Equity Indexing, Cointegration and Stock Price Dispersion:
A Regime Switching Approach to Market Efficiency

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Abstract

This paper examines the performance of a general dynamic equity indexing strategy based on cointegration, from a market efficiency perspective. A consistent return in excess of the benchmark is demonstrated over different time horizons and in different, real world and simulated stock markets. A measure of stock price dispersion is shown to be a leading indicator for the excess return, and their relationship is modelled as a Markov switching process of two market regimes. We find that the entire ‘abnormal return’ is associated with the high volatility regime, so the presence of a latent risk factor cannot be ruled out. Moreover, any market inefficiencies identified by the dynamic indexing model are temporary and occur only in special market circumstances. Our results have implications for equity fund managers: we shown how, without any stock selection, solely through smart optimisation and market timing, the benchmark performance can be significantly enhanced.

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Introduction

The phenomenon of equity indexing has attracted considerable interest in the last ten years, from both academics and practitioners. Equity indexing is the most popular form of passive investment, aiming to replicate the risk and return characteristics of a benchmark, usually a wide stock market index. The passive investment industry as a whole has witnessed a remarkable growth during the last ten years, with a huge number of funds pegging their holdings to broad market indexes such as SP500. The very reason for adopting a passive strategy rests in a belief in market efficiency, which provides the theoretical foundation of indexing. Traditional capital market theory states that the market portfolio, as defined by Fama (1970), offers the highest level of return per unit of risk and the only way that investors can beat the market over the longer term is by taking greater risks. Moreover, empirically, active management has been shown to under-perform its passive alternative most of the time, even after transaction costs and administration fees (Jensen, 1968; Elton, Gruber, Das, Hlavka, 1993; Carhart, 1997). In this framework, passive investment and, in particular, indexing, is a natural choice.

From an operational perspective, however, one needs to make the distinction between a pure index fund, managed to replicate the performance of the market portfolio or benchmark exactly, and strategies such as enhanced index tracking, that extend it into active management. The latter are constructing well-diversified portfolios that have a stable relationship with the benchmark and try to take advantage of some pockets of market inefficiency.

According to Jensen’s (1978) definition of efficient markets, a trading strategy producing consistent risk-adjusted economic gains, after properly defined transaction costs and over a sufficiently long period of time, is evidence against the efficient market hypothesis (EMH). This approach to market efficiency, as compared to earlier ones, has the advantage of testability and has subsequently generated a great deal of empirical research. Most of these studies, employing for example technical analysis and filter rules (Alexander, 1964; Fama and Blume, 1966), have shown that even if different trading strategies are successful before transaction costs, after accounting for such costs the profits vanish. Published evidence of trading profitability, after properly defined transaction costs, is rather scarce, Lakonishok and Vermaelen’s (1990) paper being one of the very few to document the profitability of some trading rules designed to exploit anomalous price behaviour.

In this paper, we investigate the performance of a dynamic indexing strategy that has recently come to the attention of many fund managers – the cointegration based index tracking,
introduced by Alexander (1999). We confirm in several real world and simulated stock market universes the finding that a portfolio constructed on a cointegration relationship with a price weighted index and comprising the same stocks produces, out of sample and after transaction costs, positive return in excess of its benchmark, which we call ‘abnormal return’. To note, the terminology employed is not meant to suggest, \textit{a priori}, market inefficiency. The aim of this paper is to investigate the statistical properties of the abnormal return and the extent to which this can be considered evidence against the EMH.

We find that the pattern of the abnormal return exhibits a pronounced time-variability: periods of stationary, zero mean returns, are alternating with periods during which positive returns are consistently accumulated. Since the only information used to construct the cointegration tracking strategy is the history of the stock prices, the cause of the abnormal return should be linked to the time-variability characteristics of the stock prices in the system. We introduce a new measure of stock price ‘cohesion’, which we call index dispersion, and find that this is a leading indicator of the abnormal return. Throughout the analysis we justify the conclusions drawn from real-world and simulated stock and index prices. Beginning with the simplest two-stock scenario, we illustrate the connection between stock prices, portfolio weights and index out-performance. In the real-world universe of the Dow Jones Industrial Average (DJIA), we document a significant non-linear relationship between the abnormal return and the lagged dispersion, which, however, has a considerable time-variability in parameters. To address this issue, we estimate a Markov switching model for the abnormal return and find strong evidence of a latent state variable, which determines the form of the linear relationship between the abnormal return and the stock prices dispersion. The Markov switching model indicates the presence of two regimes having very different characteristics. The entire abnormal return is shown to be associated to the higher volatility regime, and in this context we cannot rule out the potential presence of a hidden risk factor. Therefore, we can at most infer that the inefficiencies identified by the cointegration relationship, if any, are temporary and occur in special market circumstances.

To summarise, our contributions to existing research are in two directions: first, we provide additional empirical evidence in the EMH debate and shed some light on the potential anomalies identified through cointegration and on the mechanism producing the abnormal return. Secondly, our findings have wide implications for the passive investment industry. We show that, without any stock selection or explicit timing attempts, which are attributes of active management, solely through smart optimisation, the benchmark performance can be significantly enhanced, even after accounting for transaction costs. Moreover, the strategy can
be applied to replicate any type of value or capitalisation weighted benchmark, not only wide market indexes.

The remainder of the paper is organised as follows: section one reviews the cointegration-based tracking strategy, defines the stock price dispersion and motivates a possible relationship between dispersion and the strategy performance; section two examines the real world relationship between the abnormal return and the lagged dispersion in the DJIA universe of stocks and motivates the need for a Markov switching framework; section three discusses the difference between the stock weights in the benchmark and the cointegrating portfolio which induce the observed relationships between dispersion and the abnormal return; section four introduces the Markov switching model of dispersion; section five makes statistical inferences and examines the predictive power of the Markov switching model; section six discusses the implications for the EMH; and finally, section seven summarises and draws the main conclusions.

1. Cointegration and Index Dispersion

The focus of this paper is the performance of a very general form of an indexing model, based on cointegration, which allows the replication of all types of benchmarks, with different numbers of stocks. The rationale for constructing portfolios based on a cointegration relationship with the benchmark, rather than correlation, rests on the following features of cointegration: the price difference between the benchmark and the replica portfolio is, by construction, stationary; the stock weights, being based on a large amount of history, have an enhanced stability; finally, there is a full use of the information contained in level variables such as stock prices. Moreover, a cointegration relationship between a benchmark and a portfolio comprising all or only part of its stocks is always easy to find when benchmarks are equally weighted, price weighted or capitalisation weighted, because the benchmark is just a linear combination of stock prices.

The basic cointegration model for a tracking portfolio comprising all the stocks included in the benchmark at a given moment is a regression of the form:

\[ \ln(benchmark_{t}) = c_{t} + \sum_{k=1}^{n} c_{k} \ln(P_{k_{t}}) + \varepsilon_{t} \]  

(1)

where the benchmark is reconstructed historically based on its current membership and weights, and \( n \) is the total number of stocks included in the benchmark.\(^{ii}\) All variables in the model, apart from the error term, are integrated of order one.\(^{iii}\) The specification of the model in natural log variables has the advantage that, when taking the first difference, the expected
returns on the portfolio will equal the expected returns on the benchmark, provided that the tracking error is a stationary process.\textsuperscript{iv}

We note that the application of ordinary least squares (OLS) to non-stationary dependent variables such as $\ln(\text{benchmark})$ is only valid in the special case of a cointegration relationship. The residuals in (1) are stationary if, and only if, $\ln(\text{benchmark})$ and the tracking portfolio are cointegrated. If the residuals from the above regressions are non-stationary, the OLS coefficient estimates will not be consistent and no further inference will be valid. Testing for cointegration is, therefore, essential in constructing cointegration optimal tracking portfolios. The Engle-Granger (1986) methodology for cointegration testing is particularly appealing in this respect for its intuitive and straightforward implementation. Moreover, its well-known limitations (small sample problems, asymmetry in treating the variables, at most one cointegration vector) are not effective in our case. The estimation sample is typically set to at least three years of daily data, there is a strong economic background to treat the benchmark as the dependent variable, and identifying only one, the most important cointegration vector is sufficient for our purposes. Further to estimation, the OLS coefficients in model (1) are normalised to sum up to one, thus providing the composition of the tracking portfolio.

Alexander and Dimitriu (2002) claim to have found evidence of abnormal returns from the cointegration indexing strategy, showing that tracking portfolio comprising all the stocks in the benchmark produces positive abnormal return in certain market conditions, even after accounting for transaction costs. This is a rather counterintuitive result, as one would expect the most complete combinations of stocks, very strongly cointegrated with the reconstructed benchmark, to produce out of sample excess returns having zero mean.

To investigate whether this result can be replicated, we have constructed random subsets of stocks in the FTSE100, CAC40 and SP100 universes. For each index we have set up 100 random portfolios comprising a fixed number of stocks (50 for FTSE, 25 for CAC and 80 for SP100) and determined a price-weighted benchmark for each portfolio. Each of the 300 benchmarks was tracked with a cointegration-optimal portfolio comprising all the stocks included in that particular benchmark. The optimal weights were rebalanced every 10 trading days based on the new OLS coefficients of the cointegration regression (1), re-estimated over a fixed-length rolling calibration period of 3 years of daily data preceding the portfolio construction moment. In between re-balancing, the portfolios were left unmanaged (i.e. the number of stocks is kept constant) and evaluated, based on the daily closing prices of the stocks.
Based on the rebalancing strategy detailed above, we have determined and reported in Table 1 the average annual abnormal return over the period 1997 to 2001 for all three stock universes. For comparison, we have also reported the excess return in the real-world DJIA universe. The first observation is that for the simulated benchmarks in all four universes there is a positive average abnormal return, when measured over the entire data sample. Regarding the time distribution, the last two years in the data sample are responsible for most of this excess return, except in the CAC case. In the case of FTSE simulated indexes the abnormal return in 2000 is almost 5%, while in the case of SP100 simulated indexes, the largest abnormal return occurs during 2001, amounting to 4.6%. Comparison with the benchmark returns in Table 1B indicates that the cointegration tracking strategies are making higher returns in down markets than in up markets: the correlation between the abnormal returns and the benchmark returns in Tables 1A and 1B is –0.377. These results do provide some evidence of abnormal returns from the cointegration tracking strategy, with a consistent time-variability pattern across different markets. However, without an in-depth understanding of the mechanism producing the abnormal return, one cannot exclude the hypothesis that this abnormal return is sample dependent, despite the fact that it is identified in different stock universes.

To investigate further the pattern of the abnormal return, we restrict the analysis to the DJIA universe, for reasons of space. Using daily close prices for the thirty stocks in the DJIA as of 31-Dec-01 and a sample period from 01-Jan-90 to 31-Dec-01, we have estimated the out of sample performance of a portfolio constructed based on model (1) with a rolling 3-year calibration period and 10-day recalibration/rebalancing frequency. The cumulative daily abnormal return during the entire sample period is shown in Figure 1 and amounts to 11.6%, before transaction costs.

The issue of the transaction costs is an important one, especially in connection with the EMH. We assume an amount of 20 basis points on each trade value to cover the bid-ask spread and the brokerage commissions, which is conservative for very liquid stocks such as the ones in DJIA. The transaction costs estimated over the entire data sample sum up to no more than 2.5%. Such an amount of transaction costs can hardly be regarded as affecting the overall performance of the strategy.³

Before taking the analysis further, we ask whether there is any connection between the abnormal return from the cointegration strategy and the profitability of value strategies, which have been advocated with reference to the DJIA stock universe for some time (Dorfman, 1988; Hirschey, 2000). When compared to a price weighted benchmark, an equally weighted portfolio should capture a potential value effect. As a proxy for value we therefore use an equally weighted portfolio, rebalanced at the same frequency as the cointegration
based tracking portfolio, and investigate its relationship with the abnormal return from index tracking. An exponentially weighted moving average correlation of the two shows no evidence of statistical significance, so we exclude the value effect as the relevant source of the abnormal return generated by the cointegration strategy.

A very noticeable feature in Figure 1 is the time variability of the excess return, which is far from being uniformly accumulated throughout the data sample. Periods of stationary excess returns alternate with periods during which abnormal returns are consistently accumulated. Moreover, a visual inspection indicates that the periods during which most of the abnormal return is accumulated coincide with the main market crises during the sample period: the Asian crisis, the Russian crisis and the technology market crash. A positive correlation between the abnormal return and the volatility of the benchmark therefore seems likely. However, the contemporaneous daily correlation between excess returns and changes in an exponentially weighted moving average volatility estimate for the benchmarkvi is only –0.03, when estimated over the whole period. This is clearly not significant, and neither is the correlation between excess return and lagged benchmark volatility changes, at 0.005. Even moving average estimates of these two correlations revealed no evidence of any significant relationship between the abnormal return and volatility. These findings were also robust to changes in the volatility model (using a different smoothing constant of the exponentially weighted moving averages, or a GARCH(1,1) estimate).vii Despite this, a visual inspection of Figure 1 does indicate that a positive co-dependency between abnormal returns and a risk factor is possible. However, if it exists, the risk factor is not proxied by the benchmark volatility, and/or the co-dependency is more general than a simple, linear correlation.

The issue of time-variability in fund performance is not new. A considerable body of empirical research has shown that hedge funds and mutual funds perform better in recession periods than in boom periods. This time-variability has been associated with informational asymmetries (Shin, 2002) and changes in the investment environment, according to the phase of the business cycle (Moskowitz, 2000; Kosowski, 2001). A separate line of research concerns trend following strategies, which have been found to generate returns similar to a lookback straddle paying the owner the difference between the highest and the lowest price of the underlying asset over the observation period (Fung and Hsieh, 1997 and 2001). Trend followers appear to perform best in extreme up or down markets, and less well during calm markets. Even without highlighting the cause of this behaviour, there is considerable interest from the investment community in the fact that these funds provide a partial hedge against general market conditions (Fung and Hsieh, 1997).
However, in our case, it is a purely statistical strategy that generates this abnormal return. Having excluded other potential explanations for the abnormal return, the over-performance of the cointegration portfolio has to be connected to the portfolio weighting system and its relationship with stock price dynamics. In order to understand better the relation between the benchmark and the portfolio weights based on model (1), we investigate the theoretical example of the simplest case of a benchmark $I_n$, computed as the average of only two stock prices. For constructing a portfolio tracking $I_t$ from the two stocks, according to model (1), we need the weight $w$ such that

$$
\log(I_t) = w \log(P_{1,t}) + (1 - w) \log(P_{2,t})
$$

(2)

It follows that $w = (\log(1 + a) - \log(2))/\log(a)$ where $a = P_1/P_2$. Therefore $w > 0.5$ if and only if $P_1 > P_2$, meaning that in (2), the stock with the higher price will also have a higher weight in the portfolio. The difference between the portfolio weights in model (2) and the benchmark weights, $w^* = a/(1 + a)$, will increase with the spread between the stock prices. The further away $a$ is from unity (i.e., the larger the dispersion between the two stock prices) the larger the weight on the stock with the higher price in the portfolio, relative to its weight in the benchmark. Therefore, a significant difference between the benchmark weights and the portfolio weights will occur when the dispersion of stock prices increases.

This clear-cut result motivates our study of index dispersion, i.e. the cross sectional standard deviation of the prices across their mean (which is the price weighted benchmark), defined as:

$$
d_t = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (P_{k,t} - I_t/I_t)^2 / n}
$$

(3)

For computing the time series of index dispersion, all stock prices are rescaled to be equal to 100 at the beginning of the period, the dispersion series therefore starting from zero. In Figure 2 we plot the time series of dispersion in the DJIA. After a steady increase, the dispersion increased substantially at the beginning of the technology sector boom, due to the sharp increase in the price of technology stocks, and a relative decline in price of other sectors. The highest dispersion occurred at the beginning of 2000, but since then the dispersion has decreased, most obviously during the crash of the technology bubble. We note that index dispersion in most major equity markets (either capitalisation or equally weighted) follows a similar pattern.
2. A Basic Time Series Analysis

In this section we examine the possibility that the stock price dispersion (3) has a significant relationship with the observed excess return from cointegration tracking. Since the stock price dispersion is integrated of order one, a basic stationary specification relates the abnormal return ($AR_t$) to the daily change in stock price dispersion ($DD_t$), including also the lagged abnormal return and some lagged changes in dispersion, together with the squared lagged change in dispersion, to account for potential asymmetries:

$$AR_t = \alpha + \beta_1 AR_{t-1} + \beta_2 DD_t + \beta_3 DD_{t-1} + \beta_4 DD_{t-2} + \beta_5 DD_{t-1}^2 + \epsilon_t$$

(4)

The estimation results based on the DJIA sample from Jan-92 to Dec-01 are presented in Table 2. Significant coefficients are associated with the lag of the excess return, the first lag and the squared lagged change in dispersion. The positive coefficient of the lagged excess return accounts for the autocorrelation in the abnormal return. Additionally, there is a negative and very significant relationship between the abnormal return and the lagged change in dispersion. Thus following an increase in dispersion, there will be a relative loss in the portfolio compared with the market. The significance of the squared change in dispersion term indicates a non-linearity in the relationship: the larger the absolute change in dispersion, the higher the abnormal return. Given the asymmetry in stock markets, i.e. stock prices tend to fall faster than they rise, a sudden large change in dispersion is likely to happen during market crises rather than during stable trending markets. Thus the highly significant positive value for $\beta_5$ confirms our initial observations on a possible connection between the abnormal return and the market crises periods. The contemporaneous and the second lag of the change in dispersion are not statistically significant. Finally, the fact that the excess return is determined by the lagged change in dispersion rather than by a simultaneous variable indicates that dispersion may be a useful leading indicator of the performance of this strategy.

However, on further investigation the structural stability of this relationship seems questionable. A popular test for parameter instability is the Chow F-test, which, however, requires a-priori knowledge of the break date. A test which does not require knowledge of the potential break-point is the CUSUM test (Brown, Durbin and Evans, 1975), but this is known to have low asymptotic power (Ploberger and Kramer, 1990). A rolling version of the Chow test (Andrews, 1993), with the breakpoint set at different dates in the sample is only valid under the assumption of equal error variance in all the regressions. If there is heteroscedasticity in the restricted model, then the calculated F-statistic is biased upward and indicates greater instability in the coefficient estimates than in fact exists (Toyoda, 1974). A rolling Goldfeld-Quandt test (Goldfeld and Quandt, 1965) estimated for the period for the
period Oct-92 to Oct-01 clearly rejects the null of errors homoscedasticity so the standard Chow test cannot be used.

Provided that the sample is sufficiently large, a Wald test that remains valid in the case of heteroscedastic errors can be used. A rolling Wald test (Andrews and Fair, 1988), estimated for the period for the period Oct-92 to Oct-01 indicates that the null hypothesis of no-structural break is most significantly rejected on 16th October 2000 (Figure 3). Consequently, two separate regressions are estimated, using data before and after this point, and these have quite different results (Table 3). Note that, when the impact of the change in dispersion is separated in sub-samples, the lagged dependent variable becomes insignificant, in both cases, meaning that it initially captured part of the non-linear relationship between the abnormal return and the change in dispersion. Thus it would be naïve to conclude from Table 2 that abnormal returns are the result of an autocorrelation induced by over-lapping in-sample periods. The main difference between the two sub-samples is the sign of the coefficients of the lagged dispersion. The slope coefficient of the lagged change in dispersion is, until Oct-00, very significant and negative, but after Oct-00, the relationship between the two variables becomes even more significant, and positive. The coefficient of the squared lagged change in dispersion is not significant in the first period, but significantly positive in the second sub-sample.


The parameter stability test results demonstrated that a very significant change in the behaviour of the abnormal return occurred in October 2000. Why then? To answer this we take a closer look at the markets during the period September-December 2000. This three-month period is the time of the second great fall in the Nasdaq composite index. Index volatility reached 47.59% and the index fell 48.25%, i.e. another 745.83 points, having already fallen 425 points from March 2000. Therefore, it is reasonable to infer that October 2000 marked the end of the technology bubble. The situation depicted in Figure 4 resulted as, following the burst of the technology bubble, the technology stock prices fell below the equilibrium prices, which were based on the very high levels of dispersion observed previously.

But why should abnormal return have a negative relationship with lagged dispersion before October 2000, and a positive relationship thereafter? Figure 4 offers some intuition behind this finding. Without loss of generality we shall use the example of a higher than average priced stock: for reasons of space we leave to the reader the equivalent intuition based on a lower than average priced stock.
Figure 4, section titled ‘regime one’ shows a stable (and upwards trending) market with a smooth line representing the long-run equilibrium price of this stock, on which the cointegration weights are constructed, and a wavy line representing the actual price of this stock, determining its benchmark weight. If the actual price of the stock increases, diverging from the rest of the stock prices, its weight in the benchmark will also increase. However, its weight in the cointegration portfolio, being based on a long history of prices, will not react immediately to the increase in the stock price, which could be just noise from a long-run equilibrium perspective. Therefore, the portfolio will be relatively under-weighted on this particular stock while its price is increasing, realising relative losses compared to the benchmark. And because the stock has higher than average price, whilst these relative losses are made, the dispersion is increasing. When the price increase reverts and the stock price starts falling, returning towards its long-run equilibrium level, the dispersion in the system also decreases whilst the portfolio will make a relative profit compared with the benchmark, because it is still under-weighted (relative to the benchmark) on a stock whose price is declining. Thus, dispersion should have a negative relationship with excess return in a stable market.

Now consider Figure 4, section titled ‘regime two’, which illustrates the stock price after an upward trend in the stock price followed by a sharp price decrease. The effect of the price fall is that (a) the cointegration relationship is now based on a historical equilibrium with a price well above the actual stock price shown by the wavy line, and (b) the dispersion in the index has decreased (because before the price fall, the stock had an above average price). The cointegrating portfolio will be over-weighted in this stock, relative to the benchmark, and the portfolio will realise a relative profit if the stock price increases. So in the ‘post-crash’ regime dispersion will have a positive relationship with excess return. Thus if dispersion increases during the recovery period, the cointegration strategy will generate abnormal returns.

Note that significant abnormal returns relative to the benchmark can be made during a period of sharp trend reversals. If prices rise in a stable market, this gives time for the cointegration-based portfolio to incorporate the new price information in portfolio weights; they move towards the weights in a price weighted benchmark and the cointegrating portfolio tracks the benchmark closely during such times. No abnormal returns are made. But price falls can be sudden, and they often precipitate further price falls. A sharp reversal in trend can occur after the prices of some stocks (e.g. technology stocks) have risen more rapidly than others. At the same time as the dispersion increases, the cointegrating portfolio will be based on an out-dated equilibrium, where these stocks are under-weighted relative to the benchmark. If prices subsequently fall across the market, and if the prices of these stocks fall more than prices of
other stocks, dispersion decreases and, at the same time, relative profits will be made by the cointegrating portfolio.

In our empirical example, before the technology crash, the prices of technology stocks were far too high relative to the prices of traditional stocks. This led to a high dispersion in the early part of 2000, which persisted in the cointegrating portfolio weighting system even after the dispersion of the actual prices had decreased, following the burst of the technology bubble. Thus during crash and, since the dispersion subsequently increased again, during the upturn period following the crash, the cointegration portfolio made significant excess returns.

Of course, the symmetric situation, where unintentional abnormal losses would occur following a market ‘hike’ is theoretically possible. However, it is a well-documented stylised fact of equity markets that stock prices usually fall more rapidly than they rise. This is the result of the leverage effect, frequently documented in equity markets (Black, 1976; Christie, 1982; French, Schwert and Stambaugh, 1987) and the presence of positive feedback: an initial sell reaction to some bad news will be followed by more selling, driving the prices faster below their fundamental levels (De Long, Shleifer, Summers and Waldmann, 1990). In the presence of such asymmetry, it comes as no surprise that mostly positive abnormal returns occurred in all the real and simulated markets that we have considered.

4. A Markov Switching Model

There is a clear time-variability in the parameters of the estimated regressions in Table 3, which can be accounted for with simple tools like structural break tests, but only at the expense of inducing a significant degree of arbitrariness. There appear to be some grounds for a structural break in the relationship between the abnormal return and dispersion in October 2000, but this does not ensure that the break identified is unique. To address these issues, we employ a Markov switching framework.

Belonging to a very general class of time series models, which encompasses both non-linear and time-varying parameter models, the regime switching models provide a systematic approach to modelling multiple breaks and regime shifts in the data generating process. Increasingly, regime shifts are considered to be governed by exogenous stochastic processes, rather than being singular, deterministic events. When a time series is subject to regime shifts, the parameters of the statistical model will be time varying, but in a regime-switching model the process will be time-invariant conditional on a state variable that indicates the regime prevailing at the time.
The importance of these models has long been accepted, and the pioneering work of Hamilton (1989) has given rise to a huge research literature (Hansen, 1992 and 1996; Kim, 1994; Diebold, Lee and Weinbach, 1994; Garcia, 1998; Psaradakis and Sola, 1998, Clarida, Sarno, Taylor and Valente, 2003). Hamilton (1989) provided the first formal statistical representation of the idea that economic recessions and expansions influence the behaviour of economic variables. He demonstrated that real output growth might follow one of two different auto-regressions, depending on whether the economy is expanding or contracting, with the shift between the two states generated by the outcome of an unobserved Markov chain.

In finance, the applications of Markov switching techniques have been many and very diverse: from modelling state dependent returns (Perez-Quiros and Timmermann, 2000) and volatility regimes (Hamilton and Lin, 1996), to option pricing (Aingworth, Das and Motwani, 2002), to detecting financial crises (Coe, 2002), bull and bear markets (Maheu and McCurdy, 2000) and periodically collapsing bubbles (Hall, Psaradakis and Sola, 1999), or to measuring mutual fund performance (Kosowski, 2001). Despite their limited forecasting abilities (Dacco and Satchell, 1988), Markov switching models have been successfully applied to constructing trading rules in equity markets (Hwang and Satchell, 1999), equity and bond markets (Brooks and Persand, 2001) and foreign exchange markets (Dueker and Neely, 2002).

The Markov switching model specified for the excess return assumes the presence of a latent variable (state variable), which determines the form of linear relationship between the abnormal return and the lagged dispersion in stock prices. The advantages of using a latent variable approach instead of a pre-defined indicator have been long documented. For example, when analysing business cycles, the Markov switching model produces estimates of the state conditional probabilities, which contain more precise information about the states that are driving the process than a simple binary indicator of the states, which is prone to significant measurement errors. The estimates of the conditional probability of each state allow more flexibility in modelling the switching process. An additional motivation for using a latent variable approach in this case, is the fact that there is no obvious indicator of the states of the process generating the abnormal return – the price dispersion appears to be the leading indicator of the abnormal return, but we have no prior on the determinant of the regime switches.

In the Markov switching model of abnormal return, the intercept, regression slope and the variance of the error terms are all assumed to be state-dependent. If we let $s_t$ denote the latent
state variable which can take one of $K = 2$ possible values (i.e. 1 or 2), then the regression model can be written as:

$$y_t = z_t' \beta_{S,t} + \varepsilon_{S,t} \quad (5)$$

where

$y_t =$ vector of the excess returns;

$z_t = (1, x_t, x_t^2),$ the matrix of explanatory variables;

$x_t =$ vector of lagged change in the prices dispersion;$x_i$

$\beta_{S,t} = (\mu_{S,t}, \gamma_{S,t}, \theta_{S,t})$ is the vector of state dependent regression coefficients;

$\varepsilon_{S,t} =$ vector of disturbances, assumed normal with state dependent variance $\sigma_{S,t}^2$.

The transition probabilities for the two states are assumed to follow a first-order Markov chain and to be constant over time:

$$P(S_t = j | S_{t-1} = i, S_{t-2} = i, \ldots) = P(S_t = j | S_{t-1} = i) = p_{ij}$$

The matrix of transition probabilities can be written as:

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{matrix} = (p_{ij})$$

If we let $\xi_t$ represent a Markov chain, with $\xi_t = (1, 0)'$ when $S_t = 1$ and $\xi_t = (0, 1)'$ when $S_t = 2$, then the conditional expectation of $\xi_{t+1}$ given $S_t = i$ is given by:

$$E(\xi_{t+1} | S_t = i) = \begin{pmatrix} p_{1i} \\ p_{2i} \end{matrix} = P_{xi_t}$$

The conditional densities of $y_t$, assumed to be Gaussian, are collected in a 2x1 vector: $\eta_t = (\eta_{1t}, \eta_{2t})$ where $\eta_{1t} = f(y_t | S_t = i, z_t ; \alpha)$ is the normal density function whose parameters $\alpha$ are conditional on the state. That is, $\eta_{1t} = \frac{1}{(2\pi\sigma_i^2)^{1/2}} \exp \left(-\frac{(y_t - z_t' \beta_i)^2}{2 \sigma_i^2}\right)$. The conditional state probabilities can be obtained recursively:

$$\hat{\xi}_{t+1|t} = \frac{\hat{\xi}_{t+1|t-1} \otimes \eta_{1t}}{1' (\hat{\xi}_{t+1|t-1} \otimes \eta_{1t})} \quad (6)$$

$$\hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t}$$

where $\hat{\xi}_{t|t}$ represents the vector of conditional probabilities for each state estimated at time $t$, based on all the information available at time $t$, while $\hat{\xi}_{t+1|t}$ represents the forecast of the same conditional probabilities based on the information available at time $t$ for time $t+1$. The symbol $\otimes$ denotes element-by-element multiplication. The $i^{th}$ element of the product
\( \tilde{\xi}_{t-1} \otimes \eta_t \) can be interpreted as the conditional joint distribution of \( y_t \) and \( S_t = i \). The numerator in expression (6) represents the density of the observed vector \( y_t \) conditional on past observations. Given the assumptions made on the conditional density of the disturbances, the log likelihood function can be written as:

\[
L(\alpha, P) = \sum_{i=1}^{T} \log f(y_t | z_t; \alpha; P) = \sum_{i=1}^{T} \log \Phi(\tilde{\xi}_{t-1} \otimes \eta_t)
\]

This approach allows the estimation of two sets of coefficients for the regression and variance of the residual terms, together with a set of transition probabilities.

Considering the complexity of the log likelihood function and the relatively high number of parameters to be estimated, the selection of starting values is critical for the convergence of the likelihood estimation. To reduce the risk of data mining, we have not used any state-dependent priors as starting values. Instead, we have used the unconditional estimates of the regression coefficients and the standard error of the residual term. Additionally, we have arbitrarily set \( \xi_{11} \) to \((1 \ 0)\). A number of restrictions needed to be imposed on the coefficient values, in order to ensure their consistency with model assumptions. The transition probabilities were restricted to be between 0 and 1, while a non-negativity constraint was imposed on the standard deviation of the residuals in both states.

In Markov switching models it is essential to ensure a sufficiently long data sample for correctly identifying the time-variability of parameters. The data sample covered 10 years of daily data from 1992 to 2002. In a correctly specified switching model, i.e. one in which the entire time-variability of the parameters is captured by the regime switching and within each regime the parameter estimates are time-invariant, the use of such a long data sample should not create any difficulties.

Table 4 reports the Markov switching model estimation results over the entire data sample. The only coefficient that is not statistically significant at 1% is the regression intercept, for the second state. A noteworthy difference between the two states concerns the coefficients of the lagged and squared lagged change in dispersion: in the second state, the coefficients are positive while in the first state they are negative, thus the lagged change in dispersion has a different effect on the abnormal return in the two states. In the second state an increase in the index dispersion is followed by a relative gain from index tracking in the next period, and the larger the change, the higher the abnormal return, while in the first state, a decrease in dispersion is the one associated with relative gains. The negative coefficient of the squared
lagged change in dispersion can be the effect of a predominance, in the first state, of dispersion changes in the direction of the trend, which should be negatively related to the abnormal return. In this context, the negative sign of the squared lagged change in dispersion is highly sample dependent.

Additionally, the standard deviation of the residuals is higher in the second state. Apart from higher volatility (4% p.a., as compared to 1.8% in regime one), and higher returns, state two returns have a positive skewness (1.02) and higher excess kurtosis (5.37). State one, despite having negative mean returns, is closer to normality (skewness 0.09; excess kurtosis 3.67).

Regarding the transition probabilities, the first state appears to be more persistent than the second one: the probability of staying in state one at time t+1 provided that at time t the process was in state one is 0.98, while the probability of remaining in state two, once there, is only 0.88.

If we split the sample observations between the two states based on the criterion of estimated probability, we can determine the abnormal return associated to each state. Based on this procedure, the number of observations in state one is almost three times the number of observations in state two, which concords with the lower persistence of state two. Also, the abnormal return generated during state two is even higher than the abnormal return generated by the entire process, because the first state generates a relative loss, even if not very significant. Figure 5 shows the cumulative excess return from each state: there is a consistent abnormal return produced in state two, while the first state produces a slightly negative abnormal return.

Based on the same separation procedure as above, we observe that, as opposed to the replica portfolio, the benchmark generated smaller returns in state two than in state one (the equivalent of 8.75% p.a. as opposed to 12.93% p.a.). The notable difference concerns, however, the volatility of the benchmark during each state: state two returns are associated with an annual volatility of 19%, while the benchmark returns corresponding to state one have only 13% annual volatility. Therefore, the tracking over-performance occurs in periods with lower returns and higher volatility for the market.

The time distribution of the states is an important feature to investigate. From Figure 6, which plots the estimated probability of the second state, it becomes clear that in the first half of the sample state one is prevailing, while towards the end of the sample, state two becomes predominant. Over the entire data sample, observations in state two represent 25% of the total number of observations. However, this distribution is far from being time invariant, since in the first half of the data sample, state two accounts for only 7% of the total number of
observations, while in the last two years of the data sample, i.e. 2000 and 2001, state two occurs 87% of time.

Our conclusion is that the two states have very distinctive characteristics: state two, which occurs less frequently, but is predominant during the last few years, is responsible for producing the entire abnormal return. This state occurs in more volatile market conditions and the over-performance is subsequent to an increase in the index dispersion. In the first state there is a negative, but not significant excess return, with any positive excess return occurring further to a decrease of the index dispersion.

We remark that the presence of two distinctive regimes for the out of sample tracking error could be an indication of a regime switching cointegration relationship between the benchmark and the tracking portfolio. Recently, there has been a surge in the research on regime switching cointegration (e.g. Hall, Psaradakis and Sola, 1997; Gregory and Hansen, 1996). However, in our case, structural breaks in the cointegration relationship between a stock market index and a portfolio comprising the same stocks are conceptually unlikely. As we have shown, it is difference between the cointegration weights and the price-weighted benchmark weights that is responsible for the strategy’s overperformance, but the regime switching properties of the relationship between the abnormal return and the lagged dispersion are driven by market conditions, that is, sudden changes in the price-weighted index structure. Also, we note that, if required, building the switching approach in the cointegration based tracking portfolio model is not a straightforward task due to several reasons: (1) the portfolio calibration period would need to be substantially extended to ensure a sufficiently long data sample for a proper switching model specification, (2) the estimation procedure would be significantly more complicated and sensitive to data specifics such as outliers and (3) the portfolio stability would most likely be affected and this would result in higher transaction costs.

5. Statistical Inference and Model Predictive Power

In order to validate the above inferences about the two-state process driving the abnormal return, one needs to test and reject the null hypothesis of no switching. Even if there is evidence that the abnormal return has different patterns in the two states, this does not imply that the asymmetries between the two states are also statistically significant.

Standard testing methods such as likelihood ratio tests are not applicable to Markov switching models due to the presence of nuisance parameters under the null hypothesis of linearity, or no switching. The presence of nuisance parameters gives the likelihood surface sufficient
freedom so that one cannot reject the null hypothesis of no switching, despite the fact that the parameters are apparently significant.

A formal test of the Markov switching models against the linear alternative of no-switching, which is designed to produce valid inference, has been proposed by Hansen (1992, 1996). This method implies the evaluation of the log likelihood function for a grid of different values for the regression coefficients, standard deviation and the transition probabilities. Following Hamilton (1996), we let \( \alpha = (\mu_1 - \mu_2, \gamma_1 - \gamma_2, \theta_1 - \theta_2, \sigma_1 - \sigma_2, p_{11}, p_{22})' \) denote the regime switching parameters of model and \( \lambda = (\mu_1, \gamma_1, \theta_1, \sigma_1)' \) denote the parameters which are not state dependent. The conditional log likelihood function for the parameters will be written as \( L(\alpha, \lambda) = \log f(y_t | y_{t-1}, y_{t-2}, ..., y_1; \alpha, \lambda) \).

The null hypothesis of no switching can be written as \( \alpha = \alpha_0 = (0, 0, 0, 0, 1, 0)' \). To represent the alternative hypothesis, we have constructed a grid of 5,625 possible values for \( \alpha \), with \( A \) denoting the set comprising all values of \( \alpha \). For any \( \alpha \), \( \hat{\lambda}(\alpha) \) denotes the value of \( \lambda \) that maximises the likelihood taking \( \alpha \) as given. Hamilton (1996) defines the time series of the difference between each constraint log-likelihood function for the grid of alternatives and the constraint log-likelihood function estimated for the null hypothesis as:

\[
q_t(\alpha) = l_1(\alpha; \hat{\lambda}(\alpha)) - l_1(\alpha_0; \hat{\lambda}(\alpha_0))
\]

The likelihood ratio statistic is:

\[
LR = \max_{\alpha \in A} \frac{T \bar{q}(\alpha)}{\sqrt{\sum_{t=1}^T (q_t(\alpha) - \bar{q}(\alpha))^2}}
\]

If the null hypothesis is true, then, for large samples, the probability that the above statistic exceeds a critical value \( z \) is less than the probability that the following statistic exceeds the same value \( z \):

\[
\max_{\alpha \in A} \left( \sum_{k=0}^M \sum_{t=1}^T (q_t(\alpha) - \bar{q}(\alpha))u_{t+k} \right) / \left( 1 + M \sum_{t=1}^T (q_t(\alpha) - \bar{q}(\alpha))^2 \right)
\]

Following Hamilton and Lin (1996), we have generated Hansen’s statistic for \( M \) values of 0-4 and found that the null hypothesis is strongly rejected with a \( p \)-value of 0.0000.
An alternative approach to the Hansen statistic uses a classical log likelihood ratio test for estimating (a) the asymmetries in the conditional mean, assuming the existence of two states in the conditional volatility, and (b) the asymmetries in the conditional volatility, assuming the existence of two states in the conditional mean. Such a test follows the standard chi-squared distribution. We have tested the following hypotheses: (1) the intercept and slope coefficients are not significantly different between the two states \([H_0: \mu_1 = \mu_2; \gamma_1 = \gamma_2; \theta_1 = \theta_2]\) and (2) the standard deviations of the residuals of the two states are not significantly different \([H_0: \sigma_1 = \sigma_2]\). Both tests indicated a rejection of the null hypothesis at the highest significance, with LR statistics of 635.0 and 935.6 respectively. Therefore, we conclude that there is clear evidence of asymmetries between the two regimes identified by the model and these are not only economically, but also statistically significant.

It is well known that standard out-of-sample testing methods are not applicable to Markov switching models due to the presence of nuisance parameters (Hansen, 1992). Thus in order to test the out of sample predictive power of the model, we follow standard practice to use an operational criteria (in our case, the construction of a trading rule) instead of a statistical criteria. To this end, we extend our initial database up to Nov-02 and propose two strategies which are designed to exploit the regime dependent relationship between the index tracking out-performance and the stock prices dispersion. Their construction is based on the fact that the lag of the change in dispersion is used to explain the abnormal return, and, therefore, we have a leading indicator of portfolio performance. Also, forecasts of the latent state conditional probability can be produced for a number of steps ahead by using the unconditional transition probabilities and the current estimate of the conditional probabilities of the latent states.

The portfolio generating the abnormal return relative to the index, \(P\), is defined as the difference between the replica portfolio holdings and the benchmark holdings in each stock. Both trading rules assume active trading, with daily rebalancing according to a trading signal. The first trading rule ensures that \(P\) is held only if there is a buy/hold signal from the Markov switching model. In the second trading rule, \(P\) is held if there is a buy/hold signal, and is shorted otherwise. The ‘buy/hold’ signal occurs either after an increase in the dispersion, if the forecast of the conditional probability of the latent state indicates that the process is currently in regime one, or after a decrease in dispersion, if the forecast of the conditional probability indicates that the process is currently in regime two. Note that the trading rule is constructed solely on the sign of the change in dispersion and not also on its magnitude, which could potentially be used in setting up rebalancing filters. As the abnormal return is not
correlated with the market returns, both strategies will inherit market neutral characteristics. Moreover, they are self-financed, as the sum of all stock weights in $P$ is, by construction, zero. In order to obtain the signal for a given date, we have used only the information available at the moment of the signal estimation (the sign of the lagged change in dispersion and the one-period ahead forecast of the conditional probability of the regimes).

The returns of the trading strategies are plotted in Figure 7. Over the 11-months testing period, the first trading rule produced a cumulative return of 3.9%, with an average annual volatility of 2.2%. The second trading rule produced over the same time interval a cumulative return of 9.4%, with a slightly higher average annual volatility, i.e. 3.2% p.a.

The cumulative returns to both trading strategies demonstrate the predictive power of the Markov switching model. However these results should be treated with caution. First, 11 months is a rather short sample, and secondly, the very high profitability of the trading rules during the last part of the data sample can be the result of the predominance and persistence of regime two during this period. Moreover, the ‘trading rules’ are only designed to test the out-of-sample performance of the Markov switching model – they should not be interpreted as profitable strategies. In fact, they require daily rebalancing and this results in significant transaction costs that more than erode any apparent profitability. We have not accounted for potential transaction costs, as we only aimed to test the efficiency of the model forecasts with trading rules. The problem of potentially high transaction costs in trading rules based on Markov switching forecasts is not new and has been dealt with either by reducing the frequency of trades, or by imposing some filtering of the signals, when trades take place only if the signal exceeds a given threshold (Dueker and Neely, 2001).

To conclude, we have established the appropriateness of the Markov switching approach for modelling the abnormal return. Although the tracking portfolio was shown to replicate the index accurately most of the time and over-perform it in given market circumstances – so the cointegration model is a true enhanced tracking strategy – our attempt to time the abnormal return through a dynamic market neutral strategy based on a Markov switching model is unlikely to justify the costs involved.

6. Implications for Market Efficiency

This section discusses the implications of our findings for market efficiency. We have provided evidence of consistent over-performance from the cointegration-based tracking strategy, even after transaction costs, in different stock universes and over different time
The cointegration portfolio was shown to replicate its benchmark accurately most of the time, and to over-perform it in special, more volatile, market circumstances. Having identified the mechanism producing the abnormal return, we conclude that the over-performance is sample specific only to the extent that it is associated with specific market circumstances, indicating a transitional period for prices. Moreover, we have found a leading indicator for the abnormal return, which was shown to be predictable, if only over a short time horizon, within a Markov switching framework.

However, the success of the cointegration strategy in exploiting the information enclosed in the past stock prices and the predictive power of the Markov switching model can only be interpreted as evidence against the efficient market hypothesis in the weak form if, and only if, the abnormal return does not represent a hidden risk factor premium. As shown by Cochrane (1999), in a world in which there are multiple sources of priced risk, the multifactor efficient market portfolio will no longer be on the mean-variance efficient frontier, and will appear to be dominated to an investor interested only in mean-variance. In this context, if the Markov switching model is detecting a hidden risk factor, market inefficiency has not been proved. On the other hand, if such a hidden risk factor does not exist, then the Markov switching model is identifying pricing inefficiencies that are exploited by the cointegration strategy, albeit only temporary and occurring when the market returns are low and the volatility is high. We have shown that such circumstances can indicate a transitional period in the market, where stock prices are moving towards new equilibrium levels, after turbulent periods for the market.

It is worthwhile dwelling for a while on the question of whether the Markov switching dispersion model really does reveal the existence of a hidden risk factor explaining the abnormal return. If dispersion were proxying a permanent hidden risk factor, the observed change in sign of its relationship with the abnormal return is difficult to explain. However, such a change in sign could happen if (1) the hidden risk factor manifests itself only in the regime producing the abnormal return, and (2) it offsets and even reverses a permanent, normal, negative relationship between the abnormal return and the dispersion. Such a risk factor, proxied by the increase in dispersion, could, for example, be the uncertainty associated with the price equilibrium identified by the cointegration relationship. In this context, the apparent “abnormal” return would be in fact a risk premium for bearing the transition uncertainty.
7. Summary and Conclusions

The aim of this paper was to investigate the abnormal return generated through a dynamic indexing strategy and to analyse its implications from a market efficiency perspective. We have found a measure of stock price dispersion to be a leading indicator for the abnormal return. We also found that the relationship between the abnormal return and the dispersion, which is non-linear, due to the asymmetric behaviour of stock prices, should be modelled with breaks in regime. Consequently a Markov switching approach revealed the existence of two stock market regimes having very distinctive characteristics. We also found that almost the entire abnormal return was associated with the regime characterised by higher benchmark volatility and lower returns.

The cointegration relationship specified between the portfolio and a price weighted benchmark can be interpreted as a relative pricing model. For as long as stock prices are oscillating around the past equilibrium levels, the strategy generates accurate replicas of the benchmark. But in volatile markets, where returns are low and prices are moving towards new levels, the strategy produces consistent excess returns. We have argued that the reason why the cointegration strategy has some periods when it significantly over-performs its benchmark but no periods when it significantly under-performs it, is the asymmetric behaviour of stock prices, the fact that prices tend to fall faster than they rise. The cointegration portfolio, being based on a historical price equilibrium, happens to exploit general stock market declines and recovery periods even though it is not specifically designed for this purpose.

We have shown that the abnormal return occurs only during especially volatile periods, so we cannot exclude the presence of a hidden risk factor – related to the uncertainty arising from prices’ transition towards new equilibria – for which the abnormal return represents a risk premium. Thus we find no evidence against the EMH. Even if such a risk factor does not exist, the anomalies identified by cointegration have been shown to be only temporary and to occur only in special market circumstances.

Nevertheless, our findings do have wide implications for equity fund managers. We have shown that, without any stock selection, solely through smart optimisation, the benchmark performance can be significantly enhanced in certain market circumstances. Moreover, the strategy can be implemented to replicate any type of value or capitalisation weighted benchmark, not only wide market indexes.
References


Figure 1 Cumulative abnormal return

Figure 2 Price dispersion in DJIA

Figure 3 Wald test for parameter stability

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Figure 4 Stock prices movements in regimes one and two

![Graph showing stock prices movements in regimes one and two.]

Figure 5 Regime conditional abnormal return

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.000008</td>
<td>0.000175</td>
</tr>
<tr>
<td>Min</td>
<td>-0.007463</td>
<td>-0.009441</td>
</tr>
<tr>
<td>Max</td>
<td>0.006875</td>
<td>0.015444</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.001153</td>
<td>0.002564</td>
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<tr>
<td>Skewness</td>
<td>0.09</td>
<td>1.02</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>3.67</td>
<td>5.37</td>
</tr>
<tr>
<td>No of obs</td>
<td>1830</td>
<td>689</td>
</tr>
</tbody>
</table>

Figure 6 Estimated probability of regime two

![Graph showing estimated probability of regime two.]

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Figure 7 Cumulative returns produced by the trading rules
Table 1. A. Abnormal returns

<table>
<thead>
<tr>
<th></th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>Overall</th>
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<td>1.71%</td>
<td>-1.00%</td>
<td>1.46%</td>
<td>2.08%</td>
<td>5.61%</td>
<td>7.82%</td>
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<td>FTSE</td>
<td>-0.18%</td>
<td>-0.64%</td>
<td>0.92%</td>
<td>4.77%</td>
<td>0.34%</td>
<td>5.21%</td>
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<tr>
<td>CAC</td>
<td>1.19%</td>
<td>1.16%</td>
<td>-0.04%</td>
<td>1.22%</td>
<td>-0.13%</td>
<td>3.41%</td>
</tr>
<tr>
<td>SP100</td>
<td>0.73%</td>
<td>-1.68%</td>
<td>-1.84%</td>
<td>1.00%</td>
<td>4.59%</td>
<td>2.79%</td>
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</table>

Table 1. B. Benchmark returns

<table>
<thead>
<tr>
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</thead>
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<tr>
<td>DJIA</td>
<td>20.41%</td>
<td>14.93%</td>
<td>22.49%</td>
<td>-6.37%</td>
<td>-7.37%</td>
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<td>FTSE</td>
<td>19.86%</td>
<td>8.24%</td>
<td>8.35%</td>
<td>7.32%</td>
<td>-11.71%</td>
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<td>CAC</td>
<td>8.21%</td>
<td>27.26%</td>
<td>38.44%</td>
<td>0.45%</td>
<td>-21.79%</td>
</tr>
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<td>SP100</td>
<td>26%</td>
<td>19%</td>
<td>18%</td>
<td>1%</td>
<td>-13%</td>
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</tbody>
</table>

Table 2 Estimated coefficients of model (4)

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<thead>
<tr>
<th></th>
<th>α</th>
<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>β₄</th>
<th>β₅</th>
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<td>Coefficient</td>
<td>-0.00003</td>
<td>0.071907</td>
<td>0.001061</td>
<td>-0.01842</td>
<td>0.004874</td>
<td>0.406344</td>
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<td>Standard error</td>
<td>0.000033</td>
<td>0.019675</td>
<td>0.002336</td>
<td>0.002346</td>
<td>0.002368</td>
<td>0.052512</td>
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<td>t-statistic</td>
<td>-9.4859</td>
<td>3.654681</td>
<td>0.454261</td>
<td>-7.85479</td>
<td>2.058311</td>
<td>7.738151</td>
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<tr>
<td>P-value</td>
<td>0.3429</td>
<td>0.0003</td>
<td>0.6497</td>
<td>0</td>
<td>0.0397</td>
<td>0</td>
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Table 3 Stability test for model (4)

<table>
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<th>Probability</th>
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<tr>
<td>Wald Stability Test: 2227 (October 16, 2000)</td>
<td>358.01</td>
<td>0.000000</td>
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Sample Jan-92 to Oct-00

<table>
<thead>
<tr>
<th></th>
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<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>β₅</th>
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<tbody>
<tr>
<td>Coefficient</td>
<td>0.000036</td>
<td>0.007495</td>
<td>-0.05627</td>
<td>-0.038299</td>
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<tr>
<td>Standard error</td>
<td>0.000025</td>
<td>0.018067</td>
<td>0.001942</td>
<td>0.0045035</td>
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<td>t-statistic</td>
<td>1.40522</td>
<td>0.414814</td>
<td>-28.9779</td>
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<td>P-value</td>
<td>0.1601</td>
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Sample Oct-00 to Dec-01

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<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>β₅</th>
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<tbody>
<tr>
<td>Coefficient</td>
<td>-0.00010</td>
<td>0.047449</td>
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<td>0.596204</td>
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<tr>
<td>Standard error</td>
<td>0.000138</td>
<td>0.041945</td>
<td>0.006819</td>
<td>0.139306</td>
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<tr>
<td>t-statistic</td>
<td>-0.74385</td>
<td>1.131212</td>
<td>15.00124</td>
<td>4.279809</td>
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<tr>
<td>P-value</td>
<td>0.4576</td>
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Table 4 Estimation output for model (5)

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<th></th>
<th>μ₁</th>
<th>μ₂</th>
<th>γ₁</th>
<th>γ₂</th>
<th>θ₁</th>
<th>θ₂</th>
<th>σ₁</th>
<th>σ₂</th>
<th>p₁₁</th>
<th>p₂₂</th>
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<tbody>
<tr>
<td>Coefficient</td>
<td>4.9E-05</td>
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<td>-0.051</td>
<td>0.016</td>
<td>-0.425</td>
<td>0.664</td>
<td>0.0006</td>
<td>0.002</td>
<td>0.98</td>
<td>0.88</td>
</tr>
<tr>
<td>Std error</td>
<td>0.00001</td>
<td>0.0001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.0572</td>
<td>0.048</td>
<td>1.49E-06</td>
<td>6.99E-06</td>
<td>0.073</td>
<td>0.087</td>
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<td>Z-stat</td>
<td>3.012</td>
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<td>-41.12</td>
<td>5.167</td>
<td>-7.441</td>
<td>13.71</td>
<td>403</td>
<td>286</td>
<td>13</td>
<td>10</td>
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<td>P-value</td>
<td>0.0026</td>
<td>0.6772</td>
<td>0.000</td>
<td>0.000</td>
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Endnotes

i The portfolio in which all the assets available to investors are held in proportion to their market value.

ii For example, in our case the benchmark is reconstructed as a price weighted index of the stocks currently in the DJIA, and so the reconstructed benchmark represents a scale adjustment to the DJIA based on the value of the latest index divisor.

iii Stock prices and market indexes are usually found to be integrated of order one. The preliminary analysis of our data, using standard Augmented Dicky Fuller tests, showed that this is also our case, i.e. all stock prices and the reconstructed index are I(1). Results are available from the authors on request.

iv Hendry and Juselius (2000) show that if level variables are cointegrated, so will be their logarithms. The level variables are cointegrated by definition, since the current weighted index is a linear combination of the stock prices.

v The transaction costs computed at 30bp per trade value amount to 3.75% over the entire data period.

vi Following market convention, all exponentially weighted moving average estimates were based on the “RiskMetrics” smoothing constant of 0.94.

vii Although the periods with abnormal return are associated with market downturns over a long time horizon, on a daily basis there is no significant negative correlation with the market returns, or with the market volatility. Additionally, the 10-day no-rebalancing period is not explaining the abnormal return.

viii The ADF statistic for the dispersion series is -0.81, thus the null hypothesis of unit root cannot be rejected (5% critical value is –2.86). The ADF statistic for the first difference in dispersion, however, is –22.10, clearly rejecting the unit root hypothesis.

ix We examine a three-month period rather than only the day of October 16th because the exact date indicated by the tests as having the highest likelihood of a structural break can be an artefact of the estimation method used.

x Figure 4 illustrates the stock price to be stable upwards trending because ‘regime two’ concerns a price fall after such an upwards trend. But in fact, ‘regime one’ can be any stable market, regardless of trend, as is evident from the reasoning in this paragraph.

xi The specification including lagged excess return as an explanatory variable, as expected from our simple regression results of Table 3, is not favourable according to information criteria, and the coefficients of the lagged excess return are not statistically significant.

xii If the estimated conditional probability of regime one at time t is above 0.5, we say that the process was in regime one at time t. Alternatively, the process will be in regime two.

xiii Note that we refer to the abnormal return generated through the basic cointegration tracking strategy and not the turnover of the trading strategies investigated in the previous section, which does not account for transaction costs.

xiv The weak form of market efficiency assumes that at all times prices fully reflect the information comprised in past prices, as opposed to the other two forms of market efficiency, semi-strong and strong, which assume that prices also reflect all other public information, respectively, all other public and private information.