Multivariate GARCH Models:
Software Choice and Estimation Issues

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Abstract

A large number of important practical tasks can be accomplished using a multivariate GARCH model. This paper examines the relatively small number of software packages that are currently available for estimating such models, in spite of their widespread use. The review focuses upon estimation issues and differences in available options for controlling the optimisation, and the review then considers an application to the estimation of optimal hedge ratios. Large differences in estimated parameters and standard errors are observed, but these are found to generate only modest differences in optimal hedge ratios and virtually indiscernible differences in model performance measures.

Keywords: multivariate GARCH, hedge ratio estimation, software.

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1. Introduction

The development of multivariate generalised autoregressive conditionally heteroscedastic (MGARCH) models from the original univariate specifications represented a major step forward in the modelling of time series. MGARCH models permit time-varying conditional covariances as well as variances, and the former quantity can be of substantial practical use for both modelling and forecasting, especially in finance. For example, applications to the calculation of time-varying hedge ratios, value at risk estimation, and portfolio construction have been developed.

Whilst a number of reviews have investigated the accuracy, ease of use, availability of documentation and other attributes of the software available for the estimation of univariate GARCH models (see, for example, Brooks 1997; McCullough and Renfro, 1999; Brooks, Burke and Persand, 2001), to our knowledge none has yet conducted a comparative study of the usefulness of the various packages available for multivariate GARCH model estimation, in spite of the empirical importance of this class of models.

Brooks, Burke and Persand (2001) employed the FCP (1996) benchmark for evaluating the accuracy of the parameter estimates in the context of univariate GARCH models and stressed the importance of the development of benchmarks for other non-linear models, including others in the GARCH class. However, there are currently no benchmarks yet developed for multivariate GARCH models, and therefore, the tone of this review will be somewhat different to that of Brooks et al. (2001). Clearly, in the absence of a benchmark against which to gauge the parameter estimates from each package, it will not be possible to write in terms of one package being more or less accurate than another; rather, all that can be done is to point out the differences in results that can arise if a different package is employed. In order to determine how large are the potential practical
implications of any differences in coefficient estimates, we employ the data that were used by Brooks, Henry and Persand (2002) in their estimation of optimal hedge ratios.

The remainder of this paper is organised as follows. Section 2 briefly outlines the multivariate GARCH class of models and describes the data that we employ. Section 3 describes the packages that we examine, together with some discussion of their relevant features, while Section 4 presents the results. Finally, Section 5 offers some concluding remarks.

2. Multivariate GARCH Models and Data

Several different multivariate GARCH model formulations have been proposed in the literature, and the most popular of these are the VECH, the diagonal VECH and the BEKK models. Each of these is discussed briefly in turn below; for a more detailed discussion, see Kroner and Ng (1998).

Introducing some notation, let $H_t$ denote an $N \times N$ conditional variance-covariance matrix, $\Xi_t$ an $N \times 1$ vector of innovations, $\Psi_{t-1}$ represent the information set at time $t-1$, then the conditional variance-covariance equations of the unrestricted VECH model may be written

$$\text{VECH}(H_t) = C + A \text{VECH}(\Xi_{t-1} \Xi_{t-1}') + B \text{VECH}(H_{t-1}), \quad \Xi_t | \Psi_{t-1} \sim N(0, H_t)$$

(1)

where $C$ is an $(N(N+1)/2) \times 1$ vector containing the intercepts in the conditional variance and covariance equations, $A$ and $B$ are $(N(N+1)/2) \times (N(N+1)/2)$ matrices containing the parameters on the lagged disturbance squares or cross-products and on the lagged variances or covariances respectively. The term “VECH” arises from the use of the
VECH(\cdot) column-stacking operator applied to the upper triangle of the symmetric matrix.

A potentially serious issue with the unrestricted VECH model described by equation (1) is that it requires estimation of a large number of parameters. Even in the context of a trivariate system (\( N = 3 \)), an astonishing total of 78 parameters require estimation in the variance and covariance equations. This over-parameterisation led to the development of the simplified diagonal VECH model by Bollerslev, Engle and Wooldridge (1988), where the \( A \) and \( B \) matrices are forced to be diagonal, resulting in a reduction of the number of parameters in the variance and covariance equations to 18 for the trivariate case.

In order for an estimated multivariate GARCH model to be plausible, \( H_t \) is required to be positive definite for all values of the disturbances, but even checking this condition is a non-trivial issue for VECH or diagonal VECH models of moderate size or larger. To circumvent this problem, Engle and Kroner (1995) proposed a quadratic formulation for the parameters that ensured positive definiteness, and this became known as the “BEKK” model\(^1\).

Finally, an alternative specification proposed by Bollerslev (1990) was the constant correlation model. It does seem somewhat bizarre to allow the both the conditional variances and conditional covariances to vary over time but in a restricted way so that the conditional correlations are time-invariant; it is also not clear whether such an assumption of constant conditional correlations would be supported by the data in reality.

\(^1\) The model acronym arises from the first letters of the surnames of the authors, with Bollerslev and Krafts being co-authors on the original version of the paper.
Nonetheless, this model exists as an alternative specification that may be slightly easier to estimate than the less restrictive diagonal VECH.

In order to simplify matters as much as possible, we employ only the diagonal VECH representation, and we estimate only a bivariate system. This model is still probably more widely employed than the BEKK, and the parameters of the former model are more easily interpreted.

Although any set of data could potentially be used to compare the relative merits of the software packages, we employ a dataset that has a practical application to the estimation of optimal hedge ratios so that the full implications of the results can be highlighted. The data employed in this study are taken from Brooks, Henry and Persand (2002) and comprise 3580 daily observations on the FTSE 100 stock index and stock index futures contract spanning the period 1 January 1985 - 9 April 1999. Days corresponding to UK public holidays are removed from the series to avoid the incorporation of spurious zero returns. Letting $S_t$ and $F_t$ denote the spot (i.e. cash) and futures prices respectively, the returns series are denoted by lower case letters and are calculated as $s_t = 100 \times \left( S_t / S_{t-1} \right)$ and $f_t = 100 \times \left( F_t / F_{t-1} \right)$ in the usual fashion.

The conditional mean equations for the model that we estimate can be written as

$$ Y_t = M + \Xi_t \, , \, \Xi_t \sim N(0,H_t) $$

(2)

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2 Since Brooks, Henry and Persand (2002) estimated only BEKK models, and this paper uses the diagonal VECH representation, our results are not directly comparable with theirs.

3 Since these contracts expire 4 times per year - March, June, September and December - to obtain a continuous time series we use the closest to maturity contract unless the next closest has greater volume, in which case we switch to this contract. Extensive further details of the data can be found in Brooks, Henry and Persand.
where \( Y_t = \begin{bmatrix} s_t \\ f_{jt} \end{bmatrix} \), \( M \) is a \( 2 \times 1 \) vector of intercepts in the conditional mean (\( M = \begin{bmatrix} \mu_s \\ \mu_f \end{bmatrix} \)),

and with the conditional variance-covariance equations being given by (1) using diagonal forms for \( A \) and \( B \). The conditional variance-covariance matrix, \( H_t \), will comprise the elements \( h_{s,t} \) and \( h_{f,t} \) on the leading diagonal and \( h_{s,f,t} \) as both of the off-diagonal terms.

For clarity, the conditional mean equations can be written out separately as

\[
\begin{align*}
    s_t &= \mu_s + e_{s,t} \\
    f_t &= \mu_f + e_{f,t}
\end{align*}
\]

with the conditional variance and covariance equations as

\[
\begin{align*}
    h_{s,t} &= c_1 + a_1 e_{s,t-1}^2 + b_1 h_{s,t-1} \\
    h_{f,t} &= c_2 + a_2 e_{f,t-1}^2 + b_2 h_{f,t-1} \\
    h_{s,f,t} &= c_3 + a_3 e_{s,t-1} e_{f,t-1} + b_3 h_{s,f,t-1}
\end{align*}
\]

The purchase or sale of futures contracts provides a method for hedging exposures to movements in the price of the underlying asset. In the present context, estimating an optimal hedge ratio would involve determining the optimal number of futures contracts that should be sold per holding of the spot asset. Many studies have compared the performance of time-varying hedge ratios estimated using multivariate GARCH models with those of naïve or time-invariant hedge ratios estimated using OLS regressions. The majority of these studies have preferred the time-varying approach (see, for example, Baillie and Myers, 1991) on the grounds that they provide slightly more accurate hedge ratio estimation leading to portfolio returns with lower variances. Given the coefficients and fitted values from the estimated model, it is possible to show that the optimal hedge ratio will be given by the negative of the ratio of the one-step ahead forecast of the covariance between the spot and futures returns to the one-step ahead forecast of the futures return variance:
When the hedge ratio is expressed in this way, the returns to the hedged portfolio can be written as
\[ r_{p,j} = s_i + \beta_{i-1}^* f_j \]  \hspace{1cm} (6)

It is also possible to express the variance of the returns to the hedged portfolio as
\[ \text{var}(r_{p,j}) = h_{s,j} + \beta_{i-1}^* h_{f,j} - 2 \beta_{i-1}^* h_{s,f,j} \]  \hspace{1cm} (7)

3. The packages

3.1 Background

Brooks, Burke and Persand (2001) evaluated 9 packages for the estimation of univariate GARCH models. Of these 9, only 4 contain pre-programmed routines for the estimation of multivariate GARCH models: EViews, GAUSS, RATS and SAS. Thus, multivariate GARCH models cannot be estimated using the currently available versions of LIMDEP, MATLAB, MICROFIT, SHAZAM, or TSP\(^4\). In addition, whilst the current version of EViews (4.0) incorporates sample routines for estimating the BEKK formulation, it does not include similar instructions for estimating a diagonal VECH model. Even though code for estimating the latter model could be obtained by making relatively trivial modifications to the former, we chose not to include EViews in the review, since the resulting assessment would be a joint one of EViews estimation of the VECH model and our programming skills in that package.

\(^4\) Of course, provided that the package incorporates some sort of programming capability for users, and that it is possible to manipulate the maximum-likelihood estimation routines, a skilled programmer may be able to set up the model and estimate it herself. This may be possible with, for example, MATLAB (although multivariate GARCH models have not been already coded into the MATLAB GARCH toolbox), although it would prove impossible for a pure “click-and-point” package such as MICROFIT.
Given the widespread use of this class of models, and that they are now more than a
decade and a half old, it is rather surprising that more developers have not included
routines to estimate such models in their packages. For any package that contains a
maximum likelihood optimiser, an extension to allow for MGARCH models would not
be a difficult exercise. In addition to the packages employed by Brooks et al. (2001) that
allow for MGARCH model estimation, this review also considers the “FINMETRICS”
add-in module for S-PLUS\(^5\). Other packages, including PC-GIVE and STATA were
investigated, but these too only included the provision for estimating univariate GARCH
models.

Table 1 presents contact and version details for the four packages. Clearly, a first concern
is whether the package in question is able to estimate the model of interest for a
particular researcher, and therefore the last 4 columns of Table 1 indicate which models
from the list of full VECH, constant correlation, diagonal VECH and BEKK the
packages are able to estimate. It turns out that most of the packages are fairly flexible,
and allow the estimation of at least three of the four types of multivariate GARCH
model. The only exceptions are that the full unrestricted VECH is not available with
FANPAC or FinMetrics and the constant correlation model is not available with SAS –
although neither of these probably represent an important loss of functionality in practice.

3.2 Flexibility versus Functionality
Clearly there is an important trade off in practice between flexibility and ease of use. We
would argue that multivariate GARCH formulations are sufficiently complex that those
researchers with no programming ability at all are unlikely to be consumers of such

\(^5\) Jean-Philippe Peters and Sebastien Laurent are currently in the process of producing a new version of
their “G@RCH” add-in for OX, and it is understood that their new version will include the capability to
models, and therefore that the range of estimable models and the range of estimation options available are likely to be more important criteria for determining the usefulness of the software than how many buttons must be pressed before some results are obtained.

An important question in practice, therefore, is whether the researcher can “get at the likelihood object” in other words, can the user add exogenous variables into the conditional variance or covariance equations or can the user employ an alternative (e.g., logarithmic) specification for the equations or employ an alternative distribution for the underlying disturbances? The answer, subject to the researcher being a sufficiently adept programmer in the package concerned, is “yes” for any package where the user specifies how the equations to be estimated and the log-likelihood function are set up. This would be the case for RATS, where an exogenous variable could simply be added to the desired equation. But the range of estimable models is much more limited for GAUSS-FANPAC, SAS, or for S-Plus FinMetrics, where the researcher simply calls a sub-routine that is hard-coded and into which no access is granted. The latter packages of course therefore entail a much more compact set of instructions to estimate the model – approximately 13 and 15 lines respectively for GAUSS and SAS compared to perhaps double that for RATS. Once the data are loaded into memory, the estimation in S-PLUS FINMETRICS can be performed in one line, making it by far the most compact set of code.

3.3 Speed and Documentation

Given the computer power that is now widely available, the speed at which models are estimated is scarcely an issue worth mentioning in a software review unless one is

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6 FINMETRICS does permit the user to select $t$-distributed disturbances instead of Gaussian, and to add additional variables into the conditional mean or variance equations, and to employ higher-order terms in the conditional variance or covariance equations. Therefore, it does offer a considerable degree of flexibility, but less than the complete control users can obtain from RATS.
conducting a Monte Carlo study where such models must be estimated tens of thousands of times. For the 4 packages considered here, there was little to choose between them in terms of the time taken to estimate the models - typically 1 or 2 minutes were required on a Pentium II – 333 MHz P.C. with 196 Mb RAM and running Windows 98.

The documentation related to the estimation of multivariate GARCH models for each of the packages is adequate; ideally help should be available on-line as well as in hard copy form. Arguably, GAUSS FANPAC and S-PLUS FINMETRICS provide the most extensive written documentation on this particular class of models, and the maximum likelihood routine is also well described in the RATS manual. SAS provides less written documentation on the operation of that particular part of the software, which is somewhat disappointing given that the combined SAS manuals run to several thousand pages. However, substantially more detail on “PROC VARMAX” is available on-line.

4. Model Estimation and Results

We estimate the model parameters using as close to the default settings as possible with each package. There are two reasons for doing this. First, anecdotal evidence suggests that many researchers simply employ the default settings on the grounds of simplicity without examining whether they are optimal. Exclusive use of default settings can also occur as a result of the researcher’s lack of knowledge of the details of the package or of the technical details of how the estimation actually operates. Second, to the extent that any one approach to estimation can be considered generally superior to others, it is reasonable to assume that the developers would make the default model estimation routines the ones that are likely to be the most reliable or robust, rather than hiding them.

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7 All packages were run on the same computer and Windows platform to ensure consistency.
in a footnote in the manual. Additionally, Fiorentini et al. (1996) demonstrated via a Monte Carlo study that in the context of univariate GARCH model estimation, increased accuracy results from the use of analytic gradients and Hessian than from approaches based only on numerical approximations. Analytic information is used in computing the derivates when estimating univariate GARCH models by GAUSS and SAS but not by RATS or S-PLUS, whilst analytic information is not used in construction of the Hessian under any of the packages; only numerical procedures are used for computing the derivatives when estimating multivariate GARCH models under all packages.

Table 2 shows the results from estimating the bivariate GARCH model using the spot and futures returns described above. The parameter estimates are shown to 3 decimal places and the asymptotic $t$-ratios to 2. An interesting side-issue is the considerable variation in the apparent precision with which these numbers are reported: GAUSS-FANPAC only reports to 3 decimal places, SAS and S-PLUS FINMETRICS to 5, and RATS to 9.

The default estimations under SAS failed, and once this happens, there is no unique way to proceed. The SAS developers have stated that the PROC VARMAX procedure is “experimental rather than production” in version 8.2, as well in versions 9 and the forthcoming 9.1. SAS estimation resulted in a non-positive definite variance-covariance matrix, but a switch from the default optimisation to the Quasi-Newton approach using the “nloptions tech=quanew;” instruction for SAS results in plausible parameter and standard error estimates. Default estimation using GAUSS-FANPAC, RATS and S-

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9 The SAS developers have recommended the use of the “UDP=DDFP” and “MAXFUNC=6000” specifications for this data and model. This will estimate the model using quasi-Newton optimisation with the dual Davidson Fletcher Powell (DFP) update of the Cholesky factor of the Hessian matrix with the maximum possible number of function calls raised to 6000.
PLUS FINMETRICS results in plausible parameter and standard error estimates without user intervention\textsuperscript{10}. Note that, in the absence of a benchmark with which to compare the estimated parameters and their standard errors, it is really impossible to say any more about them other than to assess in a qualitative sense whether they seem sensible given the results of existing studies using similar models.

Examining first the parameter estimation, the degree of variation between the packages is both surprising and potentially worrying. The intercepts in the conditional mean equations are similar for GAUSS, RATS and SAS, but are almost a third higher for S-PLUS. However, it is the conditional variance and covariance equations where the differences across packages become marked. The intercept in the spot (cash) conditional variance equation \((c_1)\) is around 0.01 for RATS and SAS, but 0.08 for S-PLUS and 0.4 for GAUSS – a 40-fold gap between the highest and lowest estimate. An even bigger divergence occurs with the estimates for the same parameter in the futures conditional variance equation and in the covariance equation \((c_2, c_3\) respectively). The parameters on the lagged squared errors \((a_1, a_2)\) are also higher for GAUSS and S-PLUS than for RATS or SAS, but this time only by a factor of around 4. Finally, the parameter estimates for the lagged conditional variances and covariance are again close for RATS and SAS at around 0.95, whereas they are around 0.4 for GAUSS and 0.8 for S-PLUS.

In some senses, GAUSS is the odd one out, spreading the weight in the measure of persistence equally on \(h_t\) and \(\varepsilon_{t-1}^2\), whereas the other three packages give much bigger estimates on \(h_t\) than on \(\varepsilon_{t-1}^2\). Interestingly, the variation in estimation of the same parameter across packages is far greater than the variation in estimation for the same parameter across equations for a given package. This may arise from the tendency for a

\textsuperscript{10} Note that by “plausible”, all we mean is that the parameter estimates in the conditional variance equations are positive and non-explosive, and that the standard errors are also positive.
given package to use the same set of initial estimates for the parameters on the lagged squared error and lagged conditional variance/covariance for all equations.

Turning now to the standard error estimation, the results of which are given in the second panel of Table 2, and the $t$-ratios given in the third panel, it is evident that the differences across packages are even more marked than they were for the parameter estimates. The $t$-ratios for SAS are considerably larger than those of the other packages for all of the parameters, resulting from SAS’s orders of magnitude smaller estimates of the standard errors. Most notably, the SAS $t$-ratios are around 100 times higher than the next highest set for the intercept in the conditional mean spot equation and for the parameter on the lagged futures conditional variance. However, none of these differences are important for tests of significance: given the large sample size, all of the parameters are statistically significant at the 0.1% level under all packages.

The differences in standard error estimation are arguably unsurprising since a similar result was found by Brooks et al. (2001) in the context of the estimation of simpler univariate GARCH models. But the differences in parameter estimation are substantial, and this result is quite in contrast with Brooks et al., who found only modest differences across software. Multivariate models, by their very nature, are inherently more complex to estimate than their univariate counterparts, and this considerably increases the scope for the optimisation routine to run into problems: for example, to find only a local optimum or not to converge at all. Two obvious questions arise from these results: First, why are the parameter estimates so very different, and second, does it matter? The first of these questions could probably be answered by examining the differences in optimisation technique across packages. Differences could arise in the default settings according to the optimisation routine used (e.g. BHHH versus Newton), the use of analytic or numerical
derivatives, differences in initialisations for the error and conditional variance / covariance series, differences in parameter initial estimates, or differences in convergence criteria. A thorough examination of all of these issues is virtually impossible since the packages on the whole simply do not give sufficient detail on these points.

Ideally, a package would give as much flexibility as possible for users to specify the optimisation controls, and arguably the best package in this regard is RATS. Only RATS gives the opportunity for the user to modify all of the controls in the list above. In terms of optimisation routine, GAUSS and SAS use a version of BFGS whereas S-PLUS uses BHHH with no opportunity to use an alternative approach. GAUSS does not allow modification of the convergence criteria, the initialisations of the error and variance/covariance series or the starting values for the parameter estimates. In terms of the methods that can be used to calculate standard errors, a method based on the Hessian (default) or QML are available with GAUSS, the Hessian (default), OPG or QMLE are available with RATS, the Hessian only is available with SAS, while Hessian, OPG (default) and QMLE are available with S-PLUS.

Now addressing the issue of whether the differences in parameter estimation between packages makes a difference from a practical perspective, we calculate the (in-sample) time-varying hedge ratios using equation (5) above together with the series of fitted conditional variances and covariances for each package. Unfortunately, it is not possible to use SAS to perform this calculation since the current version of the “PROC VARMAX” procedure does not permit the user to output the fitted conditional variances or covariances. The optimal hedge ratios (OHRs) calculated in this fashion for the remaining three packages are plotted in Figure 1. Given the in some cases enormous differences in parameter estimation, the profiles of the OHRs are quite similar, although
there is clear evidence of it being considerably more variable for GAUSS and S-PLUS than for RATS. On the whole, however, the OHRs are rather unstable, ranging from below 0.4 to above 1, and thus any firm attempting to use an MGARCH model for this purpose would face substantial rebalancing costs. This range compares with a time-invariant OHR calculated using OLS (in Microsoft Excel) of 0.80.

Finally, given that OHR’s have been constructed using each of the packages, it is possible to examine how much protection these would have offered a firm in terms of reduced portfolio volatility, measured by the standard deviation of portfolio returns. These results are presented in Table 3, together with those arising from the use of the time-invariant OLS hedge and from using no hedge at all. The mean of the portfolio returns, calculated using equation (6) is not of direct interest since the objective of hedging is to reduce volatility and not to increase returns. Remarkably, in spite of the enormous differences in parameter estimates, the standard deviations of portfolio returns (calculated by taking the square root of equation (7)) are almost identical across the 3 packages (and the OLS hedge). Thus, whilst the benefit from engaging in hedging is clear, it does not matter which package you use to calculate the OHRs and you are just as well not to bother with MGARCH models at all but to stick to OLS!

Conclusions

This review has sought to compare and contrast the four packages available for estimating multivariate GARCH models: GAUSS-FANPAC, RATS, SAS and S-PLUS. Considerable differences in the resulting parameter estimates were observed, but these turned out to be relatively unimportant from a practical point of view. But how can this be the case? The answer appears to lie in the differences between the packages cancelling out to a large extent, and this cancelling out occurs on two levels. First, estimates of the
unconditional variances and covariances are much closer across the packages than the parameter estimates would suggest. For example, the unconditional variances of the spot returns implied by the model estimates are 0.82 (GAUSS), 0.92 (RATS), 0.77 (SAS), and 0.84 (S-PLUS). Further cancelling out appears to arise when both the conditional variances and covariances are over-calculated and then the latter is divided by the former in the construction of the hedge ratio.

To summarise, it is worth reiterating that in the absence of a benchmark dataset and results, it is not possible to say which set of parameter estimates arising from the various software packages is “best”, but clearly *prima facie* they represent very different characterisations of the data. There is much work to be done if this class of models is to be reliably used in practice and we argue that the development of such a benchmark would be a worthwhile activity. A further implication of our results is the indication that researchers should focus upon the end use of their model when attempting to evaluate it and not necessarily on the parameter estimates.

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References


Table 1: Details of Packages Employed and the Range of Estimable Multivariate GARCH Models

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<th>Package and Version used</th>
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<tr>
<td></td>
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<td>GAUSS 3.2.39- FANPAC 1.1.11/2</td>
<td>Aptech Systems Inc, 23804 S.E. Kent, Langley Road, Maple Vallet, WA 98038, USA <a href="http://www.aptech.com">http://www.aptech.com</a></td>
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<td>Estima, PO Box 1818, Evanston, IL 60204-1818, USA <a href="http://www.estima.com">http://www.estima.com</a></td>
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<tr>
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<td>SAS Institute, Campus Drive, Cary NC 27513 USA. <a href="http://www.sas.com">http://www.sas.com</a></td>
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</tr>
<tr>
<td>S-PLUS 6.1 FINMETRICS 1.0</td>
<td>Insightful Corporation 1700 Westlake Avenue N, Suite 500 Seattle WA 98109-3044 USA <a href="http://www.insightful.com">www.insightful.com</a></td>
<td>No</td>
</tr>
</tbody>
</table>

1 At the time of writing this review, SAS 8.2 was the most up-to-date version of the software available, although version 9 is now available. It is possible that the results for the latter version may be quite different from the former.
Table 2: Parameter Estimates for Multivariate GARCH Model using FTSE Spot and Futures Returns: 1 January 1985 - 9 April 1999

Model:

\[ s_t = \mu_s + \varepsilon_{s,t}, \]

\[ f_t = \mu_f + \varepsilon_{f,t}, \]

\[ h_{s,t} = c_1 + a_1 \varepsilon_{s,t-1}^2 + b_1 h_{s,t-1} \]

\[ h_{f,t} = c_2 + a_2 \varepsilon_{f,t-1}^2 + b_2 h_{f,t-1} \]

\[ h_{s,f,t} = c_3 + a_3 \varepsilon_{s,t-1} \varepsilon_{f,t-1} + b_3 h_{s,f,t-1} \]

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<th>( \mu_f )</th>
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<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( c_2 )</th>
<th>( a_2 )</th>
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<td>0.064</td>
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<td>0.961</td>
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<td>0.032</td>
<td>0.959</td>
</tr>
<tr>
<td>S-PLUS</td>
<td>0.073</td>
<td>0.082</td>
<td>0.076</td>
<td>0.112</td>
<td>0.798</td>
<td>0.125</td>
<td>0.134</td>
<td>0.762</td>
<td>0.099</td>
<td>0.120</td>
<td>0.773</td>
</tr>
</tbody>
</table>

Panel B: Standard Error Estimates

<table>
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<tr>
<th>Package</th>
<th>( SE(\mu_s) )</th>
<th>( SE(\mu_f) )</th>
<th>( SE(c_1) )</th>
<th>( SE(a_1) )</th>
<th>( SE(b_1) )</th>
<th>( SE(c_2) )</th>
<th>( SE(a_2) )</th>
<th>( SE(b_2) )</th>
<th>( SE(c_3) )</th>
<th>( SE(a_3) )</th>
<th>( SE(b_3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAUSS</td>
<td>0.014</td>
<td>0.016</td>
<td>0.030</td>
<td>0.013</td>
<td>0.041</td>
<td>0.044</td>
<td>0.013</td>
<td>0.039</td>
<td>0.014</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
<td>RATS</td>
<td>0.014</td>
<td>0.016</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>SAS</td>
<td>0.019</td>
<td>0.019</td>
<td>0.073</td>
<td>0.098</td>
<td>0.192</td>
<td>0.008</td>
<td>0.023</td>
<td>0.047</td>
<td>0.018</td>
<td>0.038</td>
<td>0.060</td>
</tr>
<tr>
<td>S-PLUS</td>
<td>0.013</td>
<td>0.015</td>
<td>0.005</td>
<td>0.007</td>
<td>0.010</td>
<td>0.009</td>
<td>0.007</td>
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<td>0.007</td>
<td>0.007</td>
<td>0.010</td>
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</tbody>
</table>

Panel C: Estimated t-ratios

<table>
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<tr>
<th>Package</th>
<th>( t(\mu_s) )</th>
<th>( t(\mu_f) )</th>
<th>( t(c_1) )</th>
<th>( t(a_1) )</th>
<th>( t(b_1) )</th>
<th>( t(c_2) )</th>
<th>( t(a_2) )</th>
<th>( t(b_2) )</th>
<th>( t(c_3) )</th>
<th>( t(a_3) )</th>
<th>( t(b_3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAUSS</td>
<td>4.57</td>
<td>4.00</td>
<td>12.57</td>
<td>9.85</td>
<td>10.02</td>
<td>12.86</td>
<td>11.15</td>
<td>9.36</td>
<td>33.86</td>
<td>10.67</td>
<td>31.64</td>
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<tr>
<td>RATS</td>
<td>4.51</td>
<td>4.23</td>
<td>9.24</td>
<td>17.00</td>
<td>344.79</td>
<td>9.52</td>
<td>16.25</td>
<td>407.69</td>
<td>9.51</td>
<td>15.96</td>
<td>375.43</td>
</tr>
<tr>
<td>SAS</td>
<td>313.91</td>
<td>350.61</td>
<td>13.10</td>
<td>37.65</td>
<td>496.86</td>
<td>126.45</td>
<td>134.55</td>
<td>999.00</td>
<td>52.88</td>
<td>82.96</td>
<td>999.00</td>
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<tr>
<td>S-PLUS</td>
<td>5.68</td>
<td>5.56</td>
<td>14.13</td>
<td>16.52</td>
<td>77.02</td>
<td>14.54</td>
<td>18.00</td>
<td>68.08</td>
<td>15.02</td>
<td>17.84</td>
<td>74.16</td>
</tr>
</tbody>
</table>

Note: The standard errors for SAS have been multiplied by 100 for display in the table.

Table 3: In-Sample Performance of Optimal Portfolios

<table>
<thead>
<tr>
<th>Package</th>
<th>Mean of Portfolio Returns</th>
<th>Standard Deviation of Portfolio Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAUSS – MGARCH</td>
<td>0.010</td>
<td>0.357</td>
</tr>
<tr>
<td>RATS – MGARCH</td>
<td>0.065</td>
<td>0.350</td>
</tr>
<tr>
<td>S-PLUS - MGARCH</td>
<td>0.009</td>
<td>0.355</td>
</tr>
<tr>
<td>OLS – Hedge</td>
<td>0.009</td>
<td>0.348</td>
</tr>
<tr>
<td>No hedge</td>
<td>0.046</td>
<td>0.962</td>
</tr>
</tbody>
</table>
Figure 1: Fitted Optimal Hedge Ratios