ order. This stark contrast between high noise trading (one order) and low noise trading (no orders) allows the model to capture simply the feature that with positive probability the market maker will detect the informed agent’s presence.

We assume that the probability of liquidity trade is one-half for the same reasons that Dow and Gorton (1994) do: it permits us to characterize equilibrium outcomes analytically. For other liquidity trading probabilities, analytical (closed-form) solutions for the equilibrium do not obtain, and numerical characterizations are required; Bernhardt \textit{et al.} (2002) provide those characterizations. When the probability of liquidity trade is one-half, market maker beliefs about the probability of the good state do not change following an order flow of one: $\beta_i(\beta_{i+1}, Y_i = 1) = \beta_{i+1}$.

\textbf{Informed agent:} There is at most one privately-informed agent in the market. The informed agent can trade the short-term security as often as he wishes. Of course, given positive transactions costs, he will not...
trade once his information becomes public. The informed agent and the possible liquidity trader submit their orders simultaneously. A risk neutral competitive market maker observes the aggregate order flow and sets a price equal to the conditional expected value of the short-term security given her information.

**Sequence of Events:** The sequence of events during period $i$ is as follows:

1. The market maker enters with a prior $\beta_{c,i+1}$ that reflects past trade and announcements.

2. The informed agent, having observed past order flows and public announcements, selects a trading probability. With probability one-half, a liquidity trader also submits an order.

3. The market maker observes total order flow, updates her beliefs to $\beta_{o,i}$, and sets an opening option price of $P_{o,i}$.

4. During the period, the informed agent’s private information may or may not be revealed. This information is revealed publicly with probability $\lambda_i$.

5. At period’s end, the market maker updates her prior ($\beta_{c,i}$) to reflect whether the private information was revealed publicly.

6. Outstanding options either are exercised or expire. If the good state was revealed, agents can exercise the options (and receive 1 for each option) or sell them at their closing price, $P_{c,i} = 1$. If the good state was not revealed, the options expire worthlessly, $P_{c,i} = 0$.

**Equilibrium:** The total period $i$ order flow for the short-lived security is $Y_i = X_i + Z_i$. Let $H_i = \{Y_T, Y_{T-1}, ..., Y_i\}$ and $\Theta_i = \{\theta_T, \theta_{T-1}, ..., \theta_i\}$ denote respectively, the period $i$ history of past order flows and past public announcements.

The order submission function, $\Pr \{X_i = x_i \mid v, H_{i+1}, \Theta_{i+1}\}$, is a period strategy for the informed agent, mapping the date $T$ asset value and history of order flows and announcements into a probability distribution over the set of feasible individual orders for the short-lived security for each period $i$. A period strategy for the market maker is a pair of pricing functions, $P_{o,i}(H_i, \Theta_{i+1})$ and $P_{c,i}(H_i, \Theta_i)$, for the open and close of trading respectively, that map the order flow and public announcement histories into prices.

In a sequentially rational (perfect Bayesian) equilibrium: (i) the informed agent’s order submission strategy maximizes (recursively) lifetime expected profits given correct beliefs about pricing functions; and
(ii) the pricing function is consistent with the behavior of the informed and earns the market maker zero expected profits conditional on the order flow.

We solve for the equilibrium recursively. If the total order flow in the market exceeds one or is a non-integer quantity, the market maker knows that the informed agent traded. Hence, if the informed agent submits an order, he will try to conceal his trade from the market maker by placing an order for one round lot: in equilibrium, \( X_i \in \{0, 1\} \). For simplicity, let \( \chi_i = \Pr\{X_i = 1\} \) denote the probability that the informed agent submits an order for one round lot at period \( i \); \( 1 - \chi_i \) is the probability that the informed agent defers from trading.

Given competitive pricing, in equilibrium, the history of order flow and public announcements through period \( i \) can be summarized by the market maker’s belief at the end of period \( i \) that the good state will occur, \( \beta_{c,i} \). Since the informed agent only trades when \( v = G \), the informed agent’s strategy can be summarized by \( \chi_i(\beta_{c,i+1}) \). In equilibrium, the market maker’s pricing strategy at period \( i \) can also be summarized by \( P_{o,i}(\beta_{c,i+1}, Y_i) \) and \( P_{c,i}(\beta_{c,i+1}, Y_i, \theta_i) \).

We now develop the economy formally. We first derive the beliefs the market maker forms about the probability that the good state will occur. At the beginning of period \( T \), market maker beliefs correspond to the ex ante probability that the good state occurs: \( \beta_{c,T+1} = 0.5 \). At any period \( i \leq T \), the market maker will receive either zero, one or two orders as illustrated in figure 1. In equilibrium, market maker beliefs will reflect the equilibrium trading probabilities of the informed agent.

A direct application of Bayes’ rule shows that the market maker assigns equilibrium probability

\[
\beta_{o,i}(\beta_{c,i+1}, Y_i) = \begin{cases} 
\frac{1 - \delta \chi_i(\beta_{c,i+1})}{1 - \delta \chi_i(\beta_{c,i+1}) \beta_{c,i+1}} & \text{if } Y_i = 0 \\
\beta_{c,i+1} & \text{if } Y_i = 1 \\
1 & \text{if } Y_i = 2 
\end{cases}
\]  

(2)

to the good state.\(^3\) It is important to emphasize that the updating rules reflect the equilibrium outcome resulting from the consistency of market maker beliefs and the informed agent’s actions. The probability \( \beta_{o,i}(\beta_{c,i+1}, Y_i) \) determines the opening price at which the contract is purchased in period \( i \). The closing

\(^3\)For example, this reflects that there are three possible situations in which no orders will be submitted: (a) with probability, \( 0.5\delta(1 - \beta_{c,i+1}) \), no liquidity trader enters and the informed sees bad news; (b) with probability, \( 0.5\delta \beta_{c,i+1}[1 - \chi_i(\beta_{c,i+1})] \), no liquidity trader enters and the informed agent receives good news but refrains from trading; and (c) with probability \( 0.5(1 - \delta) \), no liquidity trader enters and there is no informed agent.
price reflects whether the true state was revealed publicly. If the information is not revealed, \( \beta_{c,i} = \beta_{o,i} \). Otherwise, \( \beta_{c,i} = 1 \) if the good state is revealed and \( \beta_{c,i} = 0 \) if the bad state is revealed. To reduce notation, in what follows we denote \( \beta_{c,i} \) as simply \( \beta_i \).

### 3 Informed Agent’s Problem

By solving the informed agent’s dynamic programming problem that determines his trading decisions, we can characterize equilibrium outcomes when we substitute in consistent beliefs of the market maker. The analysis exploits the fact that prices are higher when the market maker believes the informed agent is more likely to trade, making trading less attractive. The functional equation governing the informed agent’s expected trading profits as a function of market maker beliefs is

\[
V_i(\beta_i+1) = \max_{X_i \in \{0, 1\}} E\left[ \pi_i(\beta_{i+1}, Y_i) + V_{i-1}(\beta_i(\beta_{i+1}, Y_i)) \right], \quad \text{s.t. } Y_i = X_i + Z_i,
\]

where \( \pi_i \) are period payoffs and \( V_0(\cdot) = 0 \). In equilibrium, the informed agent’s mixed trading strategy \( \chi_i \) corresponds to the probability that the market maker assigns to an informed investor with good news trading. If equilibrium is characterized by mixing, we can solve for the equilibrium value of \( \chi_i \) by finding the market maker’s belief about the trading probability implicit in the updating rule \( \beta_i(\beta_{i+1}, Y_i) \) that leaves the informed agent indifferent between trading and not.

The value of the informed agent’s private information with \( i \) periods remaining until his private information is sure to be revealed publicly is:

\[
V_i(\beta_i+1) = 0.5(1 - \lambda_i)V_{i-1}(\beta_{i+1}) + 0.5 \max_{X_i \in \{0, 1\}} \left[ \lambda_i(1 - \beta_{i+1}) - 2c, (1 - \lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0)) \right], \quad (3)
\]

where \( V_0 = 0 \). In equilibrium, the market maker’s beliefs must be consistent with the informed agent’s period trading strategies. This may require that the informed agent use a mixed trading strategy. This reflects the fact that \( \beta_i(\beta_{i+1}, Y_i = 0) \) decreases in \( \chi_i \). As a result, the market maker’s belief about the probability of the good state falls more in response to an order flow of zero when the informed agent has a pure strategy to buy than when he has a pure strategy to defer. The equilibrium is characterized by a non-degenerate mixed trading strategy when:

\[
(1 - \lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0, \chi_i = 1)) > \lambda_i(1 - \beta_{i+1}) - 2c > (1 - \lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0, \chi_i = 0)).
\]
If the informed agent mixes in equilibrium, he must be indifferent between trading and not:

\[ \lambda_i(1 - \beta_{i+1}) - 2c = (1 - \lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0)). \]  \hfill (4)

In period 1, the right hand side of (4) is simply zero and the value function is independent of \( \chi_1 \), so it is easy to compute the informed agent’s equilibrium trading probabilities: \( \chi_1(\beta_2) = 0 \) if \( \beta_2 > 1 - \frac{2c}{\lambda_1} \), and \( \chi_1(\beta_2) = 1 \) if \( \beta_2 < 1 - \frac{2c}{\lambda_1} \). In period 2, we can show that the more likely the market maker believes the good state is, the less likely the informed agent is to trade on his (good) private information:

**Lemma 1** The probability that the informed agent with good news trades with two periods remaining, \( \chi_2(\beta_3) \), decreases in \( \beta_3 \); strictly decreasing with \( \beta_3 \) if it is interior, \( 0 < \chi_2(\beta_3) < 1 \).

Before extending lemma 1 to the general period \( i \) case, it is useful to characterize how informed trading intensities vary as time passes. One might conjecture that the informed agent would trade more aggressively as the number of remaining trading opportunities falls. The following example illustrates that the analysis is more subtle. Suppose that \( \beta = 0.5, \delta = 0.4, c = 0.1, \lambda_1 = 0.8, \lambda_2 = 0.5, \) and \( \lambda_3 = 0.49 \). Then, \( \chi_3(\beta) = 0.897 > 0 = \chi_2(\beta) \); trading intensities do not rise uniformly as the end of the trading horizon nears. Intuitively, because information is far more likely to become public in period 1 than period 2, (i.e., \( \lambda_1 >> \lambda_2 \)), the gain from deferring in period 2 in hopes of profiting on trade in period 1 is high. Information, however, is only marginally more likely to be revealed at the end of period 2 than period 3, so in period 3 the choice boils down to a choice between trading in period 3 or waiting until period 1. But since his information is sufficiently likely to become public before period 1, it does not pay to defer in period 3.

This non-monotonicity appears to arise because the probability of public information revelation evolves in a convex pattern. Indeed, if the probability that the information leaks out does not vary over time, then the informed agent’s trading intensity must rise as date \( T \) is approached:

**Proposition 1** Let \( \lambda_i = \lambda, \forall i. \) Then in any period \( i \), the informed agent trades with positive probability if and only if the price is not too close to its fundamental value, i.e., \( \bar{\beta} < 1 - \frac{2c}{\lambda} \). If \( \bar{\beta} < 1 - \frac{2c}{\lambda} \), then the informed agent is more likely to submit an order as date \( T \) approaches: \( \chi_i(\bar{\beta}) \geq \chi_{i+1}(\bar{\beta}) \), and \( \chi_i(\bar{\beta}) > \chi_{i+1}(\bar{\beta}) \) if \( \chi_{i+1}(\bar{\beta}) < 1 \). Hence, as date \( T \) approaches, equilibrium prices become more sensitive to order flow.
Underlying this result is Lemma 2 that we prove in the appendix. The lemma details that if the probability of information leakage does not vary over time, then the informed agent’s expected profits rise with the number of periods available to trade on his information: \( V_i(\beta) \leq V_{i-1}(\beta) \). Thus, Proposition 1 can be interpreted as follows. When \( \lambda_i = \lambda, \forall i \), the expected return from submitting an order, \( 0.5\lambda(1 - \beta) - c \), does not vary over time. Accordingly, to keep the informed agent indifferent between trading and not as time passes, continuation profits from deferring must not fall even though fewer opportunities remain in which to benefit from any price improvement gained by deferring. Hence, as time passes, the price must fall by more following an order of zero: market maker beliefs must be revised downward by more following an order of zero. In turn, more dramatic price revisions require that the informed agent trade more aggressively on his information as date \( T \) approaches, so that a zero order flow conveys more information.

We now extend lemma 1 to the general period \( i \) case. When the probability of information leakage jumps in a non-convex way, it is difficult to rule out the possibility of multiple solutions to (4) and hence the possibility that the informed agent’s trading intensity rises with an increase in the market maker’s prior. To preclude this possibility, we assume that

**Assumption (A1):** \( \chi_i(\beta) > 0 \) implies that \( \chi_j(\beta) > 0, \forall j < i \).

Proposition 1 ensures that this assumption is satisfied if \( \lambda_i = \lambda, \forall i \). Assumption A1 precludes multiple solutions to (4). It allows us to extend lemma 1 and characterize the informed agent’s equilibrium trading strategy at all dates. The next proposition details that the more likely the market maker believes the good state is, the less likely the informed agent is to trade:

**Proposition 2** The equilibrium probability with which the informed agent submits an order, \( \chi_i(\beta_{i+1}) \), declines monotonically in \( \beta_{i+1} \); strictly decreasing with \( \beta_{i+1} \) for \( \chi_i(\beta_{i+1}) \in (0, 1) \).

This result is subtle. The more likely the market maker believes the good state is, the smaller are period trading profits. However, future market maker beliefs about the good state will also be higher if information is not revealed, so future trading profits are lower. Hence, to prove proposition 2, we must show that current profits fall more rapidly with \( \beta \) than do continuation profits. The assumption that \( \chi_i(\beta) > 0 \) implies that \( \chi_j(\beta) > 0, j < i \) ensures that continuation payoffs following an order flow of zero are a concave function of \( \beta \), while trading profits are a linear function of \( \beta \). Then, *ceteris paribus*, increasing \( \beta \) reduces the value.
the informed agent places on trading relative to manipulating market maker beliefs. To keep the informed agent indifferent, market maker beliefs must fall by less after an order flow of zero—so that the informed agent must be less likely to trade.

Corollary 1 characterizes when the informed agent will submit an order with positive probability.

**Corollary 1** A sufficient condition for there to be a non-trivial range of market maker beliefs, for which the informed agent trades with positive probability at period $i$ is:

$$
\lambda_i - 2c > \sum_{j=1}^{i-1} (0.5)^{i-j} (\lambda_j - 2c) \left( \prod_{k=j+1}^{i} (1 - \lambda_k) \right)
$$

and $\lambda_1 > 2c$. If $\lambda_i = \lambda, \forall i$, then the sufficient condition reduces to $\lambda > 2c$ — the likelihood that the private information is revealed in the next period must exceed twice the fixed trading costs.

We next characterize how the parameters describing the economy affect informed trading.

**Proposition 3**

- The more likely the informed agent’s information is to be revealed publicly, the more aggressively he trades: $\chi_i$ is weakly increasing in $\lambda_i$, strictly increasing for $\chi_i \in (0, 1)$. If $\lambda_i = \lambda \forall i$, then $\chi_i$ is weakly increasing in $\lambda$.

- The greater are fixed trading costs, the less aggressively the informed agent trades: $\chi_i$ is decreasing in $c$, strictly decreasing for $\chi_i \in (0, 1)$.

- The more likely there is to be an informed trader, the less aggressively the informed agent trades: $\chi_i$ is weakly decreasing in $\delta$, strictly decreasing for $\chi_i \in (0, 1)$.

These findings are intuitive. As $\lambda_i$ rises, the informed agent’s information is more likely to be impounded into the closing period $i$ price, raising the expected return from a period $i$ contract. Further, an increase in $\lambda_i$ reduces the expected gain from deferring from trade, as the informed agent’s information is less likely to remain private. Less obviously, this relation still holds when the probability that the private information is revealed does not vary with time. There are three direct effects: an increase in $\lambda$, (1) increases the informed agent’s expected return from submitting an order this period; and (2) reduces the expected return
from deferring since it increases the probability that information will be revealed before next period; but the potentially offsetting effect, is (3) if the information is not revealed this period, the expected return from submitting an order next period rises. We show, however, that the first two effects dominate the third.

Higher fixed trading costs \( (c) \) reduce the expected return from submitting an order. When \( c \) is high, the benefit from trading is low unless the price deviates substantially from the true value. As \( c \) rises, the informed agent is more likely to defer from trading, hoping to obtain lower prices next period, which would raise the net profit margin.

As \( \delta \) increases, the market maker believes that an informed agent is more likely, so her beliefs about the likelihood of a good state fall more sharply following an order flow of zero. Hence, as \( \delta \) increases, it becomes more profitable for the informed agent to defer from trading.

4 Time-to-Maturity

Until now, we have considered only short-dated contracts that expire after one period. In practice, options and futures contracts exist for various lengths of time. As a contract’s time until expiry increases, the model tends to that of a long-lived trader in equity (Kyle 1985), with agents able to hold positions for as long as it takes the market maker to learn the asset’s true value. In contrast to Kyle, in our model agents may not commence trading once they receive (private) good news: if the price of the asset is too close to its fundamental value an informed agent prefers to defer in the hope of manipulating the market. This result reflects that our model allows for non-convexities — fixed trading costs and round lot restrictions. Kyle’s normality assumptions preclude consideration of these important non-convexities, making submitting a sufficiently small order more attractive than completely delaying trade.

The next two sections explore different aspects of how a contract’s time-to-maturity affects outcomes. Section 4.1 details that if security contracts exist for multiple periods then the informed agent’s accumulated position affects his trading strategy. Section 4.2 provides an explanation for why markets for shorter-term contracts are far more liquid than those for longer-term contracts.
4.1 Accumulated Position

An informed agent with an accumulated position takes into consideration that submitting an order for a contract raises the probability that his information will be revealed, in which case his existing contracts will then be settled for a profit. This situation does not arise in equity because, as long as an informed agent can hold his stake, his information will eventually be incorporated into the price of a previously-accumulated position. In contrast, with short-term contracts there is a risk that the contract may expire before the information is revealed publicly. Consequently, increasing the probability that the information will be revealed has a positive value, a value that rises with the informed agent’s accumulated position.

An implication of Proposition 3 is that the longer is the holding period, the more aggressively the informed agent trades. That is, an increase in $\lambda$ captures the effects of a longer holding period, ignoring the impact of an informed agent’s accumulated position on his trading behavior. We now show that an informed agent’s accumulated position further increases the informed agent’s trading intensity.

We document this in the simplest possible context. We consider the last two periods remaining for a long-dated contract written at the beginning of period T and expiring at the end of period 1. We assume that $\lambda_1 < 1$, so that the informed agent’s accumulated position could expire before his information is incorporated into the underlying asset price. Each period, market participants can purchase contracts that expire at the end of period 1. Each period, with probability one-half, a liquidity trader places an order.

The informed agent’s accumulated position at the end of period $i+1$ is $\sum_{\iota=i+1}^{T} X_\iota$. The informed agent’s period $i$ trading strategy, which depends on the size of his accumulated position, is a probability distribution

$$\Pr \left\{ X_i = x_i \mid v, H_{i+1}, \Theta_{i+1}, \sum_{\iota=i+1}^{T} X_\iota \right\}$$

over feasible order sizes. In equilibrium, any informed order of size $X_i \notin \{0, 1\}$ reveals the informed agent’s information to the market maker, in which case the price reflects his information. Hence, without loss of generality, we need only consider the following informed period trading strategies: (1) submit an order of size two that reveals his information; (2) submit an order of size one; (3) defer from trading. The associated expected period $i$ payoffs are:

$$E \left[ \pi_i (X_i \mid H_{i+1}, \sum_{i=i+1}^{T} X_\iota) \right] = \begin{cases} 
(\sum_{\iota=i+1}^{T} X_\iota) - c & \text{if } X_i = 2 \\
0.5 (\lambda_i - P_i (Y_i = 1 | H_{i+1})) - c + (0.5 + 0.5\lambda_i) \sum_{\iota=i+1}^{T} X_\iota & \text{if } X_i = 1 \\
\lambda_i \sum_{\iota=i+1}^{T} X_\iota & \text{if } X_i = 0
\end{cases}$$
The period $i$ value function is therefore:

$$V_i\left( H_{i+1}, \sum_{\ell=i+1}^{T} X_{\ell} \right) = \max_{X_i \in \{0, 1, 2\}} \mathbb{E} \left[ \pi_i \left( Y_i | H_{i+1}, \sum_{\ell=i+1}^{T} X_{\ell} \right) + (1 - \lambda_i) V_{i-1} \left( H_i(X_i), \sum_{\ell=i}^{T} X_{\ell} \right) \right],$$

where $V_0 = 0$.

**Proposition 4** For any given market maker beliefs, $\beta_i$, the expected size of the informed agent’s order in period $i$, $i = 1, 2$ increases with his accumulated position.

Proposition 4 reveals that holding both the informed agent’s information and total past (informed plus liquidity) trade the same, future expected informed trade and hence future expected volume will be greater when the informed agent has a greater share of past trade. That is, the market maker updates prices in the same manner independently of who submitted the orders, but the informed agent trades more aggressively in the future if he traded more aggressively in the past and hence acquired a greater stake.

### 4.2 Liquidity of Distant Contract Markets

This section investigates why option and futures contracts that are relatively close to maturity are far more liquid than similar contracts with more distant expiration dates. This empirical regularity is especially puzzling given the many reasons why agents might want to use longer-term contracts to hedge against long-term risk. To do this, we extend the two-period model as follows:

- Each period, two contracts are available: a 1-period (*short-dated*) contract and a 2-period (*long-dated*) contract. The market maker observes total order flows in both markets.

- As in the basic model, each period an *exogenous* short-lived liquidity trader arrives with independent probability 0.5 and trades only once. Now, unlike the basic model, this trader randomly submits an order for either the long-dated contract or the short-dated contract with equal probability. This structure ensures that the exogenous probability of liquidity trade is the same in each contract market.

- At the beginning of period 2, a *long-lived* liquidity trader arrives with independent probability 0.5. The trading behavior of this trader is *endogenous*: he can hedge against an income shock that is negatively correlated with the good state in one of two ways: (1) buy a long-dated contract at period 2; (2) buy
a short-dated contract at period 2, and if uncertainty about the state is not resolved, purchase a short-dated contract at period 1. Let $\xi$ be the probability that the liquidity trader buys a long-dated contract.

The trader selects the hedging strategy that minimizes his expected cost.

Thus, each period there are three potential traders: an informed agent, an endogenous liquidity trader, and an exogenous liquidity trader. To focus on the long-term composition of trade in long- and short-dated contracts, we assume that private information is revealed with certainty at the end of period 1, $\lambda_1 = 1$. As a result, the informed agent has no incentive to submit a large order in period 1 in order to reveal information. This allows us to focus on the choice between trading contracts of different maturities without complicating the analysis with the issues related to the informed’s accumulated position that we detail in section 4.1.

We index short-dated contract variables by $S$ and long-dated contract variables by $L$. Bold symbols denote vectors: for example, $Y_i = (Y_i^S, Y_i^L)$ denotes total order flows in period $i$ for short- and long-dated contracts. The history of total order flow through period $i$ is: $H_i = \{Y_i^S, Y_{i-1}^S, ..., Y_i^L, Y_{i-1}^L, ..., Y_i^L\}$. The informed agent’s period $i$ strategy is a joint probability distribution over feasible orders, $\Pr\{X_i = (x_i^S, x_i^L) | v, X_{i+1}, H_{i+1}, \Theta_{i+1}\}$. The market maker selects a set of pricing functions for each contract at the open and close, $P_{io}^j(H_i, \Theta_{i+1}), P_{ic}^j(H_i, \Theta_i), j = S, L$. The price for each contract depends on order flow in both markets. Because order flow at period 2 provides the market maker information about the presence of a long-lived liquidity trader, the market maker’s updating rule at period 1 depends distinctly on both her original prior and observed past order flow.

Since $\lambda_1 = 1$, orders received at period 1 for the long- and short-dated contracts are equivalent ($X_1 = X_1^S = X_1^L$). The informed agent’s value function in the last period is:

$$V_1(Y_2, X_2) = \max_{X_1} E[\pi_1(X_1, Y_2, X_2)]$$

$$= \max_{X_1} \begin{cases} 
X_2^L & \text{if } X_1 = 0 \\
X_2^L + 0.5(1 - \Sigma)(2 - \beta_1(1, (Y_2)) - \beta_1(2, (Y_2))) + 0.5\Sigma(1 - \beta_1(2, (Y_2))) - c & \text{if } X_1 = 1 \\
X_2^L + (1 - \Sigma)(1 - \beta_1(2, (Y_2))) - c & \text{if } X_1 = 2 \\
X_2^L - c & \text{if } X_1 > 2,
\end{cases}$$
where $\Sigma = \Sigma(Y_2, X_2)$ is the probability with which the informed agent believes a long-lived liquidity trader purchased the short-dated contract.\footnote{If three orders in aggregate of short- and/or long-dated contracts are received, private information is revealed with certainty.}

The possibility of two liquidity traders in either market means that the informed agent can submit an order of size two, or submit orders to both markets, without being revealed for sure to the market maker. However, lemmas 4 and 5 in the appendix show that it is never optimal for the informed agent to do so. This result holds in period 1 even though the informed agent may know from the period 2 order flow net of his trade that there is no long-lived liquidity trader, so that the maximum liquidity trade is one, whereas the market maker cannot make such a distinction. Restricting attention to the informed agent’s three possible equilibrium period 2 trading strategies, his period 2 value function is:

$$V_2 = \max_{X_2} E \left[ \pi_2(X_2) + (1 - \lambda_2)V_1(Y_2, X_2) \right],$$

where $E[\pi_2(X_2)] = \begin{cases} 0 & \text{if } X_2 = (0, 0) \text{ (defer from trading)} \\ \lambda_2 - \frac{1}{8} [2 + (1 + 2\xi)P^T(0, 2) + (3 - 2\xi)P^T(1, 1) + 2P^T(0, 1)] - c & \text{if } X_2 = (0, 1) \text{ (buy one long-dated)} \\ \lambda_2 - \frac{1}{8} [2 + (1 + 2\xi)P^S(1, 1) + (3 - 2\xi)P^S(2, 0) + 2P^S(1, 0)] - c & \text{if } X_2 = (1, 0) \text{ (buy one short-dated)} \end{cases}$

In the appendix, we detail how the market maker updates beliefs in response to different order flows. Figure 2 illustrates the period 2 strategies from the market maker’s perspective.

Analytical characterizations are difficult because of the interaction between market maker beliefs and the strategies of the endogenous long-lived liquidity trader and the informed agent. This leads us to describe outcomes numerically. Figure 3 presents a surface diagram of the percentage of trade in long-dated contracts,

$$100(0.25 + 0.5\xi + \delta\Pr\{X^S_2 = 0, X^L_2 = 1\})/(1 + \delta(\Pr\{X^S_2 = 0, X^L_2 = 1\} + \Pr\{X^S_2 = 1, X^L_2 = 0\})),$$

for $c = 0.05$ and different values of $\delta$ and $\lambda_2$. For most values of $\delta$ or $\lambda_2$, most trade occurs in short-dated contracts. Long-dated contracts draw a majority of trade only when both $\delta$ is very small (private information
is unlikely) and $\lambda_2$ is very small (so a liquidity trader is still likely to need to hedge at period 1). Short-dated contracts draw most trade despite relatively high fixed costs, which increase the relative cost of rolling over nearby contracts. Similar outcomes hold for values of $c \leq 0.05$. Note that $c = 0.05$ represents fixed trading costs equal to 5\% of the maximum value of the contract, and more than 10\% of the maximum potential profit—so that fixed costs above 5\% are unreasonable. Thus, we find that for reasonable parameterizations, trade is concentrated in short-dated (nearby) contracts.

To understand better why trade concentrates in short-dated contracts in equilibrium, we explore how a long-lived liquidity trader’s expected total trading cost varies as we exogenously alter the probability with which he submits an order for the long-dated contract, and then compute the consistent optimal informed trading strategies and “equilibrium” zero-profit pricing. Figure 4 shows that for almost all realistic parameter ranges, the long-lived liquidity trader’s expected costs are minimized if he primarily trades short-dated contracts. Figure 5 provides the economic intuition: as liquidity trade, $\xi$, in long-dated contracts is increased, the informed agents trade the long-dated contract with increasing probability (given the “equilibrium” pricing associated with higher liquidity trade).

Long-dated contracts allow the informed agent either to accumulate a larger position or to trade less frequently (reducing the probability of being uncovered), so that adverse selection costs faced by a long-lived liquidity trader tend to rise with the probability that he hedges using the long-dated contract. Notice, however, that the liquidity trader’s expected costs are not minimized by always buying the short-dated contract ($\xi = 0$). This is because a small increase in $\xi$, say from 0 to 0.03, “raises” the (consistent) likelihood with which the informed agent trades the long-dated contract from 0.303 to 0.369. This highlights the tradeoff faced by the long-lived liquidity trader: raising $\xi$ improves prices in the short-dated contract by reducing adverse selection costs in those contracts; but raising $\xi$ also “causes” the long-lived liquidity trader to buy more long-dated contracts, which are more expensive (informed trade in these contracts is more likely). The added cost of the long-dated contracts quickly dominates as $\xi$ rises.

Figures 4 and 5 reflected parameterizations for which the informed agent always traded in period 2, mixing only over which contract to trade. Now consider parameterizations where the informed agent may defer from trading at period 2 in equilibrium, such as when $\lambda_2$ is small. In this situation, the informed mixes between deferring from trade and submitting an order for the long-dated contract. Now, if the long-
lived liquidity trader increases the frequency with which he trades the short-dated contract (i.e., reduces \( \xi \)), this may cause the informed agent to raise the probability that he defers from trading. Thus, for these parameterizations, the long-lived liquidity trader often optimally trades only the short-dated contract.

Figure 6 illustrates how the long-lived liquidity trader’s mixing probability varies with the fixed trading cost, \( c \). This figure clearly illustrates the liquidity trader’s preference for a “pooling” equilibrium outcome. For small \( c \), the liquidity trader almost always buys the short-dated contract. As \( c \) increases, the liquidity trader first begins to trade slightly more of the long-dated contract, but as \( c \) rises marginally further, he dramatically changes and trades long-dated contracts almost exclusively. Intuitively, once \( c \) is sufficiently high, the added trading cost from buying two short-dated contracts exceeds the higher adverse selection costs in the long-dated contract.

Figure 7 illustrates how the percentage of total trade in long-dated contracts varies with \( c \) when \( \delta = 0.4 \) and \( \lambda_2 = 0.8 \). For this parameterization, the percentage of trade in long-dated contracts rises with the fixed trading cost \( c \), as one might expect. Figure 8, however, reveals that trade in long-dated contracts need not always rise with \( c \). Figure 8 details outcomes when \( \lambda_2 = 0.2 \) so that private information is unlikely to be revealed at period 2. If \( c \) is small, raising \( c \) reduces trade in long-dated contracts. It is possibilities such as this that preclude analytical characterizations. Indeed, neither total expected trade across contracts, nor total expected trade in the long-dated contract need be monotone in \( c \). To understand why, consider first how liquidity trade varies with \( c \), and then consider how informed trade varies.

The long-lived liquidity trader’s strategy, \( \xi \), is generally insensitive to small changes in fixed trading costs, because he prefers to trade almost exclusively in one of the two contracts. From the long-lived liquidity trader’s perspective, increasing \( c \) raises the attractiveness of trading long-dated contracts in terms of transaction costs. For low values of \( c \), this benefit fails to offset the far higher adverse selection costs associated with moving away from a pooling equilibrium in short-dated contracts toward trading both short- and long-dated contracts, nor is it sufficient to prompt the liquidity trader to trade only the long-dated contracts. There exists a very narrow range of \( c \) such that over this range the transaction costs dominate adverse selection costs and the liquidity trader switches from almost always trading short-dated contracts to almost always trading long-dated contracts. For other values of \( c \), changes in the share of trade in short- and long-dated contracts are driven by changes in the informed agent’s behavior.
The informed agent’s behavior depends critically on whether or not his information is likely to be revealed at period 2. Consider fixed trading costs $c$ for which the long-lived liquidity trader almost always trades the short-dated contract and consider the impact of raising $c$ for:

**Case 1: information is likely to be revealed (large $\lambda_2$):** Then, the informed agent always trades at period 2, mixing between the short- and long-dated contracts. The high probability of information leakage at period 2 raises the attractiveness of trading, reducing the distinction between short- and long-dated contracts. When fixed transaction costs rise, the informed agent switches from trading short-dated contracts to trading long-dated contracts. As a result, the percentage of trade in long-dated contracts rises.

**Case 2: information is unlikely to be revealed (small $\lambda_2$):** When $\lambda_2$ is sufficiently low, then independent of market maker’s beliefs, it is never optimal for the informed agent to submit an order for the short-dated contract: if information is unlikely to be revealed during period 2, the short-dated contract purchased at period 2 is likely to expire worthlessly. When fixed transaction costs rise, the informed agent switches from trading long-dated contracts to deferring from trade at period 2. The net effect is that the percentage of trade in long-dated contracts falls.

Thus, higher fixed trading costs may not increase the share of trade in long-dated contracts. Finally, to understand why total expected trade need not fall as fixed trading costs rise, suppose that $\lambda_2$ is small, and $c$ is low enough that liquidity traders trade the short-dated contract. Then as $c$ rises, the informed agent increasingly defers from trading (as information is unlikely to be revealed), reducing total expected trade. But eventually, $c$ rises by enough that long-lived liquidity traders switch to trading long-dated contracts. Once $c$ is high enough that long-lived liquidity traders almost always trade long-dated contracts, adverse selection costs for these contracts fall so that it suddenly becomes profitable for the informed agent to trade long-dated contracts heavily; and as a result total trade rises.

These findings provide insights into the patterns of trade in long- and short-dated contracts. During the past few years, a large number of long-dated contracts have been introduced in an attempt to satisfy a demand by market participants to hedge long-term risk. Despite active promotion by exchanges and over-the-counter dealers, however, most long-dated contract markets are highly illiquid. Our characterizations reveal that for reasonable levels of fixed trading costs, long-dated contract markets are likely to thrive only in environments where informational asymmetries are slight. Thus, markets where there are minimal infor-
national asymmetries (e.g. weather derivatives) might be expected to have liquid markets at longer horizons; but markets with substantial informational asymmetries (e.g. equity derivatives) should not.

Finally, our results suggest that if exchanges hope to influence equilibrium outcomes by changing fixed trading costs, then the impact on liquidity may be more subtle than they might anticipate. To whit, when information is likely to remain privately held, increasing relative trading costs in short-dated contracts may raise trade in short-dated contracts because it reduces the attractiveness to informed agents of trading on long-term information, thereby lowering adverse selection costs.

5 Conclusion

Despite its importance, strategic trading of short-lived securities, such as option or futures contracts, has largely been ignored by the academic literature. This paper documents important differences between the strategic trading of short-lived securities and that of equity:

1. The shorter horizon in which information must be impounded for a short-lived security to pay off makes an informed agent more reluctant to trade, especially when the informed’s information is longer-term in nature. Given innocuous technical conditions, informed trading intensities rise over the trading horizon so that prices become more sensitive to order flow.

2. With short-lived securities, the greater a risk neutral informed agent’s holdings of the short-lived security, \textit{ceteris paribus}, the more aggressively he trades in the future. In contrast, in equity markets, an informed agent’s accumulated position does not affect his trading behavior.

3. For reasonable parameter ranges, liquidity traders prefer to incur extra costs to roll over their short-term positions rather than trade in distant contracts, precisely because equilibrium adverse selection costs are smaller in shorter-contracts. This allows us to reconcile the puzzling empirical finding that markets for longer-term contracts have little liquidity and large spreads.
6 Appendix

6.1 Derivations for Section 4.2

To clarify the exposition in section 4.2, we omitted the details regarding the market maker’s updating rules. For completeness, we present those updating rules here. We adopt the notation:

\[
\begin{align*}
\rho_N &= \Pr\{X^S_2 = 0, X^L_2 = 0\} = \text{probability informed agent defers from trade at period 2}; \\
\rho_L &= \Pr\{X^S_2 = 0, X^L_2 = 1\} = \text{probability informed agent buys a long-dated contract at period 2}; \\
\rho_S &= \Pr\{X^S_2 = 1, X^L_2 = 0\} = \text{probability informed agent buys a short-dated contract at period 2}.
\end{align*}
\]

If the market maker observes three (or more) orders in aggregate \((Y^S_2 + Y^L_2 \geq 3)\) in period 2, she knows the good state occurred and updates her beliefs accordingly \((\beta_2 = 1)\). The remainder of the period 2 market maker updating rules are:

\[
\begin{align*}
\beta_2(0, 0) &= \frac{1 - \delta}{2 - \delta} \\
\beta_2(1, 0) &= \frac{(3 - 2\xi)(1 - \delta + \delta \rho_N) + 2\delta \rho_S}{(3 - 2\xi)(2 - \delta + \delta \rho_N) + 2\delta \rho_S} \\
\beta_2(0, 1) &= \frac{(1 + 2\xi)(1 - \delta + \delta \rho_N) + 2\delta \rho_L}{(1 + 2\xi)(2 - \delta + \delta \rho_N) + 2\delta \rho_L} \\
\beta_2(1, 1) &= \frac{1 + 2\delta(\xi \rho_S + (1 - \xi)\rho_L)}{2 + 2\delta(\xi \rho_S + (1 - \xi)\rho_L)} \\
\beta_2(2, 0) &= \frac{(3 - 2\xi)\delta \rho_S + (1 - \xi)(1 - \delta + \delta \rho_N)}{(3 - 2\xi)\delta \rho_S + (1 - \xi)(2 - \delta + \delta \rho_N)} \\
\beta_2(0, 2) &= \frac{2\xi \rho_L + \xi \delta \rho_N + \xi(1 - \delta) + \delta \rho_L}{2\xi \rho_L + \xi \delta \rho_N + \xi(2 - \delta) + \delta \rho_L}
\end{align*}
\]

The market maker’s period 1 updating rules reflect the fact that period 2 order flow provides information about the existence of the long-lived liquidity trader:

\[
\begin{align*}
\beta_1(0, (\cdot, \cdot)) &= \frac{1 - \delta}{2 - \delta} \\
\beta_1(1, (0, 0)) &= \frac{1 - \delta(\rho_L + \rho_S)}{2 - \delta(\rho_L + \rho_S)} \\
\beta_1(1, (0, 1)) &= \frac{1 + 2\xi - \delta[(1 + 2\xi)(1 - \rho_N) - 2\rho_L]}{2 + 2\xi - \delta[(1 + 2\xi)(1 - \rho_N) - 2\rho_L]} \\
\beta_1(1, (1, 0)) &= \frac{2(1 - \xi)(1 - \delta) + 2\delta \rho_S + \delta \rho_N + 1 - \delta}{2(1 - \xi)(1 - \delta) + 2\delta \rho_S + \delta \rho_N + 1 - \delta} \\
\beta_1(1, (1, 1)) &= \frac{1 - \delta[(1 + \xi)\rho_N - 2\xi \rho_S]}{2 - \delta[(1 - \xi)\rho_N - 2\xi \rho_S]}
\end{align*}
\]
\[ \beta_1(1, (0, 2)) = \frac{2\xi\delta \rho_L + \xi \delta \rho_N + \xi (1 - \delta) + \delta \rho_L}{2\xi\delta \rho_L + \xi \delta \rho_N + \xi (2 - \delta) + \delta \rho_L} \]

\[ \beta_1(1, (2, 0)) = \frac{(1 - \xi)(1 - \delta) + \delta \rho_S}{(1 - \xi)(2 - \delta) + \delta \rho_S} \]

\[ \beta_1(2, (1, 1)) = \frac{2\xi\delta \rho_S + 3\delta \rho_L - 2\xi\delta \rho_L + \delta \rho_N + (1 - \xi)(1 - \delta) + \delta \rho_S}{2\xi\delta \rho_S + 3\delta \rho_L - 2\xi\delta \rho_L + \delta \rho_N + (1 - \xi)(2 - \delta) + \delta \rho_S} \]

\[ \beta_1(2, (2, 0)) = \frac{2(1 - \xi)\delta \rho_S + (1 - \xi)\delta \rho_N + \delta \rho_S + (1 - \xi)(1 - \delta)}{2(1 - \xi)\delta \rho_S + (1 - \xi)\delta \rho_N + \delta \rho_S + (1 - \xi)(2 - \delta)} \]

\[ \beta_1(2, (1, 0)) = \frac{(1 - \xi)\delta \rho_N + (1 - \xi)(1 - \delta) + \delta \rho_S + 0.5\delta \rho_N}{(1 - \xi)\delta \rho_N + (1 - \xi)(2 - \delta) + \delta \rho_S + 0.5\delta \rho_N} \]

If the market maker received three or more orders in aggregate in either period 2 or period 1, then she assigns \( \beta_1 = 1 \). There are three additional order flow combinations that reveal the existence of the informed agent to the market maker:

\[ \beta_1(2, (0, 0)) = \beta_1(2, (0, 1)) = \beta_1(2, (0, 2)) = 1 \]

Endogenous liquidity trader’s expected cost from buying a long-dated contract at period 2:

\[
E\left[ C^L \right] = 0.25 \left[ \delta (1 - \rho_N) + (1 + 0.5\delta \rho_N - 0.5\delta)(\beta_2(0, 2) + \beta_2(1, 1) + 2\beta_2(0, 1)) + \delta \rho_L \beta_2(0, 2) + \delta \rho_S \beta_2(1, 1) \right] + c
\]

Endogenous liquidity trader’s expected cost from buying a short-dated contract each period:

\[
E\left[ C^S \right] = 0.25 \left[ \delta (1 - \rho_N) + (1 + 0.5\delta \rho_N - 0.5\delta + \delta \rho_L) \beta_2(1, 1) \lambda_2 + (1 + 0.5\delta \rho_N - 0.5\delta) \beta_2(2, 0) \lambda_2 + 2(1 + 0.5\delta \rho_N - 0.5\delta) \beta_2(1, 0) \lambda_2 \right] + c + 0.125(1 - \lambda_2) \left[ 0.5\delta (\rho_N + 2\rho_L) + (0.5\delta \rho_N + 1 - 0.5\delta + \delta \rho_L) \beta_1(2, (1, 1)) + (1 - 0.5\delta) \beta_1(1, (1, 1)) + 0.5\delta (\rho_N + 2\rho_S) + (0.5\delta \rho_N + 1 - 0.5\delta + \delta \rho_S) \beta_1(2, (2, 0)) + (1 - 0.5\delta) \beta_1(1, (2, 0)) + \delta \rho_N + 2(1 - 0.5\delta + 0.5\delta \rho_N) \beta_1(2, (1, 0)) + 2(1 - 0.5\delta + 0.5\delta \rho_N) \beta_1(1, (1, 0)) \right] + (1 - \lambda_2)(1 - 0.25\delta (\rho_S + \rho_L))c
\]

Endogenous liquidity trader’s expected cost of hedging: \( E\left[ C^T \right] = \xi E\left[ C^L \right] + (1 - \xi) E\left[ C^S \right] \).

### 6.2 Proofs

*Proof of Lemma 1:* Let \( LHS \) and \( RHS \) be the values of the left- and right-hand sides of equation (4). Clearly, \( LHS < 0 \) for \( \beta_3 > 1 - 2c\lambda_2^{-1} \). Since \( RHS \geq 0 \), any solution to (4), if one exists, must occur for \( \beta_3 < 1 - 2c\lambda_2^{-1} \). Intuitively, the informed agent only submits an order with positive probability if the
expected one-period return from doing so is positive. We know: (a) \( \beta_2(\beta_3^2, Y_2 = 0) < \beta_3 \); (b) \( \lambda_1 \geq \lambda_2 \).

Hence, for \( \beta_3 \in \left(0, 1 - 2c\lambda_2^{-1}\right) \), it must be that \( 0.5\lambda_1(1 - \beta_2(\beta_3, Y_2 = 0)) - c > 0 \) and the RHS can be expanded as:

\[
(1 - \lambda_2)[0.5\lambda_1(1 - \beta_2(\beta_3, Y_2 = 0)) - c].
\]

Holding \( \chi_2 \) constant, the first derivative of (5) with respect to \( \beta_3 \) is

\[-0.5\lambda_1(1 - \lambda_2)(1 - \chi_2\delta)(1 - \chi_2\delta\beta_3)^{-2} < 0\]

and the second derivative of (5) with respect to \( \beta_3 \) is

\[-\lambda_1(1 - \lambda_2)\chi_2\delta(1 - \chi_2\delta)(1 - \chi_2\delta\beta_3)^{-3} < 0.\]

Thus, RHS is strictly concave for \( \beta_3 \in \left(0, 1 - \frac{2c}{\lambda_2}\right) \). The derivative of LHS with respect to \( \beta_3 \) is \( -\lambda_2 < 0 \). Hence, there is at most one solution to (4) for \( \beta_3 \in [0, 1] \). Holding \( \chi_2 \) “fixed” at a solution to (4) at period 2, it follows that LHS must fall more quickly with an increase in \( \beta_3 \) than RHS. Thus, to preserve equality, since LHS is independent of \( \chi_2 \) and RHS rises with \( \chi_2 \), the mixing probability \( \chi_2 \) must fall with \( \beta_3 \), falling strictly for \( \chi_2 \in (0, 1) \). ■

**Lemma 2** If \( \lambda_i = \lambda \forall i \), then \( V_i(\beta) \geq V_{i-1}(\beta) \forall \beta \in [0, 1], \forall i. \)

**Proof of Lemma 2:** Case 1: If \( \beta \geq 1 - 2c\lambda^{-1} \), then \( V_i(\beta) = (1 - \lambda)^{i-1}V_1(\beta) = 0 \). Then \( V_i(\beta) = V_{i-1}(\beta). \)

Case 2: If \( \beta < 1 - 2c\lambda^{-1} \), then \( V_i(\beta) = 0.5(1 - \lambda)V_{i-1}(\beta) + 0.5[\lambda(1 - \bar{\beta}) - 2c] \) and \( V_{i-1}(\beta) = 0.5(1 - \lambda)V_{i-2}(\beta) + 0.5[\lambda(1 - \bar{\beta}) - 2c] \). It follows that \( V_i(\beta) > V_{i-1}(\beta) \) if and only if \( V_{i-1}(\beta) > V_{i-2}(\beta) \).

Since \( V_1 = 0.5[\lambda(1 - \bar{\beta}) - 2c] > V_0 = 0 \), the result follows from induction. ■

**Proof of Proposition 1:** The mixing probability \( \chi_i(\bar{\beta}) \) solves \( \lambda(1 - \bar{\beta}) - 2c = (1 - \lambda)V_{i-1}(\beta_i(\bar{\beta}, 0)) \). The proposition has two parts (A and B):

**Part A:** We show that \( \bar{\beta} < 1 - 2c\lambda^{-1} \) is necessary and sufficient for \( \chi_i(\bar{\beta}) > 0 \).

**Necessity:** If \( \bar{\beta} \geq 1 - 2c\lambda^{-1} \), then \( \lambda(1 - \bar{\beta}) - 2c < 0 \). Since \( V_{i-1}(\cdot) \geq 0 \forall i \), the informed agent defers.

**Sufficiency:** Proof by induction.

**Period 1:** If \( \beta < 1 - 2c\lambda^{-1} \), then \( 0.5\lambda(1 - \bar{\beta}) - c > 0 \) and the informed agent submits an order.

**Period 2:** If \( \bar{\beta} < 1 - 2c\lambda^{-1} \) and \( \chi_1(\bar{\beta}) = 1 \), then \( \lambda(1 - \bar{\beta}) - 2c > \lambda(1) = (1 - \lambda)0.5[\lambda(1 - \bar{\beta}) - 2c] \).

24
That is, if the market maker believes that the informed agent will defer, then the informed agent’s expected return from submitting an order exceeds his expected continuation payoff. Thus, in equilibrium, \( \chi_2(\bar{\beta}) > 0 \).

**Arbitrary period** \( i \): If \( \bar{\beta} < 1 - \frac{2c}{\lambda} \) and \( \chi_j(\bar{\beta}) > 0 \) \( \forall j < i \), then

\[
\lambda(1 - \bar{\beta}) - 2c > (1 - \lambda)V_i(\bar{\beta}) = [\lambda(1 - \bar{\beta}) - 2c] \sum_{p=1}^{i-1} [0.5(1 - \lambda)]^p,
\]

since \( 1 > \sum_{p=1}^{i-1} [0.5(1 - \lambda)]^p \). Hence, \( \chi_i(\bar{\beta}) > 0 \).

**Part B:**

Case 1: If \( \chi_i(\bar{\beta}) = 1 \), then it follows immediately that \( \chi_i(\bar{\beta}) \geq \chi_{i+1}(\bar{\beta}) \).

Case 2: If \( \chi_i(\bar{\beta}) = 0 \), then from part A it follows that \( \lambda(1 - \bar{\beta}) - 2c < 0 \) and \( \chi_{i+1}(\bar{\beta}) = 0 \).

Case 3: If \( \chi_i(\bar{\beta}) \in (0, 1) \) then either \( \chi_{i+1}(\bar{\beta}) \in (0, 1) \) or \( \chi_{i+1}(\bar{\beta}) = 0 \). Then

\[
(1 - \lambda)V_i(\beta_i(\bar{\beta}, 0)) = (1 - \lambda)V_i(\beta_{i+1}(\bar{\beta}, 0)) = \lambda(1 - \bar{\beta}) - 2c
\]

Hence, \( V_{i+1}(\beta_i(\bar{\beta}, 0)) = V_i(\beta_{i+1}(\bar{\beta}, 0)) \). From lemma 2, this occurs only if \( \beta_i(\bar{\beta}, 0) < \beta_{i+1}(\bar{\beta}, 0) \). Since

\[
\frac{\partial \beta_i(\bar{\beta}, 0)}{\partial \chi_i} \leq 0,
\]

it follows that \( \chi_i(\bar{\beta}) > \chi_{i+1}(\bar{\beta}) \).

**Proof of Proposition 2:** Interior values for period \( i \) trading probabilities \( \chi_i(\beta_{i+1}) \) must solve equation (4). Let \( LHS(i) \) and \( RHS(i) \) be the period \( i \) values of the left- and right-hand sides of (4). Clearly, \( LHS(i) < 0 \) for \( \beta_{i+1} > 1 - \frac{2c}{\lambda_i} \). Since \( RHS(i) \geq 0 \), any solution to (4) must occur for \( \beta_{i+1} < 1 - \frac{2c}{\lambda_i} \). We know: (a) market maker beliefs are non-increasing in response to aggregate order flows of 0 or 1 \( (\beta_i(\beta_{i+1}, Y_i \in \{0, 1\}) < \beta_{i+1} \ \forall i) \); and (b) the probability of information leakage is non-decreasing, \( \lambda_i \geq \lambda_{i+1} \). Hence, for \( \beta_{i+1} \in \left(0, 1 - \frac{2c}{\lambda_i}\right), \)

\[
0.5\lambda_i (1 - \beta_j (\beta_{i+1}, \{Y_j, Y_{j+1}, ..., Y_i\})) - c > 0, \ \forall j < i.
\] (6)

Using an induction argument and (A1), we now show that if \( \chi_j \) declines with \( \beta_{j+1}, \forall j < i \), then \( \chi_i \) falls with \( \beta_{i+1} \).

**Periods 1 and 2:** Immediate. \( \chi_1 \) must fall with \( \beta_2 \) since

\[
\chi_1(\beta_2) = \begin{cases} 
1 & \text{for } \beta_2 < 1 - \frac{2c}{\lambda_1} \\
0 & \text{for } \beta_2 > 1 - \frac{2c}{\lambda_1} \\
[0, 1] & \text{for } \beta_2 = 1 - \frac{2c}{\lambda_1}
\end{cases}
\]
We also showed in lemma 1 that $\chi_2$ fell with $\beta_3$.

**Period 3:** For $\beta_4 \in \left(0, 1 - \frac{2\lambda_3}{\lambda_4} \right)$, expand $RHS(i = 3)$ as:

$$
.5(1 - \lambda_3) \left[ (1 - \lambda_2) (0.5\lambda_1(1 - \beta_3(\beta_4, 0)) - c) 
+ \max \left\{ \lambda_2(1 - \beta_3(\beta_4, 0)) - 2c, (1 - \lambda_2)(0.5\lambda_1(1 - \beta_2(\beta_3(\beta_4, 0), 0)) - c) \right\} \right],
$$

using (6). Let $\beta_4 \in (0, \beta_4^*)$ denote the range of $\beta_4$ such that $\chi_3(\beta_4) > 0$. From assumption (A1), if $\chi_3(\beta_4) > 0$, then $\chi_2(\beta_4) > 0$. Further $\chi_3(\beta_4) > 0$ implies $\chi_2(\beta_3(\beta_4, 0)) > 0$ because: (a) $\beta_3(\beta_4, 0) \leq \beta_4$; and (b) $\chi_2(\beta)$ is falling in $\beta$. Hence, we need only consider (7) for $\beta_4$ corresponding to $\chi_2(\beta_3(\beta_4, 0)) > 0$.

Since $LHS(i = 3) > RHS(i = 3)$ for $\beta_4 \geq 1 - \frac{2\lambda_3}{\lambda_4}$, (A1) implies that if $\chi_3(\beta_4) > 0$ then $LHS(i = 3) > RHS(i = 3)$ for $\beta_4 > \beta_4^*$. Given these observations and the fact that the derivative of $LHS(i = 3)$ with respect to $\beta_4$ is constant and equal to $-\lambda_3 < 0$, a sufficient condition to ensure at most one solution exists to (4) evaluated at period 3 is that

$$
(1 - \lambda_3) \left[ 0.5(1 - \lambda_2)(0.5\lambda_1(1 - \beta_3(\beta_4, 0)) - c) + 0.5\lambda_2(1 - \beta_3(\beta_4, 0)) - c \right]
$$

be strictly concave. Expression (8) corresponds to $RHS(i = 3)$ evaluated over $\beta_4 \in (0, \beta_4^*)$. Holding $\chi_3$ constant, the first derivative of (8) with respect to $\beta_4$ is

$$
-(1 - \lambda_3)[0.25(1 - \lambda_2)\lambda_1 + 0.5\lambda_2] \frac{\partial \beta_3(\beta_4, 0)}{\partial \beta_4} < 0
$$

and the second derivative is

$$
-(1 - \lambda_3)[0.25(1 - \lambda_2)\lambda_1 + 0.5\lambda_2] \frac{\partial^2 \beta_3(\beta_4, 0)}{\partial \beta_4^2} < 0.
$$

Thus $RHS(i = 3)$ is strictly concave for $\beta_4 \in (0, \beta_4^*)$ and there is at most one solution to (4) at period 3. Holding $\chi_3$ “fixed” at a solution to (4) at period 3, it follows that $LHS(i = 3)$ falls more quickly with an increase in $\beta_4$ than $RHS(i = 3)$. To preserve equality, since $LHS(i = 3)$ is independent of $\chi_3$ and $RHS(i = 3)$ rises with $\chi_3$, the mixing probability $\chi_3$ must decline with $\beta_4$, strictly falling if $\chi_3 \in (0, 1)$.

**Period i:** By induction. Suppose the informed’s mixing probability for periods $j < i$, falls with $\beta_{j+1}$.

Using the same logic as for period 3, we show that if $\chi_i(\beta_{i+1}) > 0$, then $\chi_j(\beta) > 0$, $\forall \beta \leq \beta_{i+1}$, $\forall j < i$.

Let $\beta_{i+1}^*$ be the maximum $\beta_{i+1}$ such that $\chi_i(\beta_{i+1}) > 0$. Hence, possible solutions for the period $i$ analog of equation (4) are in some range $\beta_{i+1} \in (0, \beta_{i+1}^*)$. Expanding $RHS(i)$ yields,

$$
\sum_{p=1}^{i-1} \left( (0.5)^{i-p}(\lambda_p(1 - \beta_p(\beta_{i+1}, 0)) - 2c) \prod_{k=p+1}^{i} (1 - \lambda_k) \right).
$$

(9)
The first derivative of (9) with respect to \( \beta_{i+1} \) is given by

\[
- \sum_{p=1}^{i-1} \left( (0.5)^{i-p} \left( \lambda_p \frac{\partial \beta_i(\beta_{i+1}, 0)}{\partial \beta_{i+1}} \right) \left[ \prod_{k=p+1}^{i} (1 - \lambda_k) \right] \right) < 0.
\]

The second derivative of (9) with respect to \( \beta_{i+1} \) is given by

\[
- \sum_{p=1}^{i-1} \left( (0.5)^{i-p} \left( \lambda_p \frac{\partial^2 \beta_i(\beta_{i+1}, 0)}{\partial \beta_{i+1}^2} \right) \left[ \prod_{k=p+1}^{i} (1 - \lambda_k) \right] \right) < 0.
\]

Thus, (9) is strictly concave over \( \beta_{i+1} \in (0, \beta^*_i) \). Since the derivative of \( LHS(i) \) with respect to \( \beta_{i+1} \) is constant and equal to \(-\lambda_i < 0\) and since, by assumption, \( LHS(i) > RHS(i) \) if \( \chi_j(\beta) = 0 \) for \( j < i \) and \( \beta < \beta_{i+1} \), there can be no more than one solution to (4) in period \( i \). Holding \( \chi_i \) “fixed” at a solution to (4) at period 3, it follows that \( LHS(i) \) falls more quickly with an increase in \( \beta_{i+1} \) than \( RHS(i) \). To preserve equality, since \( LHS(i) \) is independent of \( \chi_i \) and \( RHS(i) \) rises with \( \chi_i \), the mixing probability \( \chi_i \) must decrease with \( \beta_{i+1} \), strictly decreasing for \( \chi_i \in (0, 1) \).

**Proof of Corollary 1:** Clearly, the \( RHS(i) \) and \( LHS(i) \) of (4) are continuous and decreasing in \( \beta_{i+1} \), and \( LHS(i) > RHS(i) \) for \( \beta_{i+1} \geq 1 - \frac{2c}{\lambda_i} \). Hence, a sufficient condition for the informed agent to trade with positive probability at period \( i \) is that \( RHS(i) > LHS(i) \) evaluated at \( \beta_{i+1} = 0 \):

\[
\lambda_i - 2c > \sum_{p=1}^{i-1} \left( (0.5)^{i-p} (\lambda_p - 2c) \left[ \prod_{k=p+1}^{i} (1 - \lambda_k) \right] \right)
\]

\[\text{(10)}\]

for \( i > 1 \) and corresponds to \( \lambda_i > 2c \) for \( i = 1 \). Given (A1), if (10) holds at period \( i \), the informed agent will submit an order each period \( j, j \leq i \). When \( \lambda_i = \lambda \forall i \), (10) becomes

\[
\lambda - 2c > (\lambda - 2c) \sum_{p=1}^{i-1} [0.5(1 - \lambda)]^p.
\]

\[\text{(11)}\]

If \( \lambda - 2c > 0 \), this is always satisfied.

**Proof of Proposition 3 (Comparative Statics):**

Ia. **Change in \( \lambda_i \):** Interior values for the period \( i \) trading probabilities \( \chi_i(\beta_{i+1}) \) must solve (4). When \( \lambda_i \) decreases, \( LHS(i) \) decreases and \( (1 - \lambda_i) \) increases. In order to maintain equality, it follows that \( V_{i-1}(\beta_i(\beta_{i+1}, 0)) \) decreases and therefore \( \chi_i(\beta_{i+1}) \) decreases.
1b. Change in $\lambda$: The period $i$ analog to (4) for the case where $\lambda_i = \lambda$ is

$$
(1 - \beta_i) - 2c = (1 - \lambda)V_{i-1}(\beta_i(\beta_i+1,0)).
$$

(12)

Let $LHS^*(i)$ and $RHS^*(i)$ be the period $i$ values of the left-hand side and the right-hand side, respectively, of (12). Part A of proposition 1 implies that: (a) a solution to (12) can occur only for $\beta_i+1 \in \left(0, 1 - \frac{2c}{\lambda}\right)$; and (b) $\chi_j(\beta) > 0 \forall j \forall \beta < 1 - \frac{2c}{\lambda}$. Hence, for interior values of $\chi_i(\beta_i+1)$

$$
V_{i-1}(\beta_i(\beta_i+1,0)) = 0.5(1 - \lambda)V_{i-2}(\beta_i(\beta_i+1,0)) + 0.5\lambda(1 - \beta_i(\beta_i+1,0)) - c.
$$

The proof proceeds as follows:

**Period 1:** Recall that $\chi_1(\beta_2) = 1$ if $\beta_2 < 1 - \frac{2c}{\lambda}$ and $\chi_1(\beta_2) = 0$ if $\beta_2 > 1 - \frac{2c}{\lambda}$. Since $\frac{\partial}{\partial \lambda} \left[1 - \frac{2c}{\lambda}\right] = \frac{2c}{\lambda^2} > 0$, it follows that $\chi_1(\beta_2)$ is weakly increasing in $\lambda$.

**Period 2:** Observe that

$$
\lambda \frac{\partial LHS^*(i=2)}{\partial \lambda} - (1 - \lambda)c = \lambda(1 - \beta_3) - (1 - \lambda)c \geq \lambda(1 - \beta_3) - 2c = (1 - \lambda)V_i(\beta_2(\beta_3, 0)),
$$

and

$$
\lambda \frac{\partial RHS^*(i=2)}{\partial \lambda} - (1 - \lambda)c = \lambda \left[(1 - \lambda)\frac{\partial V_1(\beta_2(\beta_3,0))}{\partial \lambda} - V_1(\beta_2(\beta_3, 0))\right] - (1 - \lambda)c
$$

$$
< \lambda(1 - \lambda)\frac{\partial V_1}{\partial \lambda} - (1 - \lambda)c = (1 - \lambda)\left[0.5\lambda(1 - \beta_2(\beta_3, 0)) - c\right] = (1 - \lambda)V_i(\beta_2(\beta_3, 0)).
$$

It follows that $\frac{\partial LHS^*(i=2)}{\partial \lambda} > \frac{\partial RHS^*(i=2)}{\partial \lambda}$. To maintain the equality, $\chi_2(\beta_3)$ must increase.

**Period $i$:** The argument used for period 2 is extended to any period $i$. Define

$$
a_i = \sum_{k=1}^{i-1} 0.5^{k-1}(1 - \lambda)^k < 2 \quad i = 2, 3, ..., T.
$$

Observe that

$$
\lambda \frac{\partial LHS^*(i)}{\partial \lambda} - a_i c = \lambda(1 - \beta_{i+1}) - a_i c \geq \lambda(1 - \beta_{i+1}) - 2c = (1 - \lambda)V_{i-1}(\beta_i(\beta_{i+1}, 0)),
$$

and

$$
\lambda \frac{\partial RHS^*(i)}{\partial \lambda} - a_i c = (1 - \lambda) \left[\sum_{p=1}^{i-1} [0.5(1 - \lambda)]^{p-1}[0.5\lambda(1 - \beta_i(\beta_{i+1}, 0)) - V_{i-1-p} - c]\right]
$$

$$
< (1 - \lambda) \left[\sum_{p=1}^{i-1} [0.5(1 - \lambda)]^{p-1}[0.5\lambda(1 - \beta_i(\beta_{i+1}, 0)) - c]\right] = (1 - \lambda)V_{i-1}(\beta_i(\beta_{i+1}, 0)).
$$

It follows that $\frac{\partial LHS^*(i)}{\partial \lambda} > \frac{\partial RHS^*(i)}{\partial \lambda}$. To maintain the equality, $\chi_i(\beta_{i+1})$ must increase. Thus, $\chi_i(\beta_{i+1})$ is weakly increasing in $\lambda$.

2. Change in $c$: Interior values of the mixing probability $\chi_i$ are defined by (4). Differentiating $LHS(i)$ and
RHS(i) of (4) with respect to c:

\[ \frac{\partial \text{RHS}(i)}{\partial c} \bigg|_{\chi_i} = \frac{\partial V_{i-1}}{\partial c} \bigg|_{\chi_i} \geq -1 \times \frac{-2}{1 - \lambda_i} = \frac{\partial \text{LHS}(i)}{\partial c}. \]

To restore equilibrium, RHS(i) must decrease. Hence, \( \chi_i \) must fall.

3. Change in \( \delta \): For a given \( \chi_i \), an increase in \( \delta \) causes the market maker’s belief about the probability of the good state to fall more rapidly in response to observing an aggregate order flow of zero \( \left( \frac{\partial \beta_i(\beta_i + 1, 0)}{\partial \delta} \bigg|_{\chi_i} \leq 0 \right) \). As \( \delta \) increases, \( \beta_i(\beta_i + 1, 0) \) falls and \( V_i - 1(\beta_i(\beta_i + 1, 0)) \) increases. Hence, RHS(i) of (4) rises with \( \delta \). Since LHS(i) does not vary with \( \delta \), \( \chi_i \) must fall to restore the mixing equilibrium condition given by (4).

**Lemma 3**  In equilibrium, the informed agent never places an order of size two at period \( i, i \geq 2 \).

The payoff from submitting an order of size two at period \( i, i \geq 2 \), is \( \left( \sum_{i=1+1}^{T} X_i \right) - c \). The informed agent can realize a higher expected return by deferring each period from period \( i \) to period 2 and, if his information has not yet been revealed, submit an order of size two at period 1. This alternative strategy has expected return \( \sum_{i=1+1}^{T} X_i - c \left[ \prod_{j=2}^{(1 - \lambda_j)} \right] > \sum_{i=1+1}^{T} X_i - c, \lambda_j > 0, j = 2, ..., i. \)

**Proof of Proposition 4:** First observe that

\[
\frac{\partial E[\pi_1(X_1|H_2, \sum_{i=2}^{T} X_i)]}{\partial (\sum_{i=2}^{T} X_i)} = \begin{cases} 
1 & \text{if } X_1 = 2 \\
0.5 + 0.5\lambda_1 & \text{if } X_1 = 1 \\
\lambda_1 & \text{if } X_1 = 0
\end{cases}.
\]

(13)

Thus,

\[
\frac{\partial E[\pi_1(X_1 = 2|H_2, \sum_{i=2}^{T} X_i)]}{\partial (\sum_{i=2}^{T} X_i)} \geq \frac{\partial E[\pi_1(X_1 = 1|H_2, \sum_{i=2}^{T} X_i)]}{\partial (\sum_{i=2}^{T} X_i)} \geq \frac{\partial E[\pi_1(X_1 = 0|H_2, \sum_{i=2}^{T} X_i)]}{\partial (\sum_{i=2}^{T} X_i)}
\]

(strict when \( \lambda_1 < 1 \)). It follows that the informed agent is more likely to submit a larger order at period 1 as \( (\sum_{i=2}^{T} X_i) \) increases.

For period 2, Lemma 3 ensures that the informed agent never submits an order of size two. Conditional on \( \sum_{i=3}^{T} X_i \), the informed’s expected return from a round lot order at period 2 is

\[
\text{submit} \left( \sum_{i=3}^{T} X_i \right) = 0.5(1 - \lambda_2)V_1 \left( \left\{ Y_2 = 1, H_3 \right\}, \sum_{i=3}^{T} X_i + 1 \right) + 0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) - c + 0.5(1 + \lambda_2) \sum_{i=3}^{T} X_i,
\]

29
These two observations imply that

\[ \text{argmax} \left( \sum_{i=3}^{T} X_i \right) = 0.5(1 - \lambda_2) \left[ V_1 \left( \left\{ Y_2 = 1, H_3 \right\}, \sum_{i=3}^{T} X_i \right) + V_1 \left( \left\{ Y_2 = 0, H_3 \right\}, \sum_{i=3}^{T} X_i \right) \right] + \lambda_2 \sum_{i=3}^{T} X_i. \]

If \( \text{submit}(Q) - \text{defer}(Q) \geq \text{submit}(\tilde{Q}) - \text{defer}(\tilde{Q}) \), for all \( Q > \tilde{Q} \), then the relative value of submitting an order is weakly increasing in the size of the informed agent’s accumulated position. Rewriting this condition yields

\[
\left( [V_1(1, Q + 1) - V_1(1, \tilde{Q} + 1)] - [V_1(0, Q) - V_1(0, \tilde{Q})] \right) + \left( Q - \tilde{Q} - [V_1(1, Q) - V_1(1, \tilde{Q})] \right) \geq 0, \tag{14}
\]

where we suppress the order flow history, \( H_3 \), and only report \( Y_2 \) to conserve space.

Equation (13) shows that the magnitude of the increase in the value of the informed agent’s information at period 1, due to an increase in \( (\sum_{i=2}^{T} X_i) \), rises with \( X_1 \). It also ensures that for any given market maker beliefs, the informed agent is more likely to submit a larger order at period 1 as \( (\sum_{i=2}^{T} X_i) \) increases. These two observations imply that \( V_1(1, Q + 1) - V_1(1, \tilde{Q} + 1) \geq V_1(1, Q) - V_1(1, \tilde{Q}) \). To show that

\[ Q - \tilde{Q} \geq V_1(0, Q) - V_1(0, \tilde{Q}) \]

we first observe that equation (13) implies that \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] \geq \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] \). Consider each possible case:

1. If \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] = \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] \), then

\[
Q - \tilde{Q} \geq V_1(0, Q) - V_1(0, \tilde{Q}) = \begin{cases} 
\lambda_1(Q - \tilde{Q}) & \text{if } \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] = 0 \\
0.5(1 + \lambda_1)(Q - \tilde{Q}) & \text{if } \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] = 1 .
\end{cases}
\]

2. If \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] = 2 \) and \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] = 1 \), then

\[ Q - \tilde{Q} < 0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) + 0.5(1 + \lambda_1)\tilde{Q}. \]

Then \( V_1(0, Q) - V_1(0, \tilde{Q}) = Q - 0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) - 0.5(1 + \lambda_1)\tilde{Q} < Q - \tilde{Q}. \)

3. If \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] = 1 \) and \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] = 0 \), then

\[
0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) - c + 0.5(1 + \lambda)\tilde{Q} < \lambda\tilde{Q}
\]

\[ 0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) - c + 0.5(1 + \lambda)\tilde{Q} + 0.5(1 + \lambda)(Q - \tilde{Q}) - \lambda\tilde{Q} < Q - \tilde{Q} \]

Then \( V_1(0, Q) - V_1(0, \tilde{Q}) = 0.5\lambda_2 (1 - P_2(X_2 = 1, H_3)) - c + 0.5(1 + \lambda)Q - \lambda\tilde{Q} < Q - \tilde{Q}. \)
(4) Finally, if \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, Q)] = 2 \) and \( \text{argmax}_{X_1} E[\pi_1(X_1|H_2, \tilde{Q})] = 0 \), then \( \tilde{Q} - c - \lambda \tilde{Q} < 0 \). This implies directly that \( V_1(0, Q) - V_1(0, \tilde{Q}) = Q - c - \lambda \tilde{Q} < Q - \tilde{Q} \).

These observations ensure that condition (14) is satisfied, and hence, \( \chi_2(X_2 = 1|\sum_{i=3}^{T} X_i = Q, \cdot) \geq \chi_2(X_2 = 1|\sum_{i=3}^{T} X_i = \tilde{Q}, \cdot), \forall Q > \tilde{Q} \). \( \blacksquare \)

**Lemma 4** In equilibrium, the informed agent never submits an order for more than one contract at period 1.

\[
E[\pi_1(1, Y_2, X_2) - \pi_1(2, Y_2, X_2)] = (1 - \Sigma)(1 - 0.5\beta_1(1, Y_2) - 0.5\beta_1(2, Y_2)) + \Sigma(0.5 - 0.5\beta_1(1, Y_2)) - (1 - \Sigma)(1 - \beta_1(2, Y_2)) = 0.5(1 - \Sigma)(\beta_1(2, Y_2) - \beta_1(1, Y_2)) + 0.5\Sigma(1 - \beta_1(1, Y_2) > 0 \text{ where } \Sigma = \Sigma(Y_2, X_2). \]

The result then follows from \( \beta_1(2, Y_2) > \beta_1(1, Y_2) \) and \( 1 > \beta_1(1, Y_2) \). \( \blacksquare \)

**Lemma 5** In equilibrium, the informed agent never submits an order involving two contracts at period 2.

The more aggressive period 2 strategy is most profitable when \( \lambda_2 = 1 \) \( (i.e., \text{when continuation profits do not matter}) \). The following three cases show that strategies involving two contracts are still less profitable than strategies involving one contract, even for the most favorable parameter values.

**Case 1:**

\[
E[\pi_2(X_2 = (0, 2)) - \pi_2(X_2 = (0, 1))] = 1 + \frac{1}{8} [-10 + (3 - 2\xi)(\beta_2(1, 1) - \beta_2(0, 2)) + 2\beta_2(0, 1)].
\]

This is negative iff \( (3 - 2\xi)(\beta_2(1, 1) - \beta_2(0, 2)) < 2(1 - \beta_2(0, 1)) \). The desired result follows from:

\[
0.5(1 + 2\delta(\xi \rho_S + (1 - \xi)\rho_L)) < 2.5 \\
\Rightarrow 3(1 + 2\delta(\xi \rho_S + (1 - \xi)\rho_L)) < 2.5(1 + 2\delta(\xi \rho_S + (1 - \xi)\rho_L)) + 2.5.
\]

Rearranging, we have \( 3\beta_2(1, 1) < 2.5 \). Thus, given that \( \beta_2(0, 1) \leq \beta_2(0, 2) \) and \( \beta_2(0, 2) > 0.5 \), it follows that: \( 3\beta_2(1, 1) + 2\beta_2(0, 1) < 2 + 3\beta_2(0, 2) \Rightarrow 3(\beta_2(1, 1) - \beta_2(0, 2)) < 2(1 - \beta_2(0, 1)) \Rightarrow (3 - 2\xi)(\beta_2(1, 1) - \beta_2(0, 2)) < 2(1 - \beta_2(0, 1)).

**Case 2:**

\[
E[\pi_2(X_2 = (2, 0)) - \pi_2(X_2 = (1, 0))] = 1 + \frac{1}{8} [-10 + (1 + 2\xi)(\beta_2(1, 1) - \beta_2(2, 0)) + 2\beta_2(1, 0)].
\]
This is negative iff $(1 + 2\xi)(\beta_2(1, 1) - \beta_2(2, 0)) < 2(1 - \beta_2(1, 0))$. From case 1, we know that $3\beta_2(1, 1) < 2.5$. Since $\beta_2(1, 0) \leq \beta_2(2, 0)$ and $\beta_2(2, 0) > 0.5$, the desired result follows from: $3\beta_2(1, 1) + 2\beta_2(1, 0) < 2 + 3\beta_2(2, 0) \Rightarrow 3(\beta_2(1, 1) - \beta_2(2, 0)) < 2(1 - \beta_2(1, 0)) \Rightarrow (1 + 2\xi)(\beta_2(1, 1) - \beta_2(2, 0)) < 2(1 - \beta_2(1, 0))$.

**Case 3:**

$$E[\pi_2(X_2 = (1, 1)) - \pi_2(X_2 = (1, 0))] = (1 - c) + \frac{1}{8} [-10 + (3 - 2\xi)(\beta_2(2, 0) - \beta_2(1, 1)) + 2\beta_2(1, 0)].$$

This is negative iff $(3 - 2\xi)(\beta_2(2, 0) - \beta_2(1, 1)) < 2(1 - \beta_2(1, 0))$. This will always be true for $\xi > 0.5$.

For $\xi \leq 0.5$, the result follows from: $(3 - 2\xi)\delta \rho_S + (1 - \xi)(1 - \delta + \delta \rho_N) + 5\xi < 5$ implies $3((3 - 2\xi)\delta \rho_S + (1 - \xi)(1 - \delta + \delta \rho_N)) < 2.5((3 - 2\xi)\delta \rho_S + (1 - \xi)(1 - \delta + \delta \rho_N)) + 2.5(1 - \xi)$. Rearranging, we have $\beta_2(2, 0) < 2.5$.

Since $\beta_2(1, 1) \geq \beta_2(0, 1)$ and $\beta_2(1, 1) > 0.5$, it follows that: $3\beta_2(2, 0) + 2\beta_2(0, 1) < 2 + 3\beta_2(1, 1) \Rightarrow 3(\beta_2(2, 0) - \beta_2(1, 1)) < 2 - 2\beta_2(1, 0) \Rightarrow (3 - 2\xi)(\beta_2(2, 0) - \beta_2(1, 1)) < 2(1 - \beta_2(1, 0))$.■
References


6.3 Figures
Figure 1: Period trading strategies and total order flow based on the basic model outlined in section 2. The probabilities associated with each strategy reflect those used by the market maker to update her beliefs.
Figure 2: Period 2 trading strategies, total informed order flow and total liquidity order flow based on the model outlined in section 4.2. The probabilities associated with each strategy reflect those used by the market maker to update her beliefs.
Figure 3: Surface diagram of the percentage of trade in long-dated contracts given fixed transaction costs equal to $c = 0.05$ for different values of $\delta$ (the probability an informed agent has private information) and $\lambda_2$ (the probability of an information leakage in period 2). The percentage of trade in long-dated contracts is calculated as $\frac{100(0.25 + 0.5\xi + \delta \Pr\{X^S_2 = 0, X^L_2 = 1\})}{1 + \delta \Pr\{X^S_2 = 0, X^L_2 = 1\} + \Pr\{X^S_2 = 1, X^L_2 = 0\}}$. 

\[ \text{Figure 3: Surface diagram of the percentage of trade in long-dated contracts given fixed transaction costs equal to } c = 0.05 \text{ for different values of } \delta \text{ (the probability an informed agent has private information) and } \lambda_2 \text{ (the probability of an information leakage in period 2). The percentage of trade in long-dated contracts is calculated as } \frac{100(0.25 + 0.5\xi + \delta \Pr\{X^S_2 = 0, X^L_2 = 1\})}{1 + \delta \Pr\{X^S_2 = 0, X^L_2 = 1\} + \Pr\{X^S_2 = 1, X^L_2 = 0\}}. \]
Figure 4: Long-lived liquidity trader’s expected total cost \( E[C^T] \) as a function of his mixing probability \( \xi \).

The long-lived liquidity trader always purchases the short-dated contract when \( \xi = 0 \) and always purchases the long-dated contract when \( \xi = 1 \). Parameter values: \( \delta = .5, c = .01, \) and \( \lambda_2 = .8 \).
Figure 5: Informed agent’s probability of submitting an order to the long-dated contract market ($\Pr\{X_2^S = 0, X_2^L = 1\}$) as a "best response" to the long-lived liquidity trader’s mixing probability $\xi$. The long-lived liquidity trader always purchases the short-dated contract when $\xi = 0$ and always purchases the long-dated contract when $\xi = 1$. Parameter values: $\delta = .5$, $c = .01$, and $\lambda_2 = .8$. For these parameters, it is always optimal for the informed agent to trade at period 2.

Figure 6: **Liquidity trader’s preference for “pooling equilibrium”**. The long-lived liquidity trader’s mixing probability $\xi$ as a function of the fixed transaction cost $c$. The long-lived liquidity trader always purchases the short-dated contract when $\xi = 0$ and always purchases the long-dated contract when $\xi = 1$. Parameter values: $\lambda_2 = .8$, and $\delta = .4$. 

39
Figure 7: **High probability of information revelation at period 2.** The percentage of total order flow in long-dated contracts as a function of the fixed transaction cost $c$. Parameter values: $\lambda_2 = .8$, and $\delta = .4$.

Figure 8: **Low probability of information revelation at period 2.** The percentage of total order flow in long-dated contracts as a function of the fixed transaction cost $c$. Parameter values: $\lambda_2 = .2$, and $\delta = .4$. 

40