Smart Fund Managers? Stupid Money?

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Abstract

We develop a theoretical model of a mutual fund manager’s investment decision that incorporates three well-documented empirical regularities: (i) better past fund performance raises subsequent net fund inflows; (ii) fund manager compensation rises with total assets under management; (iii) trading has short-run price impacts. These features provide fund managers incentives to distort investment toward stocks in which the fund holds larger positions. We show that this leads to the empirically observed short-run persistence and long-run reversal in fund performance. It also explains why mutual funds tend to be relatively undiversified and appear to exhibit clairvoyant stock selection. Finally, we document systematic patterns in aggregate market returns consistent with the hypothesis that the presence of fund managers has grown large enough to alter end-of-quarter returns.

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Mutual fund performance offers many seemingly contradictory empirical puzzles for financial economists:

1. **Short-run persistence in performance**: There is extensive evidence of short-term persistence in performance of mutual funds (hot and cold hands).\(^1\) Zheng (1999) finds that funds with relatively high returns in one quarter have statistically significantly greater returns than the average mutual fund in the next quarter; and relatively worse performers in one quarter generate lower returns than the average mutual fund the next period.

2. **Long-run reversals in performance**: Zheng (1999) documents that the persistence in performance is short-lived. In fact, more than one quarter into the future, funds that did better in the past underperform relative to the average fund while historically poor-performing mutual funds do better. Indeed, this reversal in performance is so strong that cumulative returns of historically-better performers fall below those of worse performers within 30 months. Figure 1, taken from Zheng (1999), illustrates both the short-run persistence and long-run reversal in performance.

3. **Performance flow relationship**: Chevalier and Ellison (1997) and Rosten (1995) find investors reward better performing mutual funds with greater cash inflows.\(^2\) That is, private investors target their mutual fund investments according to recent past performance.

4. **Smaller or newer mutual funds and growth**: Chevalier and Ellison (1997) also find that relatively young funds must perform better to attract investment. Zheng (1999) documents that smaller mutual funds and growth funds particularly exhibit greater short-run persistence in performance and more dramatic long-run reversals in performance.

5. **Undiversified portfolios**: Mutual funds, especially smaller, niche mutual funds, hold very undiversified portfolios, which is reflected by their high return volatility.

6. **Clairvoyant stock selection**: Stocks purchased by mutual funds tend to outperform stocks that they sell (Wermers, Chen and Jegadeesh (2000)), but, on average, actively-managed mutual funds underperform index funds.

7. **Excess end-of-quarter returns and end-of-quarter trading**: Carhart et al. (2002) find that 80% of funds beat the S&P on the last trading day of the year (62% for other quarter-end dates), but only 37% (40% other quarters) do so on the first trading day of a new year. The difference is greater for small-cap funds (91% on year end, 70% other quarter end dates versus 34% for first quarter trading

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\(^2\)See also Ippolito (1992), Sirri and Tufano (1993), Sirri and Tufano (1998) and Goetzmann and Peles (1997).
day), and large in magnitude. The difference between year end and first day return is large, exceeding 200 basis points for small cap funds.

8. **End-of-year performance is more pronounced for better historical performers:** Carhart et al. 2002 find that better past performing funds earned 42 basis points higher returns on the last trading day, and 29 basis points lower on the first trading day than funds with worse historical performance, and they trade more aggressively at year’s end.

Many of these empirical regularities appear mutually inconsistent—mutual funds appear to do a good job selecting stocks (Wermers, Chen and Jegadeesh (2000)), but under-perform index funds; better-performing stocks tend to replicate their performance from one period to the next, but do significantly worse for longer horizons; etc. The contribution of this paper is to provide a coherent explanation for all of these empirical regularities.

One key component of our model is the performance-flow relation documented by, among others, Chevalier and Ellison (1997). That is, better-performing funds in one period generally draw more investment dollars. Because better short-run performance leads to greater investment in the fund, managers who are compensated according to funds under management have an incentive to invest so as to increase short-run returns, even if such investment strategies eventually lead to worse long-run performance. These short-run incentives are enhanced when fund manager turnover is high.

The other key component of our model is the existence of short-run price impact from share purchases or sales. Buying a significant stake in a firm drives price up; selling drives price down. Chan and Lakonishok (1995), and Keim and Madhavan (1997) both document that institutional trades have short-run price impacts. Standard explanations for this empirical regularity feature asymmetrically-informed traders (Kyle (1985), Glosten (1985), Bernhardt and Hughson (2002)) or dealer inventory balance concerns. Here we care not so much about the rationale for the short-run price impact of taking large trading positions, but rather that the price impact exists and does not vanish immediately.

Fund managers who care about short-term performance have an incentive to distort their investment of new funds toward stocks in which they hold positions. The short-run price impact from these additional purchases raises the return on their existing positions, raising short-run total portfolio returns. Consistent with this, Wermers, Chen and Jegadeesh (2000) find that stocks purchased by funds have significantly higher returns than stocks that they sell. The short-run incentives to distort investment ensures that mutual funds will, in the long-run, tend to under-perform non-managed indexes, unless fund managers have sufficiently better information than other investors.

We then explore the consequences for short- and long-run portfolio returns. We show that short-run
portfolio returns are initially an increasing function of new cash available to invest. As a result, as long as cash inflows to better past performers are not “too” great, (1) better recent past performers earn greater short-run returns, but (2) lower longer-run portfolio returns, because they distort investments toward assets in place by more. That is, short-run persistence in mutual fund performance arises because better past short-run returns raise fund flow, which, in turn, raises current period returns. Higher current returns will again lead to greater cash inflows and perhaps to another period of greater returns, but eventually this distortion in investments must catch up and cause a long-run reversal in fund performance.

The analysis can also be posed from the perspective of poorer-performing funds who have cash outflows and must decide which positions to liquidate. The negative impact of their sales on the returns on assets in place ensures that their short-run poor performance persists. Indeed, poorer short-run performance may be more persistent than short-run better performance if some of the best performers receive so much new cash that their short-run returns are reduced (see Malkiel (1995), Brown and Goetzmann (1995), and others).

The incentive to distort investment toward stocks in which the fund has a position is especially great if the persistence in the price impact is high. Thus, investment distortions should be higher for smaller niche funds in which orders tend to have more substantial and long-lasting price impacts. Similarly, investment distortion incentives are greater for newer funds, which both tend to be smaller and have more sensitive flow-fund performance relationships (Chevalier and Ellison (1997)). Consequently, such funds should exhibit more marked short-run persistence and long-run reversals in performance, as Zheng (1999) documents.

Since the price impact of trades decays over time, our theory suggests that fund managers have especially strong incentives to augment existing holdings of a stock at the end of an evaluation period. An extreme manifestation of this investment distortion is the practice of “high closing”—submitting a buy order that blows through the sell limit order book at the close of a trading day at the end of an evaluation period.

“Nearly everyone seems to agree that high closing is common. ‘It’s caused by the competitive nature of the business. They have to beat the guy across the street.’” When the Globe and Mail newspaper (July 7, 2000) examined closing prices for the final trading days, it found “a great many suspicious rises at points that determine how a fund manager’s performance will be judged. The percentage of [mutual funds] that became stars on the final trading day and turned into dogs after New Year’s was far greater than for (randomly selected mid month days)... Last-minute leaps beyond the normal market trends strongly suggested ‘portfolio pumping’.” Thomas Hirschman of the Financial Post (June 23, 2000) found that “the stocks that are manipulated are usually illiquid stocks”. He also documented the related practice of cross-trading be-

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3Another example of closing stock price manipulation is provided by Ni et al. (2003). They show that the closing prices of stocks with listed options cluster at option strike prices on option expiration dates and that this clustering is consistent with stock price manipulation by investors who hold short option positions.

between a mutual fund and a pension fund run by the same fund manager. The manager uses the pension fund, which is insensitive to short-run cash inflow performance relations, to purchase the mutual fund’s stake in an illiquid company.

Carhart et al. (2002) provide extensive empirical support for our model. As we have noted, funds earn apparently tremendous short-run returns at the very end-of-quarter/end-of-year when trading behavior has the greatest impact on quarterly or annual performance, and is especially high for small-cap funds where trading has the greatest impact on price. Funds earn these short-run returns by dramatically increasing trade in the last half hour, with volume increases in the last 30 minutes of 17% at year end, 14% at quarter end. The result is that small caps earn 100 basis point returns in the last half hour of the year, 26 basis points at other quarter ends following volume increases of 17% at year end, 14% at quarter end. Carhart et al. (2002) reject benchmark-beating hypotheses in favor of the strategic behavior motivated here. They also find that this strategic effect is higher for better past performing funds, who have more cash on hand to influence closing returns. Firms that performed better in the past year earned 42 basis points higher returns on the last trading day and 29 basis points lower on the first trading day than did worse-performing funds.

We then uncover evidence that because Carhart et al. (2002) measure fund performance relative to the S&P, they almost certainly underestimate the impact of strategic mutual fund trade. We document that strategic trade by fund managers has grown large enough to impact returns on aggregate market indexes, so that measuring the impact relative to the index understates the total effect. Specifically, we find that daily returns of the equally-weighted index on the last trading day of a quarter greatly exceed the daily returns on the first trading day of the succeeding quarter, and this return difference has risen with the share of total equity held by mutual funds. Indeed, the regression of the return difference on the mutual fund share of equity has a remarkably high adjusted $R^2$ of 0.20. Further, no significant relationship exists for the value-weighted index. This reflects that the equally-weighted index emphasizes smaller stocks, where the price impact of trades is more dramatic and longer-lasting, and where the incentives to distort investments are stronger. In sum, the data strongly support the hypothesis that mutual fund managers respond to the short-run investment incentives that we model, and that collectively they have grown large enough to significantly affect market outcomes.

The interpretation offered by Zheng (1999) for the short-run persistence in performance and the positive flow-performance relation is that investors are intelligent, and successfully predict future mutual fund performance. The long-run under-performance implies that intelligent investors must also churn their mutual fund portfolios on a quarterly basis, but this level of turnover is not observed in the data. In contrast, our model takes the fund flow relation as given and shows that this will induce optimizing fund managers to behave in such a way as to generate the persistence in fund performance. In turn, if investors devote discre-
tionary funds to past good performers, but do not churn, then in the long-run their portfolio under-performs: retail mutual fund investors are “stupid”.

Berk and Green (2002) develop a model in which rational investors learn from a fund’s performance about its manager’s ability, and allocate funds accordingly. Their model can explain both why money flows into mutual funds with recent better performance, and away from those with worse past performance, with the result that past returns do not predict future performance. Berk and Green’s model is therefore inconsistent with the long-run reversal in performance documented by Zheng (1999). Their model also cannot explain why stocks purchased by mutual funds outperform those that they sell, yet, on average, mutual funds under perform the market; and is silent with regard to the end-of-quarter returns and strategic behavior documented by Carhart et al. (2002).

The next section sets up the economic environment, characterizes how the fund managers’ existing stock holdings influence his investment decisions, and derives the consequences for persistence in fund returns. Section 2 presents supporting empirical evidence. Section 3 concludes. Proofs are in an appendix.

1 Model

Consider a risk neutral mutual fund manager who can invest in three assets: stock $A$, stock $B$ and cash. The fund manager enters date $t$ with an existing position of $S_{At}$ shares in stock $A$ and $S_{Bt}$ shares in stock $B$. The share prices at the beginning of period $t$ are $P_{At}$ and $P_{Bt}$, respectively. Without loss of generality we assume that $P_{At}S_{At} \geq P_{Bt}S_{Bt}$. Cash earns a risk-free return of $i > 0$.

Firms retain their earnings. Hence, a firm’s stock price is equal to its expected discounted lifetime cash flows. We impose no structure on the timing of cash flows. At the beginning of period $t$, each firm makes an earnings announcement and provides guidance about future cash flows, which causes market participants to update about firm values. We summarize the earnings announcement and guidance by its implications for the percentage change in firm value, $\delta_{j,t}$, so that $\delta_{j,t}P_{j,t-1}$ is the change in expected discounted cash flows.\(^5\)

Following a “signal” of $\delta_{j,t}$, investors update about cash flows, leading to a date $t$ stock price for firm $j$ of

$$P_{j,t} = (1 + i)P_{j,t-1} + \delta_{j,t}P_{j,t-1}. \quad (1)$$

We assume that in period $t$, the mutual fund manager privately learns next period’s signals, $\delta_{A,t+1}$ and $\delta_{B,t+1}$, and can trade based on this private information in period $t$ before the signals are publicly revealed with certainty at the beginning of period $t + 1$. At the end of period $t$, the fund receives net cash inflows of

\(^5\)If $\delta_{j,t+1}$ is not proportional to firm value, then pricing is sensitive to the firm’s choice of shares outstanding.
\( f(r_t) \), where \( r_t \) was the return on the mutual fund’s portfolio over period \( t \). We capture the performance-flow relation with \( f(r_{t1}^1) > f(r_{t2}^2) \) if and only if \( r_{t1}^1 > r_{t2}^2 \)—greater past fund performance raises net cash inflows from investors.

For simplicity, we suppose that at date \( t \), the fund manager invests to maximize funds under investment at the beginning of period \( t + 1 \). In general, fund manager compensation is an increasing function of funds under investment, and this induces the fund manager to choose his portfolio to maximize end-of-period return. We interpret the period’s end as the date at which investors receive information about mutual fund performance, which leads them to re-allocate their investments. Focusing on a one-period horizon for the fund manager eases the analysis. However, the incentive effects that we document remain present when fund manager horizons are longer.

We assume that a fund manager’s stock purchases have a short-term price impact: the more shares that a fund manager purchases, the greater is the short-term price impact. Specifically, if the fund manager purchases \( I_{jt} \) shares in stock \( j \), he pays

\[
\hat{P}_{jt}(I_{jt}) = P_{jt} + \Delta P_{jt}(P_{jt}, I_{jt}),
\]

per share. We make the following assumptions on the short-term price impact of share trades:

**A1.** If \( I_{jt} = 0 \), there is no price impact:

\[
\Delta P_{jt}(P_{jt}, 0) = 0.
\]

**A2.** The price impact of larger orders is greater:

\[
\frac{\partial \Delta P_{jt}(P_{jt}, I_{jt})}{\partial I_{jt}} > 0.
\]

**A3.** The price schedules are identical for the two stocks:

\[
\Delta P_{At}(\cdot) = \Delta P_{Bt}(\cdot).
\]

Assumption **A3** allows us to abstract away from how different price schedules affect a fund manager’s investment decisions. Later, we consider the possibility that price schedules differ across stocks. This may be because some stocks are more liquid, or because market makers know the institutional holdings, recognize fund manager investment incentives, and adjust price schedules accordingly.

**A4.** The price schedule is not too concave:

\[
\frac{\partial^2 \Delta P_{jt}(P_{jt}, I_{jt})}{\partial I_{jt}^2} \leq 0; \quad 2 \frac{\partial \Delta P_{jt}(P_{jt}, I_{jt})}{\partial I_{jt}} + I_{jt} \frac{\partial^2 \Delta P_{jt}(P_{jt}, I_{jt})}{\partial I_{jt}^2} > 0.
\]

In each period \( t \), the timing of events is as follows:
1. The firm announces period earnings and market participants revise their expectations about the discounted present value of the firm’s lifetime cash flows. The current stock prices are given by $P_{A,t}$ and $P_{B,t}$.

2. The fund manager learns $\delta_{A,t+1}$ and $\delta_{B,t+1}$ and invests available cash in assets $A$, $B$ and money, $\{I_{At}, I_{Bt}, M_t\}$, so as to maximize the fund’s expected period return. Available cash is composed of net cash inflows $f(r_{t-1})$ plus the present value of last period’s cash position $(1 + i)M_{t-1}$. The fund manager can sell shares, but cannot sell shares short nor borrow to finance stock investments.

3. End-of-quarter stock prices denoted $QP_{jt}$ are realized. Not all information about next period’s signal, $\delta_{j,t+1}$, necessarily leaks out by the end of the trading period.
   - With (independent) probability $\gamma$ the private information about stock $j$ is revealed, in which case
     $\quad QP_{jt} = (1 + i)P_{jt} + \delta_{j,t+1}P_{jt}$.
   - With (independent) probability $(1 - \gamma)$ the private information about stock $j$ is not revealed, in which case
     $\quad QP_{jt} = (1 + i)P_{jt} + \Delta P_{jt}(P_{jt}, I_{jt})$.

Thus, $\gamma$ captures information release between the time of purchase and the end of the period. We assume that $\gamma \in (0, 1)$, i.e., information is only sometimes revealed to the public. A smaller value of $\gamma$ reflects less leakage of information between the purchase and the end of the period. This may occur because there is less time between the purchase of the stock and the end of the quarter or because the stock is smaller and hence followed by fewer analysts.

4. End-of-period fund returns ($r_t$) are calculated. The fund receives net cash inflows of $f(r_t)$.

**Discussion.** The model takes some empirical relationships such as the performance-flow relationship as given. Presumably, underlying the performance-flow relationship are investor beliefs that fund managers differ in their abilities to identify good investments and there is persistence in manager ‘ability,’ at least in the short run (Berk and Green (2002)). In this paper, we do not want to distinguish fund managers by ability, because differences in ability would also generate persistence in performance. We abstract from the primitives not only because it simplifies the presentation, but (i) we need only a few implications of the primitives for our findings (e.g. monotonicity in the fund-flow relationship), and do not want to impose unneeded structure on the empirical relationships; and (ii) our reduced form assumptions are consistent with many possible primitive formulations and we do not want to take a stand on which one is “correct”. The pricing relationship is also a reduced-form relationship, one documented empirically by Chan and
Lakonishok (1995) and Keim and Madhavan (1997). It may be that this pricing relationship reflects the fact that trading volume contains information, or market makers must be compensated for having to re-adjust their portfolios, and it takes time to do so. The precise details underlying the reduced-form relationship are again unimportant for our qualitative conclusions. What is important is that the short-run return on the stock is affected by trading activity. This gives fund managers an incentive to purchase stocks in which they hold positions.

**Fund Manager’s Problem:** At the beginning of period \( t \), the fund manager invests so as to maximize the expected period portfolio return:

\[
\max_{M_t, I_{At}, I_{Bt}} E[r_t] = \frac{(1 + i)M_t + \sum_j (I_{jt} + S_{jt})E[QP_{jt}]}{(1 + i)M_{t-1} + f(r_{t-1}) + \sum_j QP_{jt}S_{jt}} - 1
\]

subject to

\[M_t \geq 0, \quad (3)\]
\[I_{jt} \geq -S_{jt}, \quad j = A, B \quad (4)\]
\[
\sum_j \hat{P}_{jt}I_{jt} + M_t \leq f(r_{t-1}) + (1 + i)M_{t-1}. \quad (5)
\]

where \( S_{jt+1} = S_{jt} + I_{jt} \) and

\[E[QP_{jt}] = (1 + i)P_{jt} + (1 - \gamma)\Delta P_{jt}(P_{jt}, I_{jt}) + \gamma \delta_{jt+1}P_{jt}\]

Throughout, we will assume that the short-selling constraint, \( I_{jt} \geq -S_{jt}, \quad j = A, B \) does not bind: the fund manager does not receive such a bad signal about a stock that he wants to sell more shares of the stock than he has in his portfolio. The fund manager is fully invested (i.e., investments are constrained by the inflow of new cash), as long as fund inflows are not too large relative to the manager’s private information. We will show that no information, i.e., \( \delta_{A,t+1} = \delta_{B,t+1} = 0 \), is typically “enough” private information.

The corresponding Lagrangian\(^6\) is

\[
\mathcal{L}(M_t, I_{At}, I_{Bt}) = (1 + i)M_t + \sum_j (I_{jt} + S_{jt})E[QP_{jt}] + \lambda_M M_t + \lambda_A [S_{At} + I_{At}] + \lambda_B [S_{Bt} + I_{Bt}] + \lambda_{bad} \left[ f(r_{t-1}) + (1 + i)M_{t-1} - \hat{P}_{At}I_{At} - \hat{P}_{Bt}I_{Bt} - M_t \right].
\]

Given that \( \lambda_M = \lambda_A = \lambda_B = 0 \), the associated first order condition characterizing the optimal investment in stock \( j \in \{A, B\} \) is:

\[
\frac{\partial \mathcal{L}}{\partial I_{jt}} = (1 + i)P_{jt} + (1 - \gamma)\Delta P_{jt}(P_{jt}, I_{jt}) + \gamma \delta P_{jt} + (1 - \gamma)I_{jt} \frac{\partial \Delta P_{jt}}{\partial I_{jt}} + (1 - \gamma)S_{jt} \frac{\partial \Delta P_{jt}}{\partial I_{jt}}.
\]

\(^6\)Note: maximizing portfolio returns is equivalent to maximizing the portfolio end-of-period value.
\[-\lambda_{bad} \left[ P_{jt} + \Delta P(P_{jt}, I_{jt}) + I_{jt} \frac{\partial \Delta P_{jt}}{\partial I_{jt}} \right] = 0.\]

We first impose sufficient structure on the price impact of orders such that following comparable signal patterns, the fund manager purchases more of the stock in which his holdings are greatest:

**Proposition 1** Suppose that \( \Delta P(P_{jt}, I_{jt}) = k(P_{jt}I_{jt})P_{jt} \), where \( k > 0 \). Then if the value of the fund’s initial holdings of stock \( A \) exceed the value of the fund’s holdings of stock \( B \), i.e., \( P_{At}S_{At} > P_{Bt}S_{Bt} \), the fund manager tends to distort subsequent investments toward stock \( A \). Specifically, \( \hat{P}_{At}I_{At}(\delta_{A,t+1} = \delta_1, \delta_{B,t+1} = \delta_2) > \hat{P}_{Bt}I_{Bt}(\delta_{A,t+1} = \delta_2, \delta_{B,t+1} = \delta_1) \): the manager spends more on purchasing new shares of stock \( A \) when \( A \) receives signal \( \delta_1 \) and \( B \) receives signal \( \delta_2 \), than on purchasing new shares of stock \( B \) for the opposing signal pattern.

Further, when \( \delta_{A,t+1} = \delta_{B,t+1} \), investment in stock \( A \) is a decreasing function of the probability \( \gamma \) that information about the innovations to an asset’s value is revealed before the end of the period. The structure imposed on the price impact of order flow, \( \Delta P(P_{jt}, I_{jt}) \), is sufficient to ensure that independent of initial share prices, \( P_{At} \) and \( P_{Bt} \), and initial holdings, \( S_{At} \) and \( S_{Bt} \), the fund manager always wants to distort investment toward assets in which he holds larger positions. That is, the fund manager wants to hold an increasingly undiversified portfolio. If cash inflow is so high that the fund manager chooses to hold some cash, then this result holds without the structure on \( \Delta P(P_{jt}, I_{jt}) \). If, instead, the fund manager is fully invested in the market, then the result still holds as long as either \( P_{At} \) and \( P_{Bt} \) are sufficiently close, or \( \delta_1 \) and \( \delta_2 \) are sufficiently close. Essentially, if we relax the structure on \( \Delta P(P_{jt}, I_{jt}) \), then we have to worry about second-order effects related to the relative value of purchases, and these depend on differences in ex ante share prices and differences in signals.

Proposition 1 offers several testable implications:

- If one interprets a smaller \( \gamma \) as capturing a smaller amount of time between the purchase of stocks and the end of the period, then it follows that near the end of the period, the optimal investment is more sensitive to assets in place and less sensitive to private information. The proposition suggests that fund managers should invest more heavily on the basis of private information earlier in an evaluation period (when \( \gamma \) is larger), and invest more heavily in existing holdings later in the evaluation period (when \( \gamma \) is smaller). The extreme version of this latter investment distortion is the practice of high closing.

- Re-interpreting \( \gamma \) as the probability that signals remain private information, it follows that, ceteris paribus, in larger capitalization stocks where there are more outside sources of information, investments are less sensitive to share holdings.
Thus, existing stock holdings cause a fund manager to distort investments. Despite this, we now show that if the fund manager is not fully invested in the market, the model cannot give rise to short-run persistence in fund performance:

**Proposition 2** Suppose that the expected return on equity exceeds the risk-free rate. Then, if the fund manager is not fully invested in the market, i.e., $M_t > 0$, both short-run and long-run fund returns are a declining function of funds invested.

Because, on average, fund returns exceed the risk-free rate, it follows that if the fund manager is not fully invested in stocks, then the model is inconsistent with the empirical regularity that returns exhibit short-run persistence. Indeed, in Proposition 2 we assume that $QP_{j,t-1} = P_{jt}$, so that returns are calculated under the assumption that the end of period $t - 1$ share price reflected the value of the firm at that moment, i.e., there was no past investment distortion. If $f(r_{t-1})$ was higher due to greater past investment distortion, this would lead to even more negatively-correlated short-run returns.

In practice, on average at any moment in time, cash holdings comprise but 6% of a fund’s holdings. These minimal holdings partially reflect cash inflows from new investors that have not yet been invested, and they reflect the fact that funds want to insure themselves against withdrawal demands from individual investors. About 70% of all mutual fund trades are to meet investor liquidity demands (Edelen (1999)); and due to the associated transactions costs, funds will seek to minimize them. Our model does not incorporate such investor liquidity demands, and hence the insurance role for cash holdings. As a result, in the context of our model, a 6% cash holding is best interpreted as being fully-invested.

We now show that short-run persistence in performance can arise if managers are fully invested in the market. Specifically, we now show that if the fund manager is fully invested, then short-run returns first rise with $f(r_{t-1})$.

**Proposition 3** Suppose that $\delta_{A,t+1} = \delta_{B,t+1} = \delta$, and a fund manager is fully invested in the market. Then short-run returns first rise with new funds under management, $\partial r_t / \partial f > 0$ for $f(r_{t-1})$ sufficiently small, but are a declining function of new funds for $f(r_{t-1})$ sufficiently large.

Note that $\partial r_t / \partial f > 0$ when $f$ is small even if the fund manager has no private information, so that $\delta = 0$. The intuition for Proposition 3 is cleanest when $S_{At} \sim S_{Bt}$ and $P_{At} \sim P_{Bt}$ so that $I_{At} \sim I_{Bt} \sim 0$. Then $I_{jt}$ shares are purchased at a premium of $\Delta P(P_{jt}, I_{jt})$, so the ‘cost’, $\Delta P(P_{jt}, I_{jt})I_{jt}$ is only of second order. In contrast, the price impact of the share purchase on returns has a first order positive impact of order $(1 - \gamma)\Delta P(P_{jt}, I_{jt})S_{jt}$. The rest of the proof shows that the result extends when the fund manager makes
non-trivial offsetting investments in the two stocks.\footnote{Note that as in Proposition 2 we calculate returns using $QP_{j,t-1} = P_{jt}$. Implicitly, this means that higher values of $f(r_{t-1})$ were not due to (greater) past investment distortion.}

Because short-run returns eventually fall for $f(r_{t-1})$ sufficiently large, ‘Ponzi-schemes’ cannot be supported in the long run. In $t + 1$, $\delta_{A,t+1}$ and $\delta_{B,t+1}$ are revealed and incorporated into prices, so that, ceteris paribus, greater investment distortions at date $t$ reduce returns in $t + 1$ by more. Possibly offsetting this decline is the fact that the date $t$ investment distortion induced greater cash inflows, $f(r_t)$, which, in turn, can facilitate another round of investment distortion. For returns in $t + 1$ not to fall, the second round of investment distortion must dominate the impact due to the revelation of $\delta_{A,t+1}$ and $\delta_{B,t+1}$. For longer-run returns to continue to rise, it must be that the higher short-run return from the immediate distortion of investment must more than offset lower returns due to realizing past distortions. But since short-run returns eventually decline with $f(r_{t-1})$ this cannot happen.

Combining this observation with Proposition 3, it immediately follows that

**Proposition 4** Even if the fund manager has no private information, for $f(r_{t-1})$ sufficiently small, short-run returns are an increasing function of $f(r_{t-1})$, but long-run returns are eventually a decreasing function of $f(r_{t-1})$. Hence, the model can reconcile both the short-run persistence in fund performance and the long-run reversal in fund performance documented by Zheng (1999).

Because the fund manager has no private information, from a long-run perspective, the optimal stock investment is zero. Long-run returns are a strictly declining function of $f(r_{t-1})$, because the price premium paid for each share rises with the investment.

Note that these results hold even if past returns were so bad that redemptions lead to a net outflow of money from the fund. Then the mutual fund manager must disinvest, and will do so to minimize the adverse price impacts. It follows that the fund manager will tend to sell stocks in which he has smaller positions. Further, in the environment characterized by Proposition 4, short-run returns will be lower (and negative) for fund managers with greater redemptions, but there will be a long-run reversal in performance.

Since cash inflows are more sensitive to fund performance for newer funds (Rosten (1995), Chevalier and Ellison (1997)), it follows that the persistence in short-run returns will decline as funds mature, and, in turn, there will be a smaller long-run reversal in performance for mature funds, as Zheng (1999) documents. Over time, funds with high short-run returns should under-perform the market in the long-run, as Zheng (1999) also finds.

We now show that the return-cash flow relationship characterized above emerges in the short- and long-run of the most basic economy that one could construct.
Example: Consider an economy in which the fund manager has no private information, \( \delta_{A,t+1} = \delta_{B,t+1} = 0 \), and identical holdings, \( S_{At} = S_{Bt} = S > 0 \), in two identical stocks that share an initial common price of \( P_{At} = P_{Bt} = 1 \). Further, trades have a linear price impact, \( \Delta P(1, I_{jt}) = aI_{jt}, a > 0 \), so that the purchase price for stock \( j \in \{A, B\} \) is \( \hat{P}_{jt} = 1 + aI_{jt} \).

It is straightforward to verify that there is a critical value \( \bar{f} \) such that the fund manager is fully invested if and only if cash inflows, \( f \), are less than \( \bar{f} \). Further for \( f < \bar{f} \), the fund manager does best to divide his purchases equally between stocks, purchasing \( I(f) = \sqrt{\frac{1+2af-1}{2a}} \) shares of each stock to generate an expected mutual fund return of

\[
E[r(f)] = \frac{2(I(f) + S)(1 + i + (1 - \gamma)aI(f)) - (2S + f)}{2S + f}.
\] (6)

Initially, returns from investing are increasing in \( f \),

\[
\frac{\partial}{\partial f} [E[r(f = 0)]] = \frac{a(1 - \gamma)}{2} > 0,
\] (7)

but the second derivative with respect to \( f \) is negative,

\[
\frac{\partial^2}{\partial f^2} [E[r(f = 0)]] = \frac{a[-i - (1 - \gamma)S - 1]}{2S} < 0.
\]

Figure 2 illustrates the fund flow-return relationship. The short-run portfolio return first rises and then falls with cash flow into the fund. But note that the cash inflow that maximizes short-run returns, \( \bar{f} \), is quite high, about 40% of the portfolio value. That is, although Proposition 3 only proves that short-run returns are initially increasing in cash inflow, the figure reveals that short-run returns can continue to rise even if \( f \) is reasonably large. In turn, this implies that there will be short-run persistence in portfolio performance, as better immediate past performance will draw more cash inflow, which gives rise to higher current returns. Note that the numerical finding illustrated in Figure 2 does not imply that higher returns will persist for some time. This is because the fund has to realize the negative return consequences of its immediate past investment distortions each period, as share prices incorporate the true value of past signals.

To derive how the fund flow-return relationship is affected by the parameters of the economy, we next see how returns vary with the exogenous parameters. We find that

- Short-run returns fall with the information arrival rate, \( \gamma \). That is, short-run returns are higher if information is less likely to leak out by the end of the period:

\[
\frac{\partial E[r(f)]}{\partial \gamma} = \frac{-2aI(f)(I(f) + S)}{2S + f} < 0.
\]

Thus, niche mutual funds that invest in stocks where there are fewer sources of information should exhibit stronger short-run persistence and greater long-run reversal in performance.
• Short-run returns rise with assets in place:
\[ \frac{\partial E[r(f)]}{\partial S} = \frac{(2f - 4I(f))(1 + i + (1 - \gamma)\alpha I(f))}{(2S + f)^2} = \frac{4\alpha(I(f))^2(1 + i + (1 - \gamma)\alpha I(f))}{(2S + f)^2} > 0 \]

• Greater cash inflows raise short-run returns by more if information is less likely to leak out:
\[
\begin{align*}
\frac{\partial^2 E[r(f)]}{\partial \gamma \partial f} &= \left( \frac{4\alpha I(f)}{\partial f} + 2aS \frac{\partial I(f)}{\partial f} \right) (2S + f)^{-1} + 2a(I(f)) (I(f) + S)(2S + f)^{-2} \\
&= -2a(2S + f)^{-2} \left[ I \left( 2(2S + f) \frac{\partial I(f)}{\partial f} - I(f) \right) + S \left( (2S + f) \frac{\partial I(f)}{\partial f} - I(f) \right) \right] < 0,
\end{align*}
\]

because \( \frac{\partial I(f)}{\partial f} f > I(f) \) and \( S \geq 0 \). This result can reconcile the empirical regularity that niche stocks have greater short-run persistence and larger long-run reversals in performance.

**Different Price Schedules.** We now explore how outcomes are affected if the price impact of trades differs across stocks. We first identify how fund managers’ investment incentives are affected by differences in liquidity. Specifically, we first suppose that \( \Delta P_{At}(I_{At}) = aI_{At} \) and \( \Delta P_{Bt}(I_{Bt}) = bI_{Bt} \), where \( a < b \), but that the stocks are otherwise identical, \( P_{At} = P_{Bt} = 1, S_{At} = S_{Bt} = S \), and \( \delta_{At} = \delta_{Bt} = \delta \).

We then characterize how investments are affected when, in addition, \( S_{At} > S_{Bt} \). One reason why the price impact of trade in stock \( A \) may be less than that in stock \( B \) is that \( S_{At} > S_{Bt} \) is public information. That is, market makers understand that because \( S_{At} > S_{Bt} \), a fund manager has an incentive to distort order flow toward stock \( A \), so that comparable levels of order flow in stocks \( A \) and \( B \) suggest a lower value for \( \delta_{At+1} \) than for \( \delta_{Bt+1} \). Market makers incorporate the trading behavior of fund managers into their pricing, setting a price schedule for stock \( A \) that is less sensitive to order flow than that for stock \( B \).

If the fund manager is fully invested, \( M_t = 0 \), then the first order conditions imply:
\[
(1 + i) + 2(1 - \gamma)aI_{At} + \gamma \delta + (1 - \gamma)\alpha S_{At} = \frac{(1 + i) + 2(1 - \gamma)bI_{Bt} + \gamma \delta + (1 - \gamma)\beta S_{Bt}}{1 + 2bI_{Bt}}
\]  
(8)

Substituting for \( I_{Bt} = \left( -1 + \sqrt{1 + 4b(f - I_{At}(1 + aI_{At}))} \right) / 2b \), we can express \( I_{At} \) in terms of other parameters,
\[
\frac{(i + \gamma) + \gamma \delta + (1 - \gamma)\alpha S_{At}}{1 + 2aI_{At}} = \frac{(i + \gamma) + \gamma \delta + (1 - \gamma)\beta S_{Bt}}{\sqrt{1 + 4b(f - I_{At}(1 + aI_{At}))}}
\]

The complexity of this expression leads us to characterize numerically in Figure 3 how \( I_{At} \) depends on \( a \). The figure highlights that as long as cash inflow, \( f \), is sufficiently small, investment in stock \( A \) rises with \( a \); but for higher values of \( f \), investment falls with \( a \). Further, \( S_{At} > S_{Bt} \) magnifies these investment consequences. The qualitative implications are:

• A fund manager’s investment in stock \( A \) rises with the price impact of stock \( A \) order flow, \( a \), as long as \( f \) is sufficiently small so that “painting the tape” is relatively more important; but
A fund manager’s investment in stock $A$ falls with the the price impact of stock $A$ order flow, $a$, if cash inflows, $f$ are so high that the cost of purchasing new shares dominates.

That is, as long as cash inflows are not too high, the fund manager’s incentive to distort investments toward illiquid assets rises. So, too, these results imply that if $f$ is small, investment distortions are reduced if market makers observe a fund’s holdings and consequently reduce the sensitivity of prices to order flow in stocks where the fund manager holds larger positions. However, if $f$ is sufficiently large, such pricing reinforces the direction of new investment by fund managers toward existing holdings of stock $A$.

## 2 Empirical Evidence

Our model predicts that mutual fund managers have especially large incentives to manipulate their portfolio at the end of a measurement period (e.g., quarter). If this is so, then if mutual funds are “large” players in the economy, the daily market return at the end of a measurement period may exceed daily returns on other days. We now document this fact. Indeed, we find that as the aggregate share of all equity held by mutual funds has increased over the years, the end-of-quarter daily return/other daily return difference on the aggregate market indexes has grown, strongly suggesting that fund manager portfolio manipulations have had increasingly-large systematic impacts on aggregate market outcomes.

We use the CRSP Equally- and Value-Weighted Index Returns to calculate (i) the difference in returns on the last trading day of the period (quarter or month) and the return on the first trading day of the following period, and (ii) the difference in returns on the last day of the period versus the average return on the other trading days. We obtain mutual funds holdings of corporate equity (Reference FL653064000) and market value of domestic corporations (Reference FL893064195) from the Federal Reserve US Flow of Funds accounts for the first quarter 1970 to third quarter 2001.

Table 1 presents the estimation results for the regression model

$$
(r_t - \hat{r}_t) = \beta_0 + \beta_1 \left( \frac{\text{Mutual fund holdings of corporate equity}_t}{\text{Market value of domestic companies}_t} \right) + \varepsilon_t, \quad \varepsilon_t \sim iid \left(0, \sigma^2\right),
$$

where $r_t$ is the return on the last trading day of quarter $t$ and $\hat{r}_t$ is the return on the first trading day of quarter $t + 1$. The results are presented for returns calculated using both the equally-weighted and value-weighted index. Figure 4 plots the relationship between share of mutual fund holdings of equity and the end-of-quarter abnormal return difference for the equally-weighted index. The figure clearly reveals an extremely strong and significant positive relationship between the mutual fund share of aggregate equity and the return difference on the equally-weighted index. The figure also shows that the end-of-quarter
return is far more likely than not to exceed the return on the first day of the next quarter. Indeed, the regression fit as measured by the adjusted $R^2 = 0.20$ for the equally-weighted index is remarkable. The estimate of the coefficient $\beta_1$ is significant and positive as predicted by our model. This result is robust to the exclusion of the last 5 years of data when both abnormal returns and the share of mutual fund holdings of equity were high. It is also worth noting that the share of mutual fund holdings of equity does not increase monotonically over time (as one might expect) and has considerably more explanatory power than a simple time trend variable. Contrasting the regressions for the equally-weighted and value-weighted indexes reveals that the impact of the share of mutual fund holdings on this return difference is only statistically significant for the equally-weighted index. Because the equally-weighted index emphasizes smaller, less liquid, stocks for which the short-term price impact of a trade is greater and more persistent, these are exactly the findings predicted by our model.

Table 2 presents the analogous regressions in which the dependent variable is now the difference between the end-of-quarter return and the average return on the other days in the quarter. The same qualitative pattern remains, but to a reduced degree, and the adjusted $R^2$ falls to a still substantial 0.08. The weaker relationship highlighted in Table 2 relative to Table 1 reveals that the market underperforms on the first trading day of a quarter, as it “bounces” back from the previous day’s manipulation.

Tables 3–5 investigate whether strategic mutual fund behavior explains abnormal end-of-month return differences. Tables 3 and 4 suggest that while the coefficient estimate for the share of equity held by mutual funds is significantly positive, the model has less explanatory power, as measured by the adjusted $R^2$, when estimated at monthly intervals. In fact, when the return difference for the last month in a quarter is excluded, as in Table 5, the model has no explanatory power for equally-weighted index returns. Thus, we find that the crucial measurement period is a quarter, and that the incentive for fund managers to distort investment is so strong at the end of a quarter for less liquid stocks that it significantly affects returns on the equally-weighted index. These findings reinforce those of Carhart et al. (2002) who provide additional evidence that mutual funds primarily distort behavior only at the end of quarters.

3 Conclusion

It is well known that past fund performance influences subsequent net fund flows and that fund manager compensation rises with funds under management. These observations provide fund managers the incentive to utilize short-term price impacts to “paint the tape”—in other words, to mark up their holdings at quarter end through aggressive trading of stocks they already hold. To better understand the implications of this, we model the fund manager’s investment decision and show that a fund manager the incentive to distort
investment of new cash inflows towards stocks in which the fund has larger positions. We show how this behavior leads to the empirically observed short-run persistence and long-run reversal in fund performance. Our model also explains why mutual funds tend to be relatively undiversified and exhibit persistent stock selection. It also supports the findings of Carhart et al. (2002) that trading-induced equity price inflation near the end of the last day of a quarter contributes to abnormal fund returns on those days and that the end of year performance effect is more pronounced for better historical performers (i.e., funds with more cash on hand). Finally, we find that the equally-weighted index return is significantly higher on the last trading day of a quarter, and lower on the first trading day than on other trading days; and further that this return difference has risen with the percentage of total equity that is held by mutual funds. This strong empirical evidence indicates that the incentives of fund managers to distort investments are so high at the end of a quarter that their behavior significantly alters market outcomes.
A Proofs

Proof of Proposition 1: From the first-order conditions:

\[
\lambda_{bad} = \frac{(1 + \gamma)P_{At} + (1 - \gamma)\Delta P(P_{At}, I_{At}) + \gamma P_{At} + (1 - \gamma)I_{At} \frac{\partial \Delta P}{\partial P_{At}} + S_{At}(1 - \gamma)\frac{\partial \Delta P}{\partial I_{At}}}{P_{At} + \Delta P(P_{At}, I_{At}) + I_{At} \frac{\partial \Delta P}{\partial I_{At}}} \quad (10)
\]

\[
= \frac{(1 + \gamma)P_{Bt} + (1 - \gamma)\Delta P(P_{Bt}, I_{Bt}) + \gamma P_{Bt} + (1 - \gamma)I_{Bt} \frac{\partial \Delta P}{\partial P_{Bt}} + S_{Bt}(1 - \gamma)\frac{\partial \Delta P}{\partial I_{Bt}}}{P_{Bt} + \Delta P(P_{Bt}, I_{Bt}) + I_{Bt} \frac{\partial \Delta P}{\partial I_{Bt}}} \quad (11)
\]

Substituting for \(\Delta P(P_j, I_j) = k(P_j I_j)P_j\),

\[
(1 - \gamma) + \frac{\gamma + i + \gamma \delta_1 + P_{At}S_{At}(1 - \gamma)k}{1 + 2kP_{At}I_{At}} = (1 - \gamma) + \frac{\gamma + i + \gamma \delta_2 + P_{Bt}S_{Bt}(1 - \gamma)k}{1 + 2kP_{Bt}I_{Bt}} \quad (12)
\]

Then, reversing \(P_{At}S_{At}\) and \(P_{Bt}S_{Bt}\), holding \(P_{At}I_{At}\) and \(P_{Bt}I_{Bt}\) fixed, it is clear that:

\[
\frac{\gamma + i + \gamma \delta_1 + P_{At}S_{At}(1 - \gamma)k}{1 + 2kP_{At}I_{At}} < \frac{\gamma + i + \gamma \delta_2 + P_{At}S_{At}(1 - \gamma)k}{1 + 2kP_{Bt}I_{Bt}}
\]

To restore equality, it follows that \(P_{At}I_{At}^*(\delta_1, \delta_2) > P_{Bt}I_{Bt}^*(\delta_2, \delta_1)\). Then it follows that

\[
(P_{At} + \Delta P(P_{At}, I_{At}(\delta_1, \delta_2)))I_{At}(\delta_1, \delta_2) > (P_{Bt} + \Delta P(P_{Bt}, I_{Bt}(\delta_2, \delta_1)))I_{Bt}(\delta_2, \delta_1)
\]

We now show that investment in stock A is a decreasing function of \(\gamma\). Let \(\delta_1 = \delta_2 = \delta\). Taking the derivative of the left hand side (LHS) and right hand side (RHS) of (12) we have:

\[
\frac{\partial LHS}{\partial \gamma} = -1 + \frac{1 + \delta - P_{At}S_{At}k}{1 + 2kP_{At}I_{At}} < \frac{\partial RHS}{\partial \gamma} = -1 + \frac{1 + \delta - P_{Bt}S_{Bt}k}{1 + 2kP_{Bt}I_{Bt}}
\]

since \(1 + \delta - P_{At}S_{At}k < 1 + \delta - P_{Bt}S_{Bt}k\) and \(1 + 2kP_{At}I_{At} > 1 + 2kP_{Bt}I_{Bt}\). Then, we need to show that under the same circumstances,

\[
\frac{\partial LHS}{\partial (P_{At}I_{At})} - \frac{\partial RHS}{\partial (P_{At}I_{At})} = \frac{\partial LHS}{\partial (P_{At}I_{At})} \frac{\partial P_{At}I_{At}}{\partial (P_{At}I_{At})} - \frac{\partial RHS}{\partial (P_{Bt}I_{Bt})} \frac{\partial P_{Bt}I_{Bt}}{\partial (P_{At}I_{At})} < 0
\]

But \(\frac{\partial P_{Bt}I_{Bt}}{\partial (P_{At}I_{At})} = -1\) and \(\frac{\partial P_{At}I_{At}}{\partial (P_{At}I_{At})} = \frac{\partial P_{At}I_{At}}{\partial (P_{At}I_{At})}\), thus we only need to show that

\[
\frac{\partial LHS}{\partial (P_{At}I_{At})} + \frac{\partial RHS}{\partial (P_{Bt}I_{Bt})} < 0.
\]

But this follows since

\[
\frac{\partial LHS}{\partial (P_{At}I_{At})} + \frac{\partial RHS}{\partial (P_{Bt}I_{Bt})} = -2k \left[ \frac{\gamma + i + \gamma \delta + P_{At}S_{At}(1 - \gamma)k}{(1 + 2kP_{At}I_{At})^2} + \frac{\gamma + i + \gamma \delta + P_{Bt}S_{Bt}(1 - \gamma)k}{(1 + 2kP_{Bt}I_{Bt})^2} \right] < 0.
\]
Proof of Proposition 2: If $M_t > 0$, then $\lambda_{bud} = (1 + i)$ and the marginal dollar is invested in cash ($dM_t/df = 1; \partial I_j^d/df = 0$, $j = A, B$). Differentiating short-run expected returns,

$$E[r_t] = [(1 + i)M_t + E[QP_A](I_A + S_A) + E[QP_B](I_B + S_B)] [P_A S_A + P_B S_B + f(r_{t-1})]^{-1} - 1,$$

with respect to $f = f(r_{t-1})$ yields

$$\frac{\partial E[r_t(I_A, I_B)]}{\partial f} = \left[ (1 + i) \frac{dM_t}{df} + \frac{d}{df} \left( E[QP_A](I_A^* + S_A) + E[QP_B](I_B^* + S_B) \right) \right] [P_A S_A + P_B S_B + f]^{-1} - \left[ (1 + i) M_t + E[QP_A](I_A^* + S_A) + E[QP_B](I_B^* + S_B) \right] [P_A S_A + P_B S_B + f]^{-2} = (1 + i) [P_A S_A + P_B S_B + f]^{-1} - \left[ (1 + i) M_t + E[QP_A](I_A^* + S_A) + E[QP_B](I_B^* + S_B) \right] [P_A S_A + P_B S_B + f]^{-2} = [i - E[r_t]] [P_A S_A + P_B S_B + f]^{-1} \leq 0, \quad \text{strict if } E[r_t] > i.$$

where the last equality follows from substitution. The expected mutual fund return ($E[r_t]$) is always at least as large as the risk-free rate ($i$) since the fund manager can always earn $i$ by investing new fund inflows in cash.

Proof of Proposition 3: From the budget constraint,

$$f = [P_A + \Delta P_A(P_A, I_A)] I_A + [P_B + \Delta P_B(P_B, I_B)] I_B,$$

we first prove the result that short-run returns are increasing in $f$ when $I_A$ and $I_B$ are small if $f$ is close to zero (as will be the case for $S_A \sim S_B$, and $P_A \sim P_B$). Then, it follows that:

$$1 = \frac{\partial I_A}{\partial f} P_A + \frac{\partial I_B}{\partial f} P_B.$$

If the fund manager is fully invested, then expected returns are

$$E[r_t] = \sum_{j=A,B} \{ (1 + \gamma) P_j (I_{jt} + S_{jt}) + (1 - \gamma) \Delta P_{jt}(P_{jt}, I_{jt})(I_{jt} + S_{jt}) \}.$$  

Thus

$$\frac{\partial E[r_t]}{\partial f} \bigg|_{f=0} = \left[ P_A S_A + P_B S_B \right]^{-1} \left[ (1 - \gamma) \left( \frac{\partial \Delta P_A}{\partial I_A} \frac{\partial I_A}{\partial f} S_A + \frac{\partial \Delta P_B}{\partial I_B} \frac{\partial I_B}{\partial f} S_B \right) + (1 + \gamma) \left( \frac{\partial I_A}{\partial f} P_A + \frac{\partial I_B}{\partial f} P_B \right) \right] - \left[ (1 + \gamma) \left( \frac{\partial \Delta P_A}{\partial I_A} S_A + \frac{\partial \Delta P_B}{\partial I_B} S_B \right) \right].$$

Substituting for $P_B \frac{\partial I_B}{\partial f} + P_A \frac{\partial I_A}{\partial f} = 1$ yields

$$\frac{\partial E[r_t]}{\partial f} \bigg|_{f=0} = \left[ P_A S_A + P_B S_B \right]^{-1} (1 - \gamma) \left[ \frac{\partial \Delta P_A}{\partial I_A} \frac{\partial I_A}{\partial f} S_A + \frac{\partial \Delta P_B}{\partial I_B} \frac{\partial I_B}{\partial f} S_B \right] > 0.$$
since $\frac{\partial P_{At}}{\partial I_{At}} > 0$ (by assumption) and $\frac{\partial P_{Bt}}{\partial I_{Bt}} > 0$ (since (12) can only hold if an increase in $I_{At}$ coincides with an increase in $I_{Bt}$).

More generally to show that the derivative of returns with respect to $f$ is positive recognize that this amounts to showing that

$$
\text{sign} \left( \frac{\partial \text{num}}{\partial f} - \text{num} \right) \bigg|_{f=0} > 0,
$$

where $\text{num}$ is the numerator of expected returns. Equivalently, dividing through by $P_{At}S_{At} + P_{Bt}S_{Bt}$, it amounts to showing that

$$
\text{sign} \left( \frac{\partial \text{num}}{\partial f} - (1 + E[r_t]) \right) \bigg|_{f=0} = \text{sign} \left( \lambda_{bud} - (1 + E[r_t]) \right) \bigg|_{f=0} > 0,
$$

where the equality follows from substitution of equations (10) and (11). That is, $\lambda_{bud}$ reflects the marginal return one more dollar, and if the marginal return exceeds $1 + r_t$, then $r_t$ must be increasing in $f$. The Lagrange multiplier can be interpreted as the marginal value of one more dollar of investment in stock $A$ or as the reduction in the marginal cost of selling one more dollar of investment in stock $B$. We now show that for $\delta \geq 0$, this marginal cost grows monotonically as the fund manager sells more $I_B$. Differentiating (11) with respect to $I_{Bt}$ we see that it has sign

$$
(P_{Bt} + \Delta P(P_{Bt}, I_{Bt}) + I_{Bt}\Delta P'(P_{Bt}, I_{Bt}))
\times[(1 - \gamma)(2\Delta P'(P_{Bt}, I_{Bt}) + I_{Bt}\Delta P''(P_{Bt}, I_{Bt})) + S_{Bt}(1 - \gamma)\Delta P''(P_{Bt}, I_{Bt})]
-(2\Delta P'(P_{Bt}, I_{Bt}) + I_{Bt}\Delta P''(P_{Bt}, I_{Bt}))
\times[(1 + \gamma)P_{Bt} + (1 - \gamma)\Delta P(P_{Bt}, I_{Bt}) + \gamma\delta P_{Bt} + (1 - \gamma)(I_{Bt} + S_{Bt})\Delta P'(P_{Bt}, I_{Bt})]

< (P_{Bt} + \Delta P(P_{Bt}, I_{Bt}) + I_{Bt}\Delta P'(P_{Bt}, I_{Bt}))S_{Bt}(1 - \gamma)\Delta P''(P_{Bt}, I_{Bt})
-(2\Delta P'(P_{Bt}, I_{Bt}) + I_{Bt}\Delta P''(P_{Bt}, I_{Bt}))(\gamma\delta P_{Bt} + S_{Bt}(1 - \gamma)\Delta P'(P_{Bt}, I_{Bt})) < 0,
$$

where the last inequality follows from $\delta \geq 0$ and A4. $\lambda_{bud}$ takes its value evaluated at $I_{Bt}^*$, so that it exceeds the derivative evaluated at $I_{Bt} = 0$, which we have proven exceeds $r_t$. For $\delta < 0$ an analogous exercise on (10) establishes monotonicity for $I_{At} > 0$.

That short-run returns must eventually decline with $f$, follows from A4, which implies that marginal returns from making arbitrarily large purchases of a stock must eventually become negative. This implies that once $f$ grows sufficiently large, the fund manager begins to invest in cash at the point where the marginal return on investing in stock is $i$, and where the short-run return on the portfolio exceeds $i$. It follows that for $f$ larger, short-run returns decline.
References


Table 1: Regression results: dependent variable is the difference between the return on the last trading day of the quarter $t$, denoted $r_t$, and the return on the first trading day of the next quarter, denoted $\hat{r}_t$. Return differences are expressed in percent. $t$-values are reported in parentheses below the coefficient estimates.

Number of observations: 127. The estimated regression model is:

$$(r_t - \hat{r}_t) = \beta_0 + \beta_1 \left( \frac{\text{Mutual fund holdings of corporate equity}_t}{\text{Market value of domestic companies}_t} \right) + \varepsilon_t \quad \varepsilon_t \sim iid \left(0, \sigma^2\right)$$

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<th>P-value</th>
<th>Coefficient</th>
<th>P-value</th>
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<tr>
<td></td>
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<tr>
<td>Value-weighted (inc. dividend)</td>
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<td>0.897</td>
<td>0.0182</td>
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<td></td>
<td>(-0.130)</td>
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<td>(1.13)</td>
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<td>Value-weighted (exc. dividend)</td>
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<td>0.919</td>
<td>0.0180</td>
<td>0.261</td>
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<tr>
<td></td>
<td>(-0.101)</td>
<td></td>
<td>(1.13)</td>
<td></td>
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</tr>
</tbody>
</table>
Table 2: Regression results: dependent variable is the difference between the return on the last trading day of the quarter \( t \), denoted \( r_t \), and the average daily return calculated over the remainder of the trading days in the quarter, denoted \( \bar{r}_t \). Return differences are expressed in percent. \( t \)-values are reported in parentheses below the coefficient estimates. Number of observations: 127. The estimated regression model is:

\[
(r_t - \bar{r}_t) = \beta_0 + \beta_1 \left( \frac{\text{Mutual fund holdings of corporate equity}_t}{\text{Market value of domestic companies}_t} \right) + \varepsilon_t \quad \varepsilon_t \sim iid \left(0, \sigma^2\right)
\]

<table>
<thead>
<tr>
<th>Returns</th>
<th>Share of equity held by mutual funds (in %)</th>
<th>Constant Coefficient</th>
<th>P-value</th>
<th>Coefficient</th>
<th>P-value</th>
<th>adj. ( R^2 )</th>
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<tr>
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<td>0.001</td>
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<td>(1.62)</td>
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<td>(3.46)</td>
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<td>Value-weighted (inc. dividend)</td>
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<td>0.00238</td>
<td>0.841</td>
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<td></td>
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<td>(0.560)</td>
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<td>(0.201)</td>
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<td>Value-weighted (exc. dividend)</td>
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<td></td>
<td>(0.467)</td>
<td></td>
<td>(0.162)</td>
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</table>
Table 3: Regression results: dependent variable is the difference between the return on the last trading day of month \((r_{t_i})\) and the average daily return calculated over the remainder of the trading days in the month \((\bar{r}_{t_i})\), where \(i = 1\) indicates the first month in quarter \(t\), \(i = 2\) indicates the second month in quarter \(t\), and \(i = 3\) indicates the last month in quarter \(t\). Monthly return differences are averaged over each quarter \(t\) and are expressed in percent per day. \(t\)-values are reported in parentheses below the coefficient estimates. Number of observations: 127. The estimated regression model is:

\[
\left(\frac{1}{3} \sum_{i=1}^{3} (r_{t_i} - \bar{r}_{t_i})\right) = \beta_0 + \beta_1 \left(\frac{\text{Mutual fund holdings of corporate equity}_{t}}{\text{Market value of domestic companies}_{t}}\right) + \varepsilon_t \\
\varepsilon_t \sim iid \left(0, \sigma^2\right)
\]

<table>
<thead>
<tr>
<th>Returns</th>
<th>Share of equity held</th>
<th>Constant by mutual funds (in %)</th>
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</thead>
<tbody>
<tr>
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<td>P-value</td>
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<td>Value-weighted (inc. dividend)</td>
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<td>(1.91)</td>
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</table>
Table 4: Regression results: dependent variable is the difference between the return on the last trading day of the month \((r_{ti})\) and the return on the first trading day of the next month \((\hat{r}_{ti})\), where \(i = 1\) indicates the first month in quarter \(t\), \(i = 2\) indicates the second month in quarter \(t\), and \(i = 3\) indicates the last month in quarter \(t\). Monthly return differences are averaged over each quarter \(t\) and are expressed in percent per day. \(t\)-values are reported in parentheses below the coefficient estimates. Number of observations: 127. The estimated regression model is:

\[
\left(\frac{1}{3} \sum_{i=1}^{3} (r_{ti} - \hat{r}_{ti})\right) = \beta_0 + \beta_1 \left(\frac{\text{Mutual fund holdings of corporate equity}_{t}}{\text{Market value of domestic companies}_{t}}\right) + \varepsilon_t \quad \varepsilon_t \sim iid \ (0, \sigma^2)
\]

<table>
<thead>
<tr>
<th>Returns</th>
<th>Share of equity held by mutual funds (in %)</th>
<th>Coefficient</th>
<th>P-value</th>
<th>Coefficient</th>
<th>P-value</th>
<th>adj. (R^2)</th>
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<td></td>
<td>Constant</td>
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<tr>
<td>Equally-weighted (inc. dividend)</td>
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<td>0.152</td>
<td>0.0175</td>
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<td>(1.44)</td>
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<td>(2.47)</td>
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<tr>
<td>Equally-weighted (exc. dividend)</td>
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<td>0.139</td>
<td>0.0174</td>
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<td>0.039</td>
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<tr>
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<td>(1.49)</td>
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<td>(2.46)</td>
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<td>Value-weighted (inc. dividend)</td>
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<td>(-1.61)</td>
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</table>
Table 5: Regression results: dependent variable is the difference between the return on the last trading day of the month ($r_{ti}$) and the return on the first trading day of the month ($\hat{r}_{ti}$), for months that do not occur at the end of a quarter ($i = 1, 2$), where $i = 1$ indicates the first month in quarter $t$, $i = 2$ indicates the second month in quarter $t$, and $i = 3$ indicates the last month in quarter $t$. Monthly return differences are averaged over each quarter $t$ and are expressed in percent per day. $t$-values are reported in parentheses below the coefficient estimates. Number of observations: 127. The estimated regression model is:

$$\left( \frac{1}{2} \sum_{i=1}^{2} (r_{ti} - \hat{r}_{ti}) \right) = \beta_0 + \beta_1 \left( \frac{\text{Mutual fund holdings of corporate equity}_t}{\text{Market value of domestic companies}_t} \right) + \epsilon_t \sim iid \left(0, \sigma^2\right)$$

<table>
<thead>
<tr>
<th>Returns</th>
<th>Constant by mutual funds (in %)</th>
<th>Share of equity held</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>P-value</td>
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<tr>
<td>Equally-weighted (inc. dividend)</td>
<td>0.197</td>
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<td>Equally-weighted (exc. dividend)</td>
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<td>0.041</td>
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<td>Value-weighted (exc. dividend)</td>
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<td>0.034</td>
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<td>(2.14)</td>
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</table>
Figure 1: Performance of positive and negative portfolios for different holding periods. The figure plots the monthly average $\alpha_3$ for portfolios with positive inflows (POS) and portfolios with negative inflows (NEG) for different holding periods up to 36 months. POS invests in all available funds with positive new money and weights by funds’ new money. NEG invests in all available funds with negative new money and weights by funds’ new money. $\alpha_3$ is calculated from the time series regression of the excess portfolio returns on the excess market return and the mimicking returns for the size (SMB) and book-to-market equity factors. The average monthly portfolio $\alpha_3$ are calculated for different holding periods of 1 to 36 months after the portfolio formation. **Source:** Zheng (1999), p.930.
Figure 2: Fund flow-return relationship in base example. Parameters: $S_{At} = S_{Bt} = 1$, $a = b = 0.1$, $\gamma = 0.5$, $i = 0.05$, $P_{At} = P_{Bt} = 1$, $\delta_{A,t+1} = \delta_{B,t+1} = 0$. For $f_t \leq 0.9504$, $I_{At} = I_{Bt} = ((1 + 2a f_t)^{0.5} - 1)/2a$ and $M = 0$. For $f_t > \bar{f} = 0.9504$, $I_{At} = I_{Bt} = 0.4546$ and $M_t = f_t - \bar{f}$. 

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Figure 3: Fund manager’s investment decision and differences in price sensitivity. Parameters: $i = 0.05$, $\gamma = 0.5$, $P_{At} = P_{Bt} = 1$, $\delta_{A,t+1} = \delta_{B,t+1} = 0.05$, $b = 0.1$. Balanced corresponds to $S_{At} = S_{Bt} = 1$. Unbalanced corresponds to $S_{At} = 2$ and $S_{Bt} = 1$. For both the balanced and unbalanced cases, we plot the fund manager’s investment in stock $A$, $I_{At}$, versus the price impact of stock $A$ to order flow ($a$) for low cash inflows ($f = 0.2$) and for high cash inflows ($f = 1.8$).
Figure 4: Plot of (Return on last trading day of quarter − return on first trading day of next quarter) versus (Mutual fund holdings of corporate equity / Market value of domestic companies). Returns are based on the CRSP Equally-Weighted Index (excluding dividends).