Sources of Over-Performance in Equity Markets:
Mean Reversion, Common Trends and Herding

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Carol Alexander
ISMA Centre, University of Reading, UK

Anca Dimitriu
ISMA Centre, University of Reading, UK
Abstract

In the field of optimisation models for passive investments, we propose a general portfolio construction model based on principal component analysis. The portfolio is designed to replicate the first principal component of a group of stocks, instead of a traditional benchmark, thus capturing only the common trend in the stock returns. The main advantage of this approach is that the reduction of the noise present in stock returns facilitates the replication task considerably and the optimal portfolio structure is very stable. We analyse the portfolio performance over different time horizons and in different international equity markets. The strategy over-performs both equally weighted and price weighted benchmarks, even after transaction costs. A market premium, a value premium associated with mean reversion in stock returns, and a volatility premium which give the strategy characteristics of a benchmark enhancer, all explain the over-performance, but have time-varying contributions to it. A behavioural explanation for the mean reversion mechanism leads to the conclusion that the portfolio performance is influenced by the extent of investors’ herding towards the common trend in stock returns.

Author Contacts:
Prof. Carol Alexander
Chair of Risk Management and Director of Research
ISMA Centre, School of Business, University of Reading, Reading RG6 6BA
Email: c.alexander@ismacentre.rdg.ac.uk

Anca Dimitriu (corresponding author)
ISMA Centre, School of Business University of Reading, Reading RG6 6BA
Tel +44 (0)118 9316494
Fax +44 (0)118 9314741
Email: a.dimitriu@ismacentre.rdg.ac.uk

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This discussion paper is a preliminary version designed to generate ideas and constructive comment. The contents of the paper are presented to the reader in good faith, and neither the author, the ISMA Centre, nor the University, will be held responsible for any losses, financial or otherwise, resulting from actions taken on the basis of its content. Any persons reading the paper are deemed to have accepted this.
Introduction

Comparisons between the two main equity investment styles – active and passive – have a long history, being much influenced by both academic research and the investment management industry.\(^1\) The interest in replicating market performance through a passive strategy, most frequently in the form of indexation, is substantiated by the principles of efficient markets and modern portfolio theory, where the only way that investors can beat the market over the long term is by taking greater risks (Fama, 1970). Additionally, active management has been shown to often under-perform its passive alternative (Jensen, 1968; Elton, Gruber, Das, Hlavka, 1993; Carhart, 1997) due to transaction costs and administration fees, mostly in bull, but also in bear markets. For example, the S&P active/passive scorecard for the last quarter of 2002 shows that the majority of active funds have failed to beat their relevant index even in the bear market of the last few years. As a consequence of these trends, the passive investment industry has witnessed a remarkable growth during the last ten years, with a huge number of funds pegging their holdings to broad market indexes such as SP500. Currently, it is estimated that more than $1.4 trillion are invested in index funds in the US alone (Blake, 2002).

Traditionally, indexation has targeted price weighted and value weighted indexes, which are easy to replicate with portfolios comprising the entire set of stocks and mirroring the benchmark weights. Such portfolios are self-adjusting to changes in stock prices and do not require any rebalancing, provided there are no changes in the index composition or in the number of shares in each issue. Despite the self-replication advantage, holding all the stocks in the benchmark may not always be desirable or possible.\(^2\) More involved strategies are also required for tracking equally weighted indexes, since frequent rebalancing is required in order to maintain equal dollar amounts in each stock. Larsen and Resnick (1998) provide a thorough empirical investigation of the relationship between the indexed portfolio’s composition and the tracking performance. Their results show that value weighted indexes are easier to replicate than equally weighted indexes, and capitalisation dominates other stratification criteria such as industry classification.

Given the disadvantages of direct replication, recent research has focused on developing optimisation models for passive investments. Conventionally, tracking strategies using fewer stocks are constructed on basic capitalisation or stratification considerations. Optimisation techniques have also been

\(^1\) As a consequence, the very concepts of active and passive investment styles have evolved. Now, they can only be discriminated based on their investment objective, all other features, e.g. amount of research involved, portfolio optimisation techniques, frequency of trades, being similar. The active management is seeking to over-perform the market, usually through stock selection or market timing, while passive management is aiming to replicate the market performance. Also, strategies such as enhanced index tracking, which extend a passive style into active management, have been developed.
developed using objective functions based on the correlation of the portfolio returns with the benchmark, the mean deviation of the tracking portfolio returns from the benchmark, the variance of this deviation (often referred to as ‘tracking error’) or the transaction costs. Some examples are given in Rudd (1980), Meade and Salkin (1989), Adcock and Meade (1994), Connor and Leland (1995), Alexander (1999), Larsen and Resnick (1998 and 2001). The present paper contributes to this line of research by proposing a general portfolio construction model based on principal component analysis. The model identifies, of all possible combinations of stocks with unit norm weights, the portfolio that captures the largest amount of the total joint variation of the stock returns. Such a property makes it the optimal portfolio for capturing the common trend in a system of stocks whilst filtering out a significant amount of noise.

In finance, the use of statistical techniques to model asset returns has been extensive, especially in the context of factor models. Going back to Feeney and Hester (1967) and Lessard (1973), or in more recent years, Schneeweiss and Mathes (1995) and Chan, Karceski and Lakonishok (1998), principal component and factor analysis have been used to examine the existence of common movements in stock returns. They are seen as alternatives to fundamental approaches which relate the factors influencing financial asset returns to macroeconomic measures such as inflation, interest rates and market indices, or to company specifics such as size, book to market ratio or dividend yield. A great deal of statistical factor analysis has been performed for testing the arbitrage pricing model (Ross, 1976). In this context, historical returns are used to estimate orthogonal statistical factors and their relationship with the original variables. The construction of mimicking portfolios for the statistical factors has been formalised by Huberman, Kandel and Stambaugh (1987). Furthermore, alternatives to standard principal component analysis have been developed, e.g. asymptotic PCA (Chamberlain and Rothschild, 1983, Connor and Korajczyk, 1986 and 1988) or independent component analysis (Common, 1994).

A common finding in the literature is that the first principal component of a group of stocks captures the ‘market factor’ (Chan, Karceski and Lakonishok, 1998; Connor and Korajczyk, 1988). This assessment is based on two observations. First, provided that stock returns are reasonably correlated, they will have similar loadings on the first principal component, so a shock to this factor will generate a common trend in the system. Secondly, the $R^2$ from a simple regression of an equally weighted

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2 This happens mainly because of difficulties in purchasing odd lots to exactly match the market weights, or the increased transaction costs/market impact related to trading less liquid stocks.

3 To note, throughout the paper we use the term ‘market’ to denote the specific universe of stocks targeted by the passive investment strategy, which can be anything between a selection of stocks and the true ‘market portfolio’, comprising all assets.
portfolio of all stocks on their first principal component is usually found to be high, above 0.8, the first principal component explaining to a large extent the returns on an equally weighted portfolio. This result can be extrapolated also to other type of indexes, such as price weighted, provided that the returns of price weighted and equally weighted indexes representing the same universe are generally highly correlated. Our portfolio construction model is based precisely on the resemblance of the first principal component of the stock returns to the market factor proxied by a traditional index.

The standard approach to constructing factor mimicking portfolios uses the factor loadings in the stock selection process (e.g. Fama and French, 1993). The stocks are ranked according to their loading on a particular factor, then a self-financed portfolio is set up with long positions on the stocks with the highest loadings on that factor and short positions on the stocks with the smallest loadings. Most frequently, there is no portfolio optimisation, equal dollar amounts being invested in each stock. An alternative proposed by Fung and Hsieh (1997) for factor mimicking portfolios considers, in the stock selection stage, only the stocks that are highly correlated solely to the principal component for which the replica is constructed. Having selected the stocks, their portfolio weights are optimised as to deliver the maximal correlation of the mimicking portfolio returns with the corresponding principal component.

In these two methods, principal component analysis is used as a stock selection technique and the portfolio construction is a separate stage, based either on a standard optimisation, or on an arbitrary method such as equal weighting. In this paper we propose a different approach in which a portfolio replicating the first principal component is constructed directly from the normalised eigenvectors of the covariance matrix of stock returns. Such a portfolio, by construction, captures the largest proportion of the variation in the stock returns and filters out a significant amount of noise. Therefore, it is naturally suited for a passive investment framework, requiring a fully invested portfolio of all stocks, but involving a very small amount of rebalancing trades because it captures only the major common trend in stock returns. This procedure involves a single optimisation, the one producing the principal components. Moreover, there is no arbitrary choice of the portfolio construction model, such as equal weighting of stocks.

In order to investigate the portfolio performance, we use a group of stocks included in the Dow Jones Industrial Average (DJIA) at the end of year 2002. To support the features of the strategy observed in the DJIA case, we also construct random subsets of stocks from the SP100, FTSE100 and CAC40 universes. The performance is analysed both before and after transaction costs: we examine the returns volatility and correlation (unconditional over the entire sample and also short-term time series.
estimates), and the higher order moments of returns distributions, both from an overall perspective and conditional on market circumstances. Even if a benchmark does not enter the portfolio construction model, we follow convention to use both price weighted and equally weighted indexes as benchmarks for the portfolio performance.

Unsurprisingly, our results indicate that the first principal component captures the market factor, being highly correlated with the benchmark returns. Moreover, the factor weights prove to be very stable in time, so transactions costs are minimal. However, what does come as some surprise is that, out of sample, the portfolio replicating the first principal component, while being highly correlated with its benchmarks, significantly over-performs both of them. We demonstrate that one cause of the over-performance is a mean reversion in returns for the group of stocks which are over-weighted by the portfolio. We show that these are precisely those stocks that have had higher volatility and have also been highly correlated as a group during the portfolio calibration period. Subsequently, we observe two behavioural mechanisms which could explain the mean reversion for these stocks: the attention capturing effect documented by Odean (1999) and the over-reaction based models of De Long, Shleifer, Summers and Waldmann (1990a), Lakonishok, Shleifer and Vishny (1994) and Shleifer and Vishny (1997). Separately, our results show that the abnormal return\(^4\) is related to a behavioural measure of the investors’ herding towards the market factor, driving the mean reversion in stock returns. A decomposition of the strategy’s over-performance into a market premium, a value premium and a volatility premium reveals a time-varying structure. Throughout most of the period studied, the value component dominated the other two, but during the volatile periods of the last years the strategy earned a significant volatility premium.

The remainder of the paper is organised as follows: section one introduces the statistical model for the first principal component portfolio, section two describes the DJIA data and the performance testing methodology, section three reviews the empirical properties of the first principal component, section four analyses the out-of-sample performance of the first principal component portfolio, section five reports the results of applying the strategy to other international equity markets, and finally, section six summarises and concludes.

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\(^4\) We define the abnormal return as the difference between the factor mimicking portfolio returns and the returns of a price weighted benchmark, reconstructed from the same stocks as the portfolio.
I. The common trend replication model

Principal component analysis (PCA), introduced by Hotelling (1933) in connection to the analysis of data in psychology, was recommended as an important tool in the multivariate analysis of economic data more than half a century ago (Tintner, 1946). This technique is now a standard procedure for an orthogonal transform of variables, reducing dimensionality and the amount of noise in the data.

Given a set of \( k \) stationary random variables, \( X_1, X_2, \ldots X_k \), PCA determines linear combinations of the original variables, called principal components and denoted by \( P_1, P_2, \ldots P_k \), so that (1) they explain, successively, the maximum amount of variance possible and (2) they are orthogonal. By convention, the first principal component is the linear combination of \( X_1, X_2, \ldots X_k \) that explains the most variation. Each subsequent principal component accounts for as much as possible from the remaining variation and is uncorrelated with the previous principal components.

The \( i^{th} \) principal component, where \( i = 1, \ldots, k \), may be written:

\[
P_i = w_{1i}X_1 + w_{2i}X_2 + \ldots + w_{ki}X_k
\]  

Thus, if we denote by \( \Sigma \) the covariance matrix of \( X \), then:

\[
\text{var}(P_i) = w_{i}' \Sigma w_i; \quad \text{cov}(P_i, P_j) = w_{i}' \Sigma w_j,
\]

where \( w_i = [w_{1i} \, w_{2i} \, \ldots \, w_{ki}]' \) and it is standard to impose the restriction of unit length for these vectors, i.e. \( w_i'w_i = 1 \).\(^5\) Note that these are, in fact, the eigenvectors of \( \Sigma \). The spectral decomposition of the covariance matrix is \( \Sigma = \mathbf{W} \Lambda \mathbf{W}' \), where \( \Lambda \) is a diagonal matrix of eigenvalues (ordered by convention so that \( \lambda_1 > \lambda_2 > \ldots > \lambda_k > 0 \)) and \( \mathbf{W} \) is an orthogonal matrix of eigenvectors (which have also been ordered according to the size of the corresponding eigenvalue). The principal components defined as \( \mathbf{P} = \mathbf{XW} \) observe the conditions above. Note that the variance of each principal component is equal to the corresponding eigenvalue, so the total variability of the system is the sum of all eigenvalues. To reproduce the total variation of a system of \( k \) variables, one needs exactly \( k \) principal components.

However, when the first few principal components together account for a large part of the total

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\(^5\) Eigenvectors are not unique, and so it is standard to impose the orthonormal constraint. A more natural constraint in a portfolio construction framework would be to have the sum of the eigenvectors, rather than the sum of their squares, equal to one. However, this does not ensure a balanced portfolio structure, which is essential for indexing. In order to avoid large exposures to individual stocks, we keep the unit length constraint for the eigenvectors, and then normalise them to sum up to one.
variability, the dimensionality – and much of the noise in the original data – can be significantly reduced.

Since the principal components define a $k$-dimensional space in terms of orthogonal coordinates, the distances defined in the principal components space depend on the amount of correlation in the original variables. The higher the correlation in the original system, the better a principal component can account for the original joint variation and the larger the inter-point distances will be in that dimension. The elements of the first eigenvector are the factor loadings on the first principal component in the representation of the variables in terms of principal components. In a highly correlated system, these elements will be of similar size and sign. Consequently, when portfolio weights are directly proportional to the elements of the first eigenvector, as in (2) below, the more correlated the stocks, the more evenly balanced the portfolio.

When applied to large stock universes, previous research has shown that the first principal component is capturing the market factor, explaining a very high proportion from the returns of an equally weighted portfolio of all stocks. Motivated by these results, we propose a portfolio construction model which is based on replicating the first principal component of a set of stock returns.\(^6\) For a portfolio of $k$ stocks, the portfolio weight of stock $i$ is defined as:

$$w_i = w_{i1} / \sum_{j=1}^{k} w_{j1}$$

where $w_{i1}$ is the $i^{th}$ element from the first column in the eigenvectors matrix ordered as above.

In the PCA framework the first eigenvector is obtained, independently of the others, by maximising the variance of the corresponding linear combination of stocks, under the constraint of unit norm. Therefore the portfolio based on the stock weights determined as in (2) is, of all possible combinations of $k$ stocks with unit norm, the portfolio that accounts for the largest part of the total joint variation of the $k$ stocks. This property ensures that it is the optimal portfolio for capturing the common trend in a system of stocks. Considering that the model maximises the variance of the portfolio under some constraint, it

\(^6\) We note that, often, the original stationary variables are standardised to have zero mean and unit variance before the principal component analysis – that is, that the eigenvectors of the correlation matrix are used to construct the principal components, rather than the eigenvectors of the covariance matrix. This ensures that the variable with the highest volatility does not dominate the first principal component. However, in a realistic portfolio construction setting, the assumption of equal volatilities for all assets is not feasible. Such an assumption would result in the portfolio model being constructed solely on the correlation structure of the assets, rather than the complete covariance structure of the data. Therefore, for the purpose of our model, we do not standardise the stock returns.
will over-weight, relative to benchmark, the stocks that were both highly correlated and had higher than average volatility over the estimation period.

The common trend replication model is different from the traditional approaches to portfolio optimisation (Markowitz, 1952; Chan, Karceski and Lakonishok, 1999; Jagannathan and Ma, 2002) in more than one respect. Firstly, it is maximising and not minimising portfolio variance, and this might appear counterintuitive at a first glance. However, when combined with unit norm constraint on the factor loadings, the result is a balanced portfolio with a stable structure which also explains most of the joint variance in the system of stocks.7 Secondly, it is not aiming at stock selection, but rather at diversifying over the entire universe of stocks. All stocks will be represented in the portfolio replicating the common trend and the portfolio will be fairly evenly balanced if there is a high level of correlation in the stock returns. Finally, despite being a passive investment model, the benchmark does not enter into the methodology anywhere. This eliminates the problems associated to using an inappropriate benchmark in the portfolio construction, but also limits the relevance of traditional indexing performance measures such as tracking error, so caution is needed when interpreting such results.

II. Data, benchmarks and portfolio ‘out-of-sample’ performance measurement

In order to examine the properties of the portfolio replicating the first principal component, we use a main data set comprising daily closing prices on the 25 of the stocks currently included in the DJIA which have a history available for the period Jan-80 to Dec-02. Four out of the five stocks which are currently in the DJIA, but which do not have a history going back to Jan-80, are technology stocks. Therefore, our portfolio has a lower loading on technology than the current DJIA and the latter cannot be considered the relevant benchmark because of a ‘technology’ bias. Also, the stock selection methodology may raise the concern of performance biases such as survivorship and look-ahead, because we are selecting the stocks which had a history of at least 23 years of data available. We deal with all these potential biases by creating benchmarks from exactly the same stocks as our portfolio, so that the benchmarks are affected by the same biases as the portfolio.8 Subsequently, we analyse all performance on a relative basis.

7 The unit norm constraint ensures a balanced portfolio structure, without large exposures to individual stocks. This constraint can also be interpreted as a Bayesian approach to limiting the effect of outliers in the historic stock returns – a large weight on an individual stock results when the stock has a very high in-sample volatility, but this could simply be due to measurement errors or single outliers (Jagannathan and Ma, 2002).

8 The alternative would be to include in our portfolio at time t the stocks that were in the benchmark at time t. However, this would necessitate a complex dynamic back-test procedure and the underlying principle, that the stocks in the benchmark are the same as the stocks in the portfolio, is the same. Actually, over the entire data sample the price weighted benchmark had a
Despite the fact that a benchmark does not formally enter the portfolio construction model, it is needed to evaluate its performance. By restricting the information used in the benchmark construction to the information used in the portfolio construction (i.e. the history of stock prices, no capitalisation figures) there are two alternative benchmarks: a price weighted benchmark (PW) and an equally weighted benchmark (EW). The first implies no trading as long as the universe of stocks does not change and it is self-adjusting to price changes. Therefore, PW is a natural choice as benchmark for a passive investment strategy. However, the returns differential between EW and PW will also enter our performance analysis, as a proxy for a value portfolio: by construction, PW places more weight on growth stocks, so their returns difference can be interpreted as a value premium.

For the purpose of performing principal component analysis, we are particularly interested in the average correlation of the stock returns, as this has a strong influence on the effectiveness of principal component analysis. We find that the average correlation of the daily stock returns from the DJIA set is in the range of 0.3 to 0.4, occasionally going to as low as 0.2. The highest average correlation in stock returns occurs in down, volatile markets, such as 1987, 1990, or 2001-2002, this being a common finding for stock markets.

Regarding the general market conditions during the sample period, it is worth mentioning that, in 10 out of the 23 years, the stocks in DJIA had average returns above 20%. By contrast, in only 5 years out of 23, the average return was negative, which, however, was the case for the last 3 years in the sample. The average volatility stayed in the range of 20%-30%, increasing significantly in the last part of the data sample. The year 1987 stands out from the sample, in terms of returns correlation, volatility, excess kurtosis and negative skewness, because of the October crash.

For the out-of-sample returns analysis, the portfolio optimisation and rebalancing procedure is as follows: at each rebalancing moment, the stock weights are determined from the eigenvectors of the covariance matrix of the stock returns estimated from the most recent 250 observations prior to the moment of the portfolio construction.\textsuperscript{9} For the out-of-sample performance assessment, the portfolio cumulative return of 203% and the actual DJIA returned 215%. Therefore, the survivorship bias due to the difference in technology loadings is in fact negative.\textsuperscript{9} The ‘rolling sample’ PCA raises the issue of consistent identification of the factor loadings because the choice of the sign of the eigenvectors is arbitrary. Choosing a particular normalisation is not relevant if the estimation of the principal components is performed over the entire data sample. However, when the optimisation is performed over a rolling sample, in order to have consistent principal component estimates from successive estimations, one needs to ensure that the same normalisation is used throughout the entire data sample. To this end, following Chan, Karceski and Lakonishok (1998), we impose an additional...
constructed in the previous step is left unmanaged for the next 10-trading days, and then rebalanced based on the new stock weights from principal component analysis. We have used a 10-day rebalancing period having in mind an institutional investor, for which this trading frequency is standard. However, the rebalancing frequency could be easily reduced, without affecting the portfolio performance because the portfolio weights are stable over time. In order to account for transaction costs we assume an amount of 20 basis points on each trade value to cover the bid-ask spread and the brokerage commissions.\textsuperscript{10}

In addition to DJIA stocks, we use several sets of daily closing prices of stocks included at the end of year 2002 in the CAC40, FTSE100 and SP100 indexes. The length of the data sample ranges from 1,600 daily observations for SP100 (Apr-96 to Jun-02) to 2,100 daily observations for FTSE100 (Jul-94 to Dec-02).

\textbf{III. Empirical properties of PCA}

This section examines the empirical results of estimating the first principal component on a rolling sample of 250 observations on daily returns to DJIA stocks. First, we show that the in-sample properties of the first principal component justify the use of the PC1 portfolio to capture a ‘market’ factor. Subsequently we examine the size and the stability of the factor loadings on the first principal component, as this will determine the structure of the PC1 portfolio and the associated transactions costs.

The price weighted benchmark (PW) and the PC1 portfolio have very similar information ratios, as shown in Figure 1. Each point in Figure 1 represents the information ratio over the last 250 observations. The main exceptions are the periods 1985-1986 and 1995-1996, during which the information ratios of the PC1 portfolio are significantly higher. Additionally, their returns are also highly correlated. The correlation coefficient ranges from 0.7 to 0.98. Lower correlation occurs between 1992 and 1996, but most of the time it is still above 0.9. A standard regression of the benchmark returns on the first principal component, estimated over the entire sample, has an $R^2$ of 0.8. Therefore, we can safely conclude that the first principal component largely captures the market factor.\textsuperscript{11}

\textsuperscript{10} We do not account for potential tax implications for individual investors, assuming that the strategy is primarily designed for institutional investors.

\textsuperscript{11} The term ‘market’ refers to the specific universe of stocks included in our analysis. As this universe increases, it will converge to the true ‘market portfolio’.

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The amount of total variation explained by the first principal component in our system turns out to be in the range of 30-40%, in line with previous research on this issue (Chan, Karceski and Lakonishok, 1998; Connor and Korajczyk, 1988). This is directly related to the amount of correlation in the original system of returns. Figure 2 reports the proportion of variance explained by the first principal component and the average correlation of returns, both based on a rolling sample of the last 250 observations. Clearly, the average correlation in the original data is the single most important determinant of the proportion of variance explained by the first principal component. The lowest average correlation (and, consequently, amount of variation explained by the first principal component) occurs between 1992 and 1997, and again in 1999 and 2000. These times were relatively calm periods for the developed stock markets, and correlations are generally higher during more volatile periods.

Of central interest to our analysis are the eigenvectors of the covariance matrix of stock returns, as these will determine the stock weights in the portfolio replicating the first principal component. The eigenvector corresponding to the first principal component comprises the sensitivity of each stock to changes in the first principal component, the so-called ‘factor loading’. If the stock returns were perfectly correlated, the first principal component would capture the entire variation of the system and the factor loadings would all be equal. More generally, in a highly but not perfectly correlated system, the factor loadings on the first principal component will be similar but not identical so that a change in the first principal component generates a nearly parallel shift in the original variables. In this case we can associate the first principal component with the existence of a ‘common trend’ in stock returns.

In the DJIA case, the factor loadings on the first principal component are largely in the same range: during periods of high average correlation (e.g. after the 1987 crash) the factor loadings are high and very similar but more recently they tend to be lower and less similar. This observation is justified by Figure 3, which plots the standard deviation of the factor loadings. The similarity of the factor loadings is an important feature of the model, as it allows the construction of balanced portfolios, without extreme exposures to individual stocks. In consequence we observe that even though there are no short-sale restrictions imposed on the model, short positions occur very rarely.

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12 A ‘ghost feature’ caused by the October-87 crash can be identified in both of them: the correlation and the percentage of variance explained remain very high for as long as the October crash stays in the estimation sample and drop immediately after excluding that observation from the sample. This is an artefact of the equal weighting in returns and would not be evident if exponential weighting of the covariance matrix were applied.
The dispersion of the factor loadings has been used as a measure of herding behaviour in recent research in behavioural finance (Hwang and Salmon, 2001), and we shall return to the implications of this in the next section when we analyse the out-of-sample performance of the model. Apart from the cross-sectional variability of the factor loadings, a very attractive feature is their low time variability. The factor loadings are very stable in time, which, in a portfolio construction setting, is translated into a reduced amount of re-balancing trades and low transaction costs.

**IV. Out-of-sample performance of the statistical factor equity portfolio**

To recap, the portfolio replicating the first principal component (PC1) is constructed from the 25 stocks that were both included in DJIA at the end of 2002 and had a history that goes back as far as Jan-80. The benchmarks for the performance assessment are a price weighted portfolio (PW) and an equally weighted portfolio (EW) both with all 25 stocks. For the out-of-sample analysis, PC1 is first set up in Jan 81, based on the principal component analysis performed on the 250 observations preceding the portfolio construction moment and further rebalanced every 10-trading days. In between rebalancing, the number of stocks in each portfolio is kept constant and the out of sample portfolio performance recorded.

The performance statistics for out-of-sample daily returns series generated by the PC1 portfolio and the two benchmark portfolios are reported in Table I. In terms of annual returns, the PC1 portfolio over-performs PW by an average of 5% per year, with only 1% extra volatility. EW is also over-performed by the PC1 portfolio with an annual average of 2.70%. The superior performance results in an information ratios of 0.75 for PC1, compared with 0.5 for PW and 0.63 for EW. The PC1 portfolio returns appear to be marginally closer to normality than the returns on the benchmark portfolios, but all three portfolios have heavy tailed and negatively skewed returns distributions. There are other strong similarities between the three portfolios: when out-of-sample returns are analysed period by period all portfolios are affected by the main market crises during the period in observation: Oct-87, the Gulf War, the Asian Crisis, the burst of the technology bubble and Sep-01. They all have a strong January effect, which, however, is less evident in the case of the PC1 portfolio. The correlation between the PC1 and the both benchmark portfolios returns is very high, indeed the PC1 portfolio and the benchmarks have very similar short-term volatility and correlation properties. The exponentially weighted moving average (EWMA) volatilities and correlation for PC1 and PW with a smoothing parameter of 0.96 are shown in Figure 4. The volatility of the PC1 portfolio is slightly higher,
especially during the last part of the sample, but is closely following the benchmark volatility. With very few exceptions, the EWMA correlation is high, staying above 0.8 most of the time and it is particularly high during market crises such as Oct-87 or Sep-01. Finally, the transaction costs are almost negligible, amounting to an average of 0.24% per year for implementing the PC1 strategy. As our target is to explain the ‘pure’ over-performance, i.e. the difference between the PC1 portfolio return and the PW benchmark return, after establishing that the overall profitability of the strategy does not disappear after transaction costs, we will perform the analysis of the portfolio returns before transaction costs.

Considering the PC1 portfolio over-performance with respect to the price weighted benchmark, the abnormal return can be thought of as being produced by a self-financed strategy which, at each moment in time, is long on the PC1 portfolio and short on PW. In this case (see Table I) the 5.19% annual return is associated with an annual volatility of 6.3%. Its information ratio is 0.82, higher than those of the benchmarks and PC1 portfolio. Moreover, the abnormal return is uncorrelated with the benchmark return and much closer to normality than the latter. The fact that the over-performance of the PC1 portfolio is not caused by singular events is evident from Figure 5, which shows the cumulative returns difference between the PC1 portfolio and the two benchmarks.

Any strategy that over-performs in the DJIA stock universe should lead one to question if there is any connection with the famous ‘Dogs of the Dow’ or ‘Fool’s Four’ value strategies. Indeed, we shall see that the PC1 strategy has a significant value tilt. But apart from this, there are no other similarities. The ‘Dogs of the Dow’ and ‘Fool’s Four’ strategies pick a small number of stocks from DJIA (ten and four respectively) and rebalance as rarely as once a year. They are known to be a classic case of data mining, that is, an extensive search through a large number of trading strategies for the ones which have historically over-performed the benchmark. Apart from the low diversification and increased volatility, these strategies have been shown not to be robust to out-of-sample tests and associated transaction costs (Hirschey, 2000; McQueen and Thorley, 1999). In contrast, the PC1 portfolio model is not the result of a blind search through the historical performance of different trading rules. It is constructed on strong theoretical foundations, to capture the common trend in stock returns. Moreover, it is not betting on few stocks, having a similar industry and stock diversification to its benchmarks. Finally, it is robust to out-of-sample tests and inclusion of transaction costs.

13 We have also analysed the performance of a portfolio comprising all 30 stocks currently included in DJIA, over the period Jan-91 to Dec-02. The results are very similar to the ones obtained with the 25-stocks portfolio. For reasons of space, we have
In order to explain the performance of the portfolio replicating the first principal component, we shall construct a simple model which is based on the relationship between the PC1 portfolio and the benchmarks. Given the high correlation between PW and EW, in order to avoid near multicollinearity, we include in the model only the price weighted benchmark as a proxy for the ‘market’, and, separately, the returns differential between EW and PW, as a proxy for a value factor. Over the entire data sample there is a negative but not very significant correlation between the two explanatory variables. This is to be expected, given that most of the value over-performance has been documented in negative market circumstances. Thus, we estimate the following model on daily data covering the period Jan-1981 to Feb-2003 (from which we have eliminated the two outliers representing the market crashes from October 1987 and September 2001), using ordinary least squares:

\[
\text{PC1}\_\text{return}_t = \alpha + \beta_1 \times \text{PW\_return}_t + \beta_2 \times (\text{EW\_return} - \text{PW\_return})_t + \varepsilon_t
\]

This very simple specification is not robust to heteroskedasticity tests, the pattern in the autocorrelation of squared residuals indicating a GARCH(1,1) as the alternative, this being a common specification for stock market index volatility. Additionally, the model does not pass specification error tests, indicating a possible non-linear relationship between PC1 returns and one of the explanatory variables. These specification tests improve when squared returns on the price weighted benchmark are included as follows:

\[
\begin{align*}
\text{PC1}\_\text{return}_t &= \alpha + \beta_1 \times \text{PW\_return}_t + \beta_2 \times (\text{EW\_return} - \text{PW\_return})_t + \beta_3 \times \text{PW\_return}^2_t + \varepsilon_t \\
\varepsilon_t &\sim N(0, \sigma^2_t) \\
\sigma^2_t &= \omega + \alpha_t \varepsilon_{t-1}^2 + \beta_t \sigma^2_{t-1}
\end{align*}
\]

The estimation results for model (3) covering the sample are reported in Table II.14 The very high R squared, above 0.98, comes as no surprise, considering the strong correlation between the portfolio return and its benchmarks. All coefficients, except for the mean regression intercept, are positive and highly significant at 1% significance level. The variance regression model estimates show an almost integrated vanilla GARCH model for the variance of the PC1 portfolio, with the persistence coefficient (0.96) and reaction coefficient (0.037) in the usual range for stock market volatility during

---

14 This version of the model passes the autocorrelation and ARCH tests, having slightly non-normal residuals, probably due to the presence of outliers. The information criteria clearly favour this specification as compared to the initial one.
this sample period. The mean regression model provides a complete decomposition of the PC1 portfolio performance into risk factor premia, given that the intercept term is not statistically significant.

The coefficient of the price weighted benchmark returns, which measures the portfolio’s sensitivity to the market factor, is above unity (i.e. at 1.04). Therefore, part of the portfolio over-performance can be attributed to a higher loading on the market risk factor. Also, the positive relationship between the portfolio return and the value factor proxy indicates that part of the over-performance is due to a higher loading on value stocks. Finally, the positive and significant coefficient of the squared price weighted benchmark returns can be interpreted as the portfolio sensitivity to a volatility factor, proxied by the squared market returns.

The portfolio premium from each factor is defined as the product of the portfolio sensitivity to that factor times the factor premium. If we combine the market premium earned by the PC1 portfolio with the volatility premium, the returns differential between the two and the price weighted benchmark has a straddle pattern: the PC1 portfolio over-performs large negative and large positive benchmark returns, and marginally under-performs small negative market returns. This feature of the strategy is important, as it allows the investor to reduce the portfolio exposure to negative market circumstances, while increasing the exposure to positive ones. From this point of view, the strategy acts like a benchmark enhancer.

Another important issue to investigate is the evolution of the individual contributions to the PC1 portfolio over-performance. Over the entire data period, the contribution of the three sources to the total over-performance of the PC1 portfolio is the following: market premium 11%, value premium 60% and volatility premium 29%. However, this distribution is far from being stationary, as shown by Figure 6. This figure shows the contributions of the three over-performance sources estimated from model (3) on a rolling window of 500 daily observations. Each premium is computed as the portfolio sensitivity to the factor times the annualised mean factor premium over the estimation sample.

- The market premium has the most stable, but also the smallest contribution to the portfolio over-performance, becoming negative from 2001 as the stock market generally declined. The PC1 portfolio, having a high market beta, will under-perform in down markets.

- The value premium accounts for the largest part of the portfolio over-performance during most of the ‘80s and ‘90s. This finding is consistent with abnormal returns being generated by a mean
reversion mechanism, as we shall argue below. However, the value premium falls sharply, becoming negative, around the time of the October ’87 crash, after the Gulf War, during the Asian crisis and again at the end of the sample. An explanation for this change is that the ‘normal’ mean reversion cycle is broken around the time of market crises, because investors’ behaviour changes significantly.

- Given the increased stock market turbulence during the last few years, the volatility premium increased markedly during 2001 and 2002. It accounts for 70% of the total over-performance by the end of the sample.

Having identified the sources of portfolio over-performance, we now explain a mechanism through which this over-performance may be achieved. As shown in section I, the stock weights in the PC1 portfolio are chosen to maximise the portfolio variance, subject to the constraint of unit norm for the factor loadings. Since portfolio variance increases with both individual asset variance and the covariance between assets, the portfolio will over-weight, relative to the PW benchmark, stocks that have higher volatility over the estimation period and which are also highly correlated as a group. Separately, the PW benchmark is under-weighting stocks that have recently declined.

Now, if it does hold true that markets tend to be more turbulent after a large price fall than after a similar price increase (i.e. the ‘leverage effect’ that is commonly identified in stock markets, as in Black, 1976; Christie, 1982; French, Schwert and Stambaugh, 1987), then the same group of stocks will be impacted through the over-weighting of volatile, correlated stocks in the PC1 portfolio and the under-weighting of declining stocks in the PW benchmark. These stocks have had a volatile, declining period over the estimation sample. From this perspective, the over-performance of the PC1 portfolio must be due to a mean reversion in stock returns over the one-year estimation period used for our portfolio. The portfolio over-weights stocks that have recently declined in price, relative to the benchmark, so the relative profit on the portfolio has to be the result of a consequent rise in price of these stocks. The hypothesis that mean reversion takes place over a period of one-year is supported by the fact that when the PC1 estimation sample is reduced, the in-sample over-performance of the first principal component with respect to the PW benchmark disappears.

These results are in line with the research on short-term momentum and long-term reversals that has frequently been identified in stock returns. For example, De Bondt and Thaler (1985), Lo and MacKinlay (1988), Poterba and Summers (1988) and Jagadeesh and Titman (1993) identify positive
autocorrelation in stock returns at intervals of less than one year and negative autocorrelation at longer intervals. In behavioural finance, two explanations are usually proffered for long-term reversals and short-term momentum in stock markets. The first explanation focuses on relatively volatile stocks, which capture the attention of ‘noise traders’ for whom they are the best buy candidates (Odean, 1999). The trading behaviour of noise traders creates an upward price pressure on these volatile stocks, forcing mean reversion when their high volatility was associated with a recent decline in price. The same explanation is not applicable to a selling decision, creating symmetrically downward price pressure on volatile stocks, because the range of choice in a selling decision is usually limited to the stocks already held (Barber and Odean, 2002). Additionally, we note that volatile stocks which have recently experienced a price decline, also qualify as value stocks, so this explains the value premium previously observed.

A second behavioural explanation of the short-term momentum followed by mean reversion has been provided by De Long, Shleifer, Summers and Waldmann (1990a), Lakonishok, Shleifer and Vishny (1994) and Shleifer and Vishny (1997). This explanation is based on investors’ sentiment, over-reactions and excessive optimism/pessimism. The occurrence of some bad news regarding one stock creates an initial excess volatility and, according to these models, some investors will become pessimistic about that stock and start selling. If there is positive feedback in the market, more selling will follow and the selling pressure will drive the price below its fundamental level. However, the arbitrageurs (sometimes called ‘smart money’, or ‘rational’ investors) will not take positions against the mispricing either because (1) the mispricing is too small to justify arbitrage after transaction costs, or (2) there is no appropriate replica available for that stock, so the fundamental risk cannot be hedged away, or (3) there is a ‘noise trader risk’ arising from positive feedback, where the excessive investors’ pessimism will drive the price even further down over the short term. In the presence of positive feedback, De Long, Shleifer, Summers and Waldmann (1990b) show that the arbitrageurs will initially join the noise traders in selling, in order to close their positions when the mispricing has become even larger. This type of investor behaviour justifies both short-term momentum and longer-term mean reversion.

In addition to the above explanations which justify the over-performance of the PC1 portfolio by a mean reversion in stock returns, we also find that there is a connection between the abnormal returns

\[ \text{\textsuperscript{15}} \text{Noise traders are usually defined in the literature as not fully rational investors, making investment decisions based on beliefs or sentiments which are not fully justified by fundamental news, or which are subject to a systematic biases.} \]
generated by the PC1 portfolio and another behavioural phenomenon that is well documented in stock markets – investors’ herding. We shall show that the more intense the herding behaviour, as measured by a decrease in the cross sectional standard deviation of the factor loadings, the higher the abnormal returns generated by the PC1 portfolio.

The use of the cross sectional distribution of stock returns as an indication of herding was first introduced by Christie and Huang (1995) in the form of the cross sectional standard deviation of individual stock returns during large price changes. Hwang and Salmon (2001) build on this idea but instead advocate the use of a standardised standard deviation of PCA factor loadings to measure the degree of herding. Their measure has the advantage of capturing ‘intentional’ herding towards a given factor, such as the market factor, rather than ‘spurious’ herding during market crises. Following Hwang and Salmon (2001), we assume that the standard deviation of the factor loadings (shown in Figure 3) captures the intentional herding of the investors towards the first principal component of the stocks, or their common trend. An intense herding of the investors towards the common trend of the stocks should reduce the differences in the individual stocks loadings on the first principal component. Therefore, we interpret a low standard deviation of the factor loadings as an indication of herding. From Figure 3 we see that more intense herding appears to happen before 1993, and then again before 1998, which supports the findings in Hwang and Salmon (2001) that this type of herding occurs especially during quiet periods for the market. During the market crises of the last five years, the herding behaviour appears to be significantly reduced.

Given the interpretations of mean reverting behaviour presented above, an intense herding towards the first principal component, indicated by a sharp reduction in the standard deviation of the factor loadings, should enhance and speed up the mean reversion. Therefore the standard deviation of the factor loadings should be negatively related to the over-performance of the PC1 portfolio. Indeed, the correlation between the standard deviation of the factor loadings and the abnormal return, estimated over all non-overlapping sub-samples of 120 observations, is negative (-0.33) and significant at 5%. From the three different sources of over-performance, this measure of herding is mostly related to the value premium. The smaller the standard deviation of the factor loadings, the more similar the PC1 portfolio will be to an equally-weighted portfolio, and the closer its over-performance will be to the way we have defined the value premium. From Figures 3 and 6 we see that the highest value premium coincides with the periods when the standard deviation of the factor loadings was the smallest.
Provided that most of the over-performance of the PC1 portfolio comes from the value premium, we conclude that the more intense the herding towards the first principal component, the more effective the mean reversion in stock returns and the higher the over-performance of the PC1 portfolio.

V. Results on SP100, FTSE100 and CAC40

Our results on the relationship between herding behaviour and common trends, mean reversion and over-performance are not specific to the DJIA universe. To show this we have constructed 100 random subsets of stocks in each of the SP100, FTSE100 and CAC40 indexes. Each subset comprises 75% of the total number of stocks available for that index, so that in the CAC universe, the subset has 23 stocks, in the FTSE universe there are 52 stocks and in the SP universe there are 75 stocks.16 The common sample of data available covers 6 years, from Apr-96 to Jun-02. For each subset, we have constructed a PW index, an EW index and a portfolio replicating the first principal component in the system of stock returns, and compared their performance.

Our first observation is that, within each stock universe, the performance of the strategy across randomly selected sets of stocks is very similar. The correlation of the portfolio returns within each index is very high, in the range of 0.8 to 0.9. These results are not surprising, as it is to be expected that the performance of portfolios based on any unique strategy, which always comprise 75% of the stocks in a limited universe, exhibits similar features. Moreover, the similarity should be even more pronounced because the strategy is constructed on a common trend, rather than on the individual stock returns.

In order to compare the results obtained for different markets, we average the returns of all 100 portfolios in each universe. Figure 7 reports the cumulative average abnormal return for the three markets and, for reference, the cumulative abnormal return for the DJIA, measured against the PW benchmark. One interesting feature of this figure is the similarity of the two average returns series for the European markets, CAC and FTSE. Both strategies over-perform their benchmarks until Aug-00, when there is a steady abnormal return. After this date, the abnormal return becomes very volatile and eventually erodes the previous gains. A relatively similar pattern is identified also for the abnormal return in the SP100 and DJIA stock universes. The abnormal returns in DJIA are, however, much less eroded than the one in SP100. This can be due to an increased inertia in the DJIA stocks, and also to the fact that our reduced DJIA universe was not much affected by the technology boom and bust. We

16 There is always a trade-off between the diversity of portfolios within one universe and the number of stocks selected in each portfolio. 75% of all stocks in each portfolio ensures a relative balance of the two.
also note a significant difference in the magnitude of returns in the European and US markets. Even before Aug 2000, the abnormal returns in SP100 and DJIA are steadier and less volatile than in the European counterparts. After Aug 2000, the decrease in the average abnormal return in the US markets is less spectacular than in the case of the European markets.

The average contribution of the risk factors to the portfolio over-performance, estimated according to model (3), is presented in Table III for the period Apr 97 to Jun 02. We also present the results obtained in the DJIA framework over the same period, as a reference. The first observation is that for all stock universes the volatility premium dominates the other two. These results come as no surprise, considering the high volatility in these equity markets during most of this period. Also, there is also a significant market premium, which was also previously identified in the case of DJIA. However, the contribution of the value premium is either negative, for CAC40 and SP100, or marginally positive, in the case of FTSE100. The only universe that still exhibits a significantly positive value premium between Apr 97 and Jun 02, even if considerably reduced when compared to previous periods, is the DJIA.

The similarity in the performance of portfolios constructed in different stock universes can be interpreted as evidence of common trends in the international stock markets. Usually, such evidence has been produced as a result of examining the properties of different market indexes and/or groups of stocks, e.g. cointegration, correlation in different market circumstances, etc. The evidence of similarities in the performance of a strategy, as a dynamic combination of stocks, in different markets is equally relevant for the hypothesis of common movements, even if indirect. Moreover, we have shown that there are significant similarities also in the contribution of the risk factors to the strategy overperformance. The differences in the patterns of the US and European results, however small, present a potential for diversification.

**VI. Summary and conclusions**

Following an extensive academic and practical interest in passive investment and indexing models, we have proposed a portfolio construction model based on the principal component analysis of stock returns. As opposed to traditional approaches to indexing, which aim to replicate the performance of a standard benchmark, our model is based on the replication of only the common trend of the stocks included in that benchmark. The model is identifying, of all possible combinations of stocks with unit norm weights, the portfolio that captures the largest part of the total joint variation of the stock returns. By so doing, the strategy manages to filter out a significant amount of the noise present in stock returns,
which facilitates the replication task considerably. On these grounds, the PC1 portfolio structure turns out to be very stable over time, requiring only a minimal amount of rebalancing which results in negligible transactions costs, amounting to less than ¼% p.a.

Moreover, we have shown that the PC1 portfolio, while being highly correlated with its benchmarks, has significantly over-performed them. The sources of over-performance have been shown to be a market premium, a value premium and volatility premium. The straddle pattern created by the volatility premium is particularly appealing for investors, reducing the exposure to large negative benchmark returns and increasing the exposure to positive benchmark returns. A mechanism explaining the over-performance is the mean reversion in returns for the stocks which are over-weighted by the portfolio, that is stocks that have had higher volatility and have also been highly correlated as a group, during the portfolio calibration period. We pointed out two behavioural phenomena that could be driving the mean reversion for these stocks: the attention capturing effect and investors’ over-reaction, both of them resulting in different forms of herding behaviour. Indeed, we found a close relationship between the abnormal return and a measure of investors herding towards the market factor.

The distribution in time of the over-performance sources has been shown to evolve, from a value-dominated over-performance towards an increased volatility premium associated to the volatile years towards the end of our data sample. This finding supports the behavioural mechanisms thought to be driving the mean-reversion in stock returns, to the extent that during volatile periods investors tend to herd less and this prevents mean reversion from taking place.

Finally, these findings are not restricted to the Dow Jones index. We have found a common pattern in the strategy performance applied to three major stock markets: there is a high correlation between the strategy results for the two European indices, and a high correlation between the results for the two US indices, but a lower correlation between the results on the European and US indices. The differences in the patterns of the US and European results, however small, present a potential for diversification. Extending the analysis to other stock markets, less correlated with the US and European ones, could uncover even better diversification opportunities.
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Figure 1 Information ratio for PC1 and the benchmark portfolio

Using the 25 stocks currently in DJIA and which have data available from Jan-1980, the graph illustrates the similarity in the information ratios of a price weighted portfolio of these stocks and the first principal component of their returns, estimated on the covariance matrix. The information ratios, i.e. annualised returns divided by annualised volatility, are estimated over a rolling window of 250 trading days.

Figure 2 Proportion of variance explained by PC1 and the average correlation of stock returns

This graph illustrates the connection between the average correlation of the 25 stock returns and the proportion of total variance explained by the first principal component of the stock returns. The two variables are estimated over a rolling window of 250 trading days.
**Figure 3 Cross-sectional standard deviation of the stock factor loadings**

This graph plots the time series of the cross-sectional standard deviation of the elements of the first eigenvector of the covariance matrix of the 25 stock returns, which are also the loadings of each stock on the first principal component. Principal component analysis is performed on a rolling window of 250 trading days. A small standard deviation of the stock loadings on the first principal component indicates that a change in the first principal component will have an equal effect on individual stocks, generating a parallel shift in the system. This is also interpreted as an indication of investors’ herding towards the common trend.

**Figure 4 EWMA volatilities and correlations**

The exponentially weighted moving average volatilities and correlations are estimated for the time series of PC1 portfolio and price weighted benchmark returns, estimated out of sample and before transaction costs. We have used a smoothing parameter of 0.96, which is related to the high persistence in the volatility of the two series (see Table II).
Figure 5 Cumulative over-performance in DJIA framework

The over-performance is estimated as the difference between the PC1 portfolio return and the price weighted benchmark return, respectively the equally weighted benchmark return, of the same group of 25 stocks in DJIA. The returns are estimated out-of-sample, based on the following rebalancing procedure: the most recent 250 observations are used to calibrate the PC1 portfolio, which is then left unmanaged (that is the number of stocks is kept constant) over the next 10 trading days, during which the performance of the PC1 portfolio and benchmarks are monitored. The results are reported before transaction costs, but these are less than ¼% p.a.

Figure 6 Time distribution of the over-performance sources

The time distribution of the contributions to over-performance of the three sources, market premium, value premium and volatility premium, is obtained by estimating model (3) on a rolling window of 500 observations. Each portfolio premium is the product of its sensitivity to the factor times the annualised average premium on that factor, over the estimation sample.
Figure 7 Average cumulative over-performance in FTSE, CAC and SP100 universes

Using the stocks in FTSE, CAC and SP100, we have generated sets of 100 random portfolios in each index, and constructed for each subset a price weighted index and a portfolio replicating the first principal component. The cumulative over-performance is estimated for each subset out-of-sample, based on the following rebalancing procedure: the most recent 250 observations are used to calibrate the PC1 portfolio, which is then left unmanaged (that is, the number of stocks is kept constant) over the next 10 trading days, during which the performance of the PC1 portfolio and benchmark are monitored. The series of abnormal returns were then averaged within each index and the results are presented in the graph below.
Table I Performance statistics over the period Jan-81 to Jan-03

This table summarises the out-of-sample performance of the price weighted and equally weighted benchmarks of the 25 stocks currently in DJIA and which have data available from Jan-1980 and the PC1 portfolio of the same stocks. The over-performance (excess return) is estimated as the difference between the PC1 portfolio returns and the benchmark returns. We are presenting the annual average return, volatility, the information ratio, as well as the third and fourth moments of the three portfolio returns. In addition, we present the relative volatility of the PC1 portfolio with respect to the benchmarks, its correlation with the benchmark returns, as well as the correlation of the excess return with the benchmarks returns.

<table>
<thead>
<tr>
<th></th>
<th>PW benchmark</th>
<th>EW benchmark</th>
<th>PC1 portfolio</th>
<th>Xs return PW</th>
<th>Xs return EW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual return</td>
<td>8.97%</td>
<td>11.34%</td>
<td>14.16%</td>
<td>5.19%</td>
<td>2.70%</td>
</tr>
<tr>
<td>Annual volatility</td>
<td>17.91%</td>
<td>17.84%</td>
<td>18.96%</td>
<td>6.30%</td>
<td>2.59%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.99</td>
<td>-1.91</td>
<td>-1.54</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>46.85</td>
<td>40.66</td>
<td>32.45</td>
<td>5.95</td>
<td>18.82</td>
</tr>
<tr>
<td>Portfolio relative volatility</td>
<td>1.06</td>
<td>1.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio correlation with benchmark returns</td>
<td>0.94</td>
<td>0.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess return correlation with benchmark returns</td>
<td>-0.004</td>
<td>-0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II Estimated coefficients of model (3)

The model estimated for the PC1 portfolio returns includes the price weighted benchmark returns, the returns differential between the equally weighted and price weighted benchmarks as a value factor proxy, and the squared returns on the price weighted benchmark, as a proxy for the portfolio intrinsic alpha. The model estimated on daily data for the period Jan-81 to Jan-03 is the following:

\[
P_{\text{C1 return}} = \alpha + \beta_1 \cdot P_{\text{W return}} + \beta_2 \cdot (E_{\text{W return}} - P_{\text{W return}}) + \beta_3 \cdot P_{\text{W return}}^2 + \epsilon_i
\]

\[
\epsilon_i \sim N(0, \sigma_i^2)
\]

\[
\sigma_i^2 = \omega + \alpha_v \cdot \sigma_{i-1}^2 + \beta_v \cdot \alpha_v
\]

<table>
<thead>
<tr>
<th></th>
<th>(\alpha)</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(\omega)</th>
<th>(\alpha_v)</th>
<th>(\beta_v)</th>
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<tr>
<td>Coefficient</td>
<td>-4.6E-09</td>
<td>1.04432</td>
<td>1.07232</td>
<td>0.43170</td>
<td>2.29E-09</td>
<td>0.03765</td>
<td>0.96148</td>
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<td>Std error</td>
<td>1.18E-05</td>
<td>0.00108</td>
<td>0.00311</td>
<td>0.01675</td>
<td>5.47E-10</td>
<td>0.00279</td>
<td>0.00278</td>
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<td>t-statistic</td>
<td>-0.00039</td>
<td>964.390</td>
<td>344.251</td>
<td>25.7725</td>
<td>4.178975</td>
<td>13.4734</td>
<td>345.436</td>
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<td>P-value</td>
<td>0.99</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table III Contribution of the risk factors to the strategy over-performance in FTSE100, CAC40, SP100 and DJIA universes

The contributions to over-performance of the three sources, market premium, value premium and volatility premium, are obtained by estimating model (3) on each of the 300 randomly selected stock subsets from FTSE100, CAC40, SP100 and DJIA universes, and averaged across each index. The results are reported for the estimation period Apr 97 to Jul 02.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Market</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTSE100</td>
<td>0.14%</td>
<td>0.85%</td>
<td>2.19%</td>
</tr>
<tr>
<td>CAC40</td>
<td>-0.40%</td>
<td>0.89%</td>
<td>1.72%</td>
</tr>
<tr>
<td>SP100</td>
<td>-0.82%</td>
<td>0.88%</td>
<td>2.21%</td>
</tr>
<tr>
<td>DJIA</td>
<td>1.57%</td>
<td>0.47%</td>
<td>1.42%</td>
</tr>
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</table>