Better Cross Hedges with Composite Hedging?

Hedging Equity Portfolios

Using Financial and Commodity Futures

Fei Chen
ICMA Centre, University of Reading

Charles Sutcliffe
ICMA Centre, University of Reading

10 May 2007

ICMA Centre Discussion Papers in Finance DP2007-04

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Abstract

Unless a direct hedge is available, cross hedging must be used. In such circumstances portfolio theory implies that a composite hedge (the use of two or more hedging instruments to hedge a single spot position) will be beneficial. Surprisingly, the study and use of composite hedging has been neglected; possibly because it requires the estimation of two or more hedge ratios. This paper demonstrates a statistically significant increase in out-of-sample effectiveness from the composite hedging of the Amex Oil Index using S&P500 and Nymex crude oil futures. This conclusion is robust to the technique used to estimate the hedge ratios, and to allowance for transactions costs, dividends and the maturity of the futures contracts.

Key Words: Composite hedging, cross hedging, Amex oil index, contract maturity, transactions costs

Author Details:

Ms Fei Chen
The ICMA Centre,
University of Reading,
PO Box 242,
Reading RG6 6BA,
Email: F.Chen@icmacentre.reading.ac.uk

Professor Charles Sutcliffe
The ICMA Centre,
University of Reading,
PO Box 242,
Reading RG6 6BA,
Email: C.M.S.Sutcliffe@rdg.ac.uk

The authors wish to thank John Board (University of Reading) and Thorsten Zimmermann and Ghasan Shamsan (University of Southampton) for their input to earlier drafts of this paper.
Hedging is a major reason for trading in derivative markets, and has been extensively studied. It usually involves a single spot position being hedged with a single hedging instrument. However, in a substantial number of situations a more effective hedge may be available - a composite hedge. This is when a single spot position is hedged using two or more hedging instruments. This paper argues that composite hedging has been neglected by both practitioners and academics, and shows that in appropriate circumstances it can lead to a substantial improvement in risk reduction. The spot position to be hedged may consist of a single asset (or liability), or a portfolio of different assets (liabilities). The present paper concentrates on hedging a single asset, contrasting the performance of composite hedging with that of single hedging.

Depending on the nature of the hedging instrument, single hedges can be classified into perfect, direct and cross hedges.

1. A perfect hedge means that the hedge portfolio has virtually zero risk. An example of a perfect hedge is when the value of an asset at time $T$ is hedged by taking a long position in a futures contract on this asset for delivery at time $T$. The value of the future converges to that of the hedged asset at time $T$.

2. A direct hedge occurs when the hedging instrument is a derivative based on the price of the spot position, but the time to maturity of the hedging instrument ($J$) differs from the holding period of the underlying asset ($K$). If $J > K$, a single hedging instrument can be used, and there will be basis risk. If $J < K$, the hedging instrument must be rolled over, leading to roll over risk, as well as basis risk.

3. In comparison with the number of spot assets, the number of available hedging instruments is relatively small. Therefore a direct hedge may not exist, and hedging must be conducted using instruments based on a correlated asset, i.e. a cross hedge. A cross hedge usually involves considerably more risk than a direct hedge because the hedging instrument is not based on the underlying asset being hedged. This means that, even if $J = K$, there is still risk because the price of the hedging instrument does not converge to the price of the asset being hedged at the delivery date.

Composite hedging is relevant when there is no available direct hedge, so that only cross hedges
are possible. Composite hedging is also known as basket hedging, multiple hedging, portfolio hedging and, confusingly, cross hedging. It will be most beneficial when the spot asset has two or more important sources of risk for which hedging instruments are available. For example, a spot position may involve some combination of commodity price risk, credit risk, interest rate risk, stock market risk, exchange rate risk, freight rate risk and weather risk. Circumstances in which a composite hedge has proved beneficial include:-

1. The hedging instrument is denominated in a different currency from the spot asset, introducing foreign exchange risk, in addition to price risk.
2. The hedging instrument and spot position are in the same currency, which is different to that of the trader.
3. Hedging the cost of a forward commitment that includes freight costs.
4. Hedging high yield bonds that are exposed to both interest rate and stock market or credit risk.
5. Hedging long term forward commitments, where repeated roll over is necessary.
6. Hedging the shares of a commodity-based industry.
7. Hedging an intermediate product with futures on the final output and the finishing costs.

Hedging is analogous to finding the risk-minimizing portfolio, where investment in the spot asset is fixed. Hedging using a single hedging instrument corresponds to finding the risk-minimizing two asset portfolio. As for portfolio theory, a risk-minimizing hedge portfolio with many assets usually has lower risk than a two asset portfolio due to the diversification effect and less than perfect positive correlations between the spot position and hedging instruments. Composite hedging exploits this diversification effect by using two or more hedging instruments.

After discussing possible reasons for the neglect of composite hedging in section 1, the theory of composite hedging is summarized in section 2. The previous empirical literature on composite hedging is reviewed in section 3. Section 4 describes the data used in this study - a spot position in the Amex Oil Index, hedged using crude oil and S&P500 futures. Section 5 explains the alternative procedures employed for estimating the hedge ratios, while section 6 sets out the methodology used in this study. Sections 7 to 10 have the results for hedging effectiveness, the
hedge ratios, and the inclusion of transactions costs and dividends, respectively. Section 11 investigates the effects of allowing for changes in the maturity of the hedging instruments, and section 11 contains the conclusions.

1. The Neglect of Composite Hedging

There have been thousands of empirical studies of hedging, but very few have used multiple hedging instruments, e.g. composite hedging. This may be because the vast majority of previous studies have chosen to investigate hedging a spot position with a future on that spot position, making composite hedging unnecessary. However, in many real world hedging situations there is no direct hedging instrument, a cross hedge is necessary, and composite hedging may well be the most effective hedging strategy. Therefore composite hedging is relevant to many hedging problems. In addition, the concept of composite hedging is based on portfolio theory and diversification; making its use a straightforward extension of well known techniques. Therefore, the lack of attention given to composite hedging in the academic literature, and the absence of examples of its use by practitioners, is surprising.

Possible reasons for the neglect of composite hedging by practitioners include:-
• minimal incremental risk reduction from adding a second hedging instrument.
• the associated increase in transactions costs.
• increased indivisibility problems for the hedging instrument, as two or more instruments are used.
• lack of awareness of composite hedging.
• the reluctance of practitioners to formally apply portfolio theory.

Section 3 of this paper provides a literature review which demonstrate a range of applications where the gains from composite hedging are substantial. Later sections of this paper present a new empirical application of composite hedging which also investigates the associated transactions costs, relative to the reduction in risk; and argues that there is a strong case for composite hedging after allowing for transactions costs. The indivisibility problem is a function of the absolute size of the spot position to be hedged, relative to the contact size of the hedging
instrument. Since contract size does not apply in forward markets, indivisibilities are not a problem for currency hedges; yet there is little evidence of substantial composite hedging using currency futures. Since knowledge of portfolio theory is widespread, this leaves an unwillingness of practitioners to formally apply portfolio theory, even in the three variable case, as the remaining explanation for the minimal usage of composite hedging.

Michaud (1989) suggests seven reasons why practitioners may not formally apply portfolio theory:- (a) the need to forecast a large number of parameters, together with estimation error maximization, because portfolio theory concentrates money in the outliers; (b) the model ignores market impact; (c) the efficient frontier has unstable optimal portfolios due to ill conditioning; (d) counter intuitive portfolios are generated (investment in only about 10% of the available assets when short selling is prohibited; and investment in every asset, often with very large short positions, when it is not); (e) poor performance, (f) politics - the effective use of portfolio theory requires a change in the organisational structure; and (g) an unwillingness or inability of fund managers to grapple with the technicalities of portfolio theory.

However, these reasons for not formally using portfolio theory apply with much less force to composite hedging. The error maximization problem is much reduced. Forecasts of only three correlations and three variances are required for a composite hedge using two hedging instruments. The failure to allow for market impact, unstable optimal portfolios and counter intuitive portfolios are not substantive problems for composite hedging. Poor performance is contradicted by the empirical results presented in this and other studies of composite hedging. This leaves just politics, and an unwillingness to grapple with the technicalities of portfolio theory.

When engaged in single hedging, practitioners usually do not formally estimate hedge ratios, but select a hedge ratio of around unity, without the explicit application of portfolio theory. For a composite hedge it is much more difficult to guess a set of appropriate hedge ratios. In this case practitioners will probably be required to formally estimate the multiple hedge ratios. This may be an important reason for the reluctance of practitioners to use composite hedging, which may
be reduced if “standard” composite hedge ratios were available from academic studies.

2. Composite Hedging

Composite hedging occurs when the risk of a single spot position (e.g. an equity portfolio) is hedged using more than one hedging instrument (Anderson and Danthine, 1980, 1981). The formulae for composite hedging will be derived using a simple model, expressed in terms of equity returns.

For simplicity, assume that there are two sources of risk, \( G_1 \) and \( G_2 \). The analysis below can be generalized to \( n \) sources of risk and hedging instruments. Returns on the spot portfolio whose risks are to be hedged over period \( t \) to \( t+k \) are given by \( s_t = (S_{t+k} - S_t + D_{t+k})/S_t \), where \( D_{t+k} \) is the value at time \( t+k \) of any dividend entitlements received during the period \( t \) to \( t+k \), and \( S_t \) is the price of one unit of the spot asset (e.g. the portfolio of shares being hedged) at time \( t \). Let spot returns have a linear response to these two sources of risk:

\[
S_t = \alpha_s + \beta_{1s} G_{1t} + \beta_{2s} G_{2t} + \varepsilon_{st}
\]

where \( \alpha_s \) is a constant, \( \beta_{1s} \) and \( \beta_{2s} \) are slope coefficients and \( \varepsilon_{st} \) is a disturbance term. Assume that two other instruments, \( X \) and \( Y \) (with returns \( x_t \) and \( y_t \), respectively), exist and that their returns are also linear functions of the two sources of risk.

\[
x_t = \alpha_x + \beta_{1x} G_{1t} + \beta_{2x} G_{2t} + \varepsilon_{xt}
\]

\[
y_t = \alpha_y + \beta_{1y} G_{1t} + \beta_{2y} G_{2t} + \varepsilon_{yt}
\]

where \( \alpha_x \) and \( \alpha_y \) are constants, \( \beta_{1x}, \beta_{2x}, \beta_{1y} \) and \( \beta_{2y} \) are slope coefficients, and \( \varepsilon_{xt} \) and \( \varepsilon_{yt} \) are disturbance terms. Equations (1) and (2) imply a linear relationship between spot returns and returns on the other two instruments.

\[
s_t = \gamma_0 + \gamma_1 x_t + \gamma_2 y_t + \omega_t
\]

where \( \gamma_0 \) is a constant, \( \gamma_1 \) and \( \gamma_2 \) are slope coefficients and \( \omega_t \) is a disturbance term. The two instruments \( X \) and \( Y \) can be used to hedge the portfolio against risk sources \( G_1 \) and \( G_2 \), and the coefficients \( \gamma_1 \) and \( \gamma_2 \) measure the sensitivity of spot returns on the portfolio to returns on the hedging instruments.
The rate of return on a futures contract is not well defined because the denominator is unclear. Different authors have used as the denominator: the current value of the underlying asset, the current futures price, the current value of the hedged portfolio, and other values. Letting $Z_t$ represent the denominator, it may be measured by $S_t$, $X_t$, $Y_t$ or the current value of the spot asset underlying the futures contract. Note that if returns are defined in terms of the current spot value, $S_t$, then $\lambda_X = \lambda_Y = 1$, and $b_X = \gamma_1$, and $b_Y = \gamma_2$.

In the traditional risk minimising approach to the estimation of hedge ratios, the hedged portfolio is designed not to change in value over the hedge period. The change in the value of the hedged portfolio over the period from $t$ to $t+k$ is given by:

$$S_{h,t+k} - S_h = Q_X [(S_{t+k} - S_t - D_{t+k})] + Q_X [(X_{t+k} - X_t)] + Q_Y [(Y_{t+k} - Y_t)]$$

where $Q_X$, $Q_Y$ and $Q$ are the quantities of the spot asset (in terms of units of the index basket) and the hedging instruments $X$ and $Y$ respectively, while $S_{h,t}$ is the value of the hedged position at time $t$. If $X$ and $Y$ are futures contracts, $Q_X$ and $Q_Y$ are measured in terms of the spot asset underlying the futures contract.

Letting $x_t = (X_{t+k} - X_t)/Z$, $y_t = (Y_{t+k} - Y_t)/Z$ be the returns on the hedging instruments, $s_{h,t} = (S_{h,t+k} - S_{h,t})/S_t$ be the return on the hedged position (since the initial investment in the hedging instruments is assumed to be zero), $\lambda_X = Z_y/S_p$, $\lambda_Y = Z_y/S_p$ and denoting the hedge ratios for the two hedging instruments as $b_X = Q_X/Q$, and $b_Y = Q_Y/Q$, then, dividing equation (4) through by $S_t$, allows it to be rewritten as:

$$s_{h,t} = Q_X [x_t + b_X \lambda_X x_t + b_Y \lambda_Y y_t]$$

The traditional objective is to select hedge ratios, $b_X$ and $b_Y$ to minimise the variance of $s_{h,t}$. Minimising $\text{Var}(s_{h,t})$ in equation (5) is equivalent to minimising $\text{Var}(\omega_t)$ in equation (3) (see Sutcliffe, 2006, ch.9) which in turn is just the standard regression problem. The coefficients $\gamma_0$ and $\gamma_1$ in equation (3) are related to the traditional hedge ratios implicit in equation (4):

$$b_X = \frac{Q_X}{Q} = \gamma_1 \frac{S_t}{Z_t}, \quad b_Y = \frac{Q_Y}{Q} = \gamma_2 \frac{S_t}{Z_t}$$

Assuming that the disturbance term in equation (3) is serially independent and normally distributed with zero mean and constant variance, the coefficients $\gamma_1$ and $\gamma_2$ in equation (3) can be estimated by ordinary least squares (OLS). The estimates are:

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1 The rate of return on a futures contract is not well defined because the denominator is unclear. Different authors have used as the denominator: the current value of the underlying asset, the current futures price, the current value of the hedged portfolio, and other values. Letting $Z$ represent the denominator, it may be measured by $S_t$, $X_t$, $Y_t$ or the current value of the spot asset underlying the futures contract. Note that if returns are defined in terms of the current spot value, $S_t$, then $\lambda_X = \lambda_Y = 1$, and $b_X = \gamma_1$, and $b_Y = \gamma_2$. 

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However, if \( \omega_t \) is not independent over time (or fails to satisfy other of the Gauss-Markov conditions), the OLS estimators in (7) will not be the best linear unbiased estimator, and alternative methods of calculating the hedge ratios are available. If the sources of risk are independent, so that \( \text{cov}(G_j, G_k) = 0 \), then equation (7) simplifies to the expressions for a single risk-minimizing hedge:

\[
\gamma_1 = \frac{\text{cov}(s, x) \text{var}(y) - \text{cov}(s, y) \text{cov}(x, y)}{\text{var}(x) \text{var}(y) - \text{cov}(x, y)^2},
\]

\[
\gamma_2 = \frac{\text{cov}(s, y) \text{var}(x) - \text{cov}(s, x) \text{cov}(x, y)}{\text{var}(x) \text{var}(y) - \text{cov}(x, y)^2}.
\]

3. Literature Review

Composite hedging has previously been applied to hedging the risks of spot positions in commodities, interest rates, foreign exchange and equities.

**Commodities.** Wilson (1984) examined hedging eight types of US wheat using up to three US wheat futures. He found that composite hedging using two futures improves hedging effectiveness by 1% to 2% compared with single hedging; while composite hedging using three futures offers little additional improvement in effectiveness. Braga (1990) hedged Canadian soyabean contracts with soyabean futures denominated in US$ to hedge the price risk, and US$-Canadian $ futures to hedge the exchange risk on the soyabean futures contracts. Using monthly data, he found this composite hedge removes 92% of the out-of-sample risk. Braga and Martin (1990) hedged Italian soyabean meal in lira using US soyabean meal futures and US$-DM futures. The composite hedges are about 15% more effective than the best single hedges. Grant and Eaker (1989) examined hedging spot positions in corn, wheat or oats with single and composite hedges. When the future on the underlying spot commodity is included in the composite hedge, there is little improvement in out-of-sample effectiveness, relative to a direct hedge.

Miller (1982b) considered hedging feeder pigs, which are young pigs that will be fattened and slaughtered. Two important determinants of the price of feeder pigs are the price of slaughtered
pigs (positive effect) and the cost of feeding them (negative effect). Therefore Miller used a composite hedge of live hog futures and corn futures, and found that this is superior to any single hedge. Distillers dried grains with solubles is a byproduct of ethanol, and Miller (1982a) found that hedging it with a composite hedge of soyabean meal and corn futures leads to a reduction in risk. Millfeed is a by-product of milling wheat into flour, and Miller (1985) examined the use of corn, wheat, oat and soyabean futures to hedge its risk. He found that composite hedges with up to four futures are superior to the best single hedge (corn).

Haigh and Holt (2000) investigated hedging the spot position of a US-based trader exporting wheat or soyabees to Rotterdam. Two hedging instruments were used: futures on either wheat or soyabees, and BIFFEX freight futures on route 1 (US Gulf to Rotterdam). They found that, for utility maximizing hedgers, composite hedging is superior. Haigh and Holt (2002) examined hedging a spot position in US wheat, corn or soyabees purchased by a German-based trader. They used three hedging instruments: futures on the US dollar price of the underlying wheat, corn or soyabees; US dollar-Deutschmark futures, and BIFFEX freight futures on route 1. For utility maximizing hedgers, composite hedging using all three futures is preferable to that using only one or two futures.

Herbst and Marshall (1994) studied hedging jet fuel using futures on crude oil, heating oil and unleaded gasoline; and found that composite hedging with the three futures improves effectiveness over single hedges. Neuberger (1999) hedged a commitment to supply crude oil 9 to 72 months in the future, using up to three Nymex crude oil futures of different maturities under 9 months. In this case the maturity of the hedging instrument is less than that of the forward commitment, the hedging instrument had to be rolled over, and a highly effective single hedge is impossible due to rollover risk. Neuberger demonstrated that hedging with two or three crude oil futures of different maturities substantially reduces risk.

**Interest Rates.** Pennings and Leuthold (2001) investigated hedging 10 year Dutch Treasury bonds with up to four financial futures, and concluded that composite hedging leads to greater effectiveness. Bookstaber and Jacob (1986) hedged 82 high yield corporate bonds with US
Treasury bonds and shares in the corresponding company, and found the composite hedges to be superior. Grieves (1986) examined hedging spot positions in portfolios of US Treasury bonds, industrial bonds and utility bonds with US Treasury bond futures and S&P500 futures. As expected, there is no improvement from the use of composite hedging for US Treasury bonds because US Treasury bond futures provide a direct hedge. Composite hedging improves effectiveness for industrial bonds by 2% for high grade bonds, and 15% for low grade bonds. The corresponding improvements for utility bonds are 0% and 2%. Marcus and Ors (1996) repeated the analysis by Grieves (1986) of hedging a portfolio of industrial bonds with Treasury and S&P500 futures, and confirmed that composite hedging is appropriate for industrial bonds. Ramaswami (1991) tested the ability of US Treasury bond futures and futures on the firm’s equity (with the future replicated using options) to hedge the risk of high yield bonds issued by Chrysler, RJR Nabisco and Kroger. Composite hedging is of minimal benefit for Chrysler, but is effective for RJR Nabisco and Kroger.

*Foreign Exchange.* Eaker and Grant (1987) examined hedging the risk of nine currencies against the US dollar using up to three currency futures. When the hedging instruments include the future on the spot currency being hedged, as expected, composite hedging do not provide any increase in hedging effectiveness. But when there is no direct hedge, composite hedging increases effectiveness by 3% to 14%. DeMaskey (1997) hedged the risk of six minor currencies (lira, peseta, drachma, Korean won, Singapore $ and Hong Kong $) with futures on five major currencies (sterling, mark, yen, Swiss franc and Canadian $). In each case the rate was against the USS. She found that composite hedging of the European minor currencies with futures on the major currencies is on average 14% more effective than single hedging; and that there is little difference between the effectiveness of composite hedges with two and all five futures. DeMaskey and Pearce (1998) hedged five minor currencies (Indonesia, Malaysia, Philippines, Singapore and Thailand) with six commodity futures and five futures on major currencies. For composite hedges of up to six futures, the out-of-sample results are disappointing. Only for Malaysia is the composite hedge superior to the best single hedge. Mun and Morgan (1997) hedged the currency risk of five countries (Indonesia, Korea, Malaysia, Singapore and Thailand) against the US$ with futures for five currencies against the US$ (sterling, Swiss franc, mark, yen,
Canadian $). They find that in each case composite hedging using all five futures is superior to the best single hedge.

Harris and Shen (2006) studied the benefits of hedging a cash position in Australia, Canada, the EU, Japan, New Zealand, Norway, Singapore, Sweden, Switzerland and the USA for a sterling investor. The hedging instruments were spot positions in one or two of the other nine currencies. For seven currencies, composite hedging is more effective; but for New Zealand, Norway and Singapore, composite hedging is inferior to single hedging. Bowman (2005) hedged the risk of the Australian dollar and the Papua New Guinea kina with 11 commodity futures, and found that composite hedges with four futures are superior to hedges using one or two futures. Lien and Luo (1993) investigated what they call “spreading”. They used futures on the German mark and the Swiss franc to hedge the risk of a spot position in the Swiss franc. The average hedge ratios are 0.95 for the Swiss franc and 0.05 for the German mark. Since the spot asset corresponds exactly to the asset underlying one of the futures, the composite hedge ratios of approximately one and zero are not surprising. In such cases, composite hedging will not be of benefit.

Equities. Lien and Luo (1993) hedged the risk of the NYSE Composite index using NYSE Composite and MMI futures. They found, as might be expected, that the average hedge ratios are 0.96 for NYSE Composite futures and 0.01 for MMI futures. Butterworth and Holmes (2001) hedged the risks of the FTSE 100, FTSE Mid 250, FTSE 350 and FT Investment Trust Indices using a combination of FTSE 100 and FTSE Mid 250 futures. Composite hedging increases effectiveness by between 0% and 5.5%, and this low level of improvement is expected because futures on the spot asset being hedged are used. Brooks, Davies and Kim (2007) hedged spot positions in 438 US stocks which did not have individual stock futures or options using up to three of 97 individual stock futures, and S&P500 futures. They concluded that the best hedging performance is achieved by composite hedging using the market index and one individual stock future. Sholund (1985) tested the feasibility of hedging changes in the weighted average cost of capital for General Electric and IBM using S&P500 and Treasury bond futures. For both firms he found that hedging with just Treasury bond futures is more effective than the composite hedge.
This literature survey found only 26 applications of composite hedging, supporting the view that it is a neglected technique. These empirical studies of composite hedging demonstrate that it can be applied to a wide range of real-world hedging situations, and in the right circumstances leads to substantial improvements in effectiveness.

4. Data
This paper examines the effectiveness of hedging an equity portfolio - an asset where there have been few previous studies of composite hedging. A portfolio of oil company shares (the Amex Oil Index) is hedged for one month using monthly data from January 1985 to February 2006 on S&P500 futures and Nymex crude oil futures (254 months).

The Amex Oil Index is a price weighted index of companies involved in the exploration, production and development of oil. In December 2006 the 13 index constituents were Marathon Oil, Exxon Mobil, Chevron Corp, Conocophillips, Total 'B' ADS, Royal Dutch Shell ADS, BP ADS, Sunoco Inc, Valero Energy, Hess Corp, Occidental Petroleum, Anadarko Petroleum and Repsol ADS. (The Amex Oil Index is treated as a single asset, rather than 13 separate assets.) The Amex Oil Index ignores dividends, and in section 10 these are added back to convert this price index into a total return index. Amex trades options on the price index, but not the corresponding total return index; and no futures or forwards are traded on either the price or total return indices. Therefore, no direct hedge exists and composite hedging is relevant; particularly for hedgers who wish to use futures or forwards.

In any composite hedge, it is necessary to select the number and identity of hedging instruments used.

A. Number of Hedging Instruments. There are diminishing returns to diversification, and the initial diversification, e.g. moving from one to two hedging instruments, is much more beneficial than moving from (say) 11 to 12 instruments. Wilson (1984), DeMaskey (1997) and Brooks, Davies and Kim (2004) found that composite hedging using two instruments is preferable to using more instruments; while Haigh and Holt (2002) and Bowman (2005) found some improvement from using additional hedging instruments. However, there comes a point where
the extra benefits from including additional hedging instruments are outweighed by the increase in transactions costs from trading more instruments. Just as investors limit the number of assets in their portfolio due to the various transactions costs of holding many different assets; so composite hedgers limit the number of hedging instruments they use.

B. Identity of Hedging Instruments. According to Gunnin (1984), risk in the oil industry can be divided into macro risk and micro risk. Macro risk refers to worldwide concerns such as OPEC and other political and economic factors that affect all oil companies. Micro risk, on the other hand, concerns the individual transactions of an oil company; and this firm-specific risk is diversified by the Amex Oil Index, which contains 13 oil companies. The macro risk can be split into oil market risk and stock market risk. Because there are two main sources of risk for the Amex Oil Index, this paper uses two hedging instruments to demonstrate the benefits of composite hedging. Two heavily traded US futures have been chosen to hedge each of these risks. Since the international oil industry operates in US$, and the spot and futures used in this study are also expressed in US$, there is no pressing need to hedge currency risk. Stock market risk was hedged with S&P500 futures traded on the Chicago Mercantile Exchange (CME), while oil market risk was hedged using crude oil futures traded on the New York Mercantile Exchange (Nymex).

S&P500 futures trade on a March, June, September and December cycle, and the nearest futures contract, which is the most liquid, is used. Monthly returns are computed as the difference in the logarithms of end of month prices. Every three months the near contract is rolled into the next near contract on the Thursday prior to the third Friday of the delivery month. To compute returns for roll months the price of the next near contract for the previous month is used, so that the return computation do not involve prices for futures with different maturities.

Nymex crude oil futures trade with a new contract every calendar month, and the contract used in this study is rolled on a monthly basis. A study of Nymex crude oil futures by Ripple and Moosa (2005) found that rolling the hedge every month is preferable to holding a six month position, supporting the strategy of monthly rollovers. The Nymex contract expires on the third
business day prior to the 25th calendar day of the month preceding the delivery month. Monthly logarithmic returns are computed in the same way as for S&P500 futures, except that the roll to the next near contract occurs every month.

The Amex Oil Index, S&P500 and crude oil futures prices are all taken at the close of trading on the last day of the month, and so any non-synchronicity problem is small in relation to monthly returns. The unconditional correlation of returns for the Amex oil index and crude oil futures is 0.359; for the Amex oil index and S&P500 futures it is 0.546; and for crude oil futures and S&P500 futures it is −0.108. These modest positive correlations between the spot asset to be hedged and each of the hedging instruments; and a negative correlation between the two hedging instruments support the use of composite hedging. Any single hedge will result in only a modest risk reduction; while the negative correlation indicates that the hedging instruments address different sources of risk, and so are worth using in combination.

5. Estimation of the Hedge Ratios

There are two important choices to be made when implementing a hedging strategy: - how to estimate the risk-minimizing hedge ratio; and whether to use a static or dynamic hedge ratio. There has been a long running academic debate on these two questions, and a large number of econometric techniques have been examined; e.g. Lee, Yoder, Mittelhammer and McCluskey (2006) and Moosa (2003). The static-dynamic choice is considered in section 6, while the choice of estimation technique is discussed now.

The focus of the present paper is the benefits of composite hedging, not the methodology for estimating hedge ratios; and so three popular estimation techniques are selected to demonstrate the robustness of the conclusions on composite hedging to the estimation method - (a) Ordinary Least Squares (OLS), (b) Multivariate GARCH (1,1) (M-GARCH), and (c) OLS or M-GARCH with an Error Correction Mechanism (ECM).

While the data does not comply with the OLS assumptions, this estimation technique is very widely used in the estimation of risk minimizing hedge ratios, and generally gives good hedging
effectiveness results (although this may be due to the use of Ederington’s effectiveness measure; see Lien, 2005a, 2005b). Therefore OLS is used in the analysis below.

The Engle (1982) and White (1980) tests found significant ARCH effects in the data, and so M-GARCH is also used to estimate the hedge ratios. The presence of GARCH effects is confirmed by the significant conditional variance coefficients for the M-GARCH models estimated in sections 7 to 11 below.

The Augmented Dicky Fuller (ADF) test with MacKinnon (1996) one-sided $p$-values found that Amex Oil Index and S&P500 futures prices are non-stationary, while crude oil futures prices are stationary. After first differencing, all three series are stationary. The residuals from the cointegrating regressions for the Amex Oil Index and crude oil futures, the Amex Oil Index and S&P500 futures, and the Amex Oil Index and the two hedging instruments were tested for stationarity. These ADF tests cannot not reject the null hypothesis that each of these sets of residuals has a unit root. Therefore, there is no long run relationship between the spot position to be hedged and the two hedging instruments, and the inclusion of an error correction mechanism is unnecessary. In view of the absence of any co-integrating relationships, the ECM approach is not used as one of the methods for estimating the hedge ratios.

Based on equation (3), the OLS regression to estimate the hedge ratios for composite hedges is:-

$$s_t = \alpha + \beta_1 f_{t-1} + \beta_2 f_{nt} + \epsilon_t \quad (9)$$

where $s_t$ is the logarithmic return on the Amex Oil Index for time period $t$, $f_t$ is the logarithmic return on S&P500 futures, $f_{nt}$ is the logarithmic return on crude oil futures, $\alpha$ is a constant term, $\beta_1$ and $\beta_2$ are the slope parameters and $\epsilon_t$ is the disturbance term (assumed for the OLS model to be independently and normally distributed). The corresponding hedge ratios are computed using equation (6), where $Z_{nt}$ is the relevant current futures price. For single hedges, only one of the futures is included in equation (9).

Multivariate GARCH involves $n$ simultaneous means equations. For the composite hedging case the means equations for the M-GARCH model are:
\[ s_t = a_s + \epsilon_{st} \]
\[ f_{ct} = a_c + \epsilon_{ct} \]
\[ f_{nt} = a_n + \epsilon_{nt} \]  \hspace{1cm} (10)

where \( a_s, a_c \) and \( a_n \) are constants; and \( \epsilon_{st}, \epsilon_{ct} \) and \( \epsilon_{nt} \) are disturbances for time period \( t \). In M-GARCH models the variance of each of the disturbance terms depends on the lagged squares and cross-products of all the disturbance terms. Using the restricted BEKK (Engle and Kroner, 1995) parameterization, the disturbance terms in equation 10 have a conditional variance-covariance matrix \( (H_t) \) given by:-

\[ H_t = C'C + A'E_{t-1}E'_{t-1}A + B'H_{t-1}B \]  \hspace{1cm} (11)

where \( E_t \) is a vector of the disturbance terms \( (\epsilon_t) \) in equation (10), and \( C, A \) and \( B \) are parameter matrices of order \( n \times n \), where \( n + 1 \) is the number of hedging instruments. \( C \) is a lower triangular matrix, while \( A \) and \( B \) are diagonal matrices. The BEKK representation guarantees that \( H_t \) is positive definite. It also requires the estimation of only 12 parameters in the conditional variance equation for the case of trivariate GARCH (i.e. the composite hedge), and 7 parameters for bivariate GARCH (i.e. the two single hedges). For the M-GARCH estimation, equations 10 and 11 were estimated using maximum likelihood. The risk minimizing hedge ratios are computed by using the estimated equation 11 to forecast the conditional variances and covariances for the next period. These forecasts are then inserted into equation 7 or 8.

6. Methodology

The main aim is to compare the effectiveness of risk-minimizing hedges using a single hedging instrument (either S&P500 futures or Nymex crude oil futures) with that of a composite hedge using both hedging instruments. Since the Amex Oil Index is price weighted, it can be replicated using a buy-and-hold strategy, and so does not create any necessity for the use of dynamic hedge ratios. However, the hedging literature contains many examples of hedge ratios that change over time, and the oil sector is notorious for large changes in price volatility, which then feed through to changes in the hedge ratio. Therefore, to allow for this possibility, the risk-minimizing hedge ratio is permitted to change over time. To this end, a rolling windows approach is adopted. The first 60 months (January 1985 to December 1989) are used as the estimation period to provide the hedge ratios for use in hedging the Amex Oil Index during the next month (January 1990).
The estimation window is then rolled forwards by one month (February 1985 to January 1990), and the hedge ratios re-estimated to give hedge ratios for the next month (February 1990). This process is repeated, with the last hedge ratio estimated for February 2006, producing 194 out-of-sample estimates of the hedge ratio.

Futures contracts come in integer quantities, leading to rounding problems when hedging small positions, particularly for composite hedging. This problem is avoided by assuming that the spot position to be hedged is large, relative to the size of the futures contracts. Since the crude oil futures are rolled every month, the maturity of the future used in the hedge is constant. However, the S&P500 contract is rolled once every 3 months, and so its maturity when the hedge is placed is 11 weeks, 7 weeks or 3 weeks. Herbst, Kare and Marshall (1993) and Herbst and Marshall (1994) derived a method of allowing for variations in the maturity of the hedging instrument when estimating the hedge ratio, and this will be investigated in section 11.

For the entire sample period both S&P500 futures and crude oil futures have a highly significant effect on the Amex Oil Index. However, for the much shorter 60 month window used in the regressions to estimate the hedge ratios, it is possible these relationships are less clear, and so the in-sample statistical significance of these estimated coefficients is examined for the OLS regressions. S&P500 futures hedge ratios are all significant at the 5% level, while the corresponding figures for crude oil futures are 68% for single hedges, and 97% for composite hedges. Corresponding numbers for the M-GARCH regressions are less clear cut as the hedge ratios are non-linear functions of 7 (single hedges) and 12 (composite hedges) parameters.

At the end of each month the investor’s wealth is computed for the hedged and unhedged portfolios. The out-of-sample effectiveness of the single and composite hedges is quantified by a measure analogous to the Ederington (1979) in-sample effectiveness measure:

\[
e = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)}
\]

(12)

where \(\text{Var}(U)\) and \(\text{Var}(H)\) are the variances of the 194 values of the unhedged and hedged portfolios, respectively. Lien (2005a, 2005b, 2006 and forthcoming) has shows that the Ederington effectiveness measure has various shortcomings. First, it is biased downwards, so
understating the benefits of hedging, and second, it favours OLS hedges. The primary focus of this paper is on comparing the effectiveness of composite hedges with single hedges, where both hedge ratios have been estimated in the same way. The two problems with the Ederington measure are not important for such comparisons.

7. Hedging Effectiveness Results

Table 1 shows the values of $Var(U)$, $Var(H)$ and $e$ for the two estimation techniques. For both the OLS and M-GARCH hedges, the composite hedge is substantially more effective than either of the single hedges. Indeed it is more effective than the sum of the two single hedges. This suggests that the risk hedged by crude oil futures is orthogonal to that hedged by S&P500 futures. So far as the estimation procedure is concerned, the OLS hedges are more effective than the M-GARCH hedges. However, the effectiveness measure is biased in favour of OLS (Lien, 2005a, 2005b), making such comparisons unreliable.

<table>
<thead>
<tr>
<th>Hedging Instruments</th>
<th>OLS</th>
<th>M-GARCH (1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>$e$</td>
</tr>
<tr>
<td>Unhedged</td>
<td>0.003</td>
<td>-</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>0.002</td>
<td>13.04%</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.002</td>
<td>24.14%</td>
</tr>
<tr>
<td>S&amp;P500 &amp; Crude Oil</td>
<td>0.001</td>
<td>42.71%</td>
</tr>
</tbody>
</table>

Table 1: Hedging Effectiveness of Single and Composite Hedges

While composite hedging has substantially higher effectiveness than single hedging, the superiority of composite hedging may not be statistically significant. Jarque-Bera tests show that returns on two hedged positions (OLS-S&P500 and M-GARCH-S&P500) are non-normal, while normality is accepted for the other four hedges. $F$-tests comparing a single hedge using crude oil futures with a composite hedge, both estimated using either OLS or M-GARCH, find that the composite hedge has a lower variance than the single hedge at the 5% significance level. However, due to non-normality, $F$-tests cannot be used to compare the other hedges, and so bootstrapping is employed to test whether composite hedging generates out-of-sample portfolios with a lower variance than does simple hedging. Table 2 sets out the bootstrapping results, where
1,000 replications are performed. The third column shows the ratio of the variance of returns on the composite hedge to the variance of returns on the simple hedge, while the fourth column has the one-tailed 5% significance levels. In every case the variance ratio is less than the required number, showing that the variance of returns on the composite hedges is significantly less than that on the corresponding single hedge.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Var(x) ÷ Var(y)</th>
<th>α = 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS-Composite</td>
<td>OLS-Crude Oil</td>
<td>0.6589</td>
<td>0.7531</td>
</tr>
<tr>
<td>OLS-Composite</td>
<td>OLS-S&amp;P500</td>
<td>0.7552</td>
<td>0.8562</td>
</tr>
<tr>
<td>M-GARCH-Composite</td>
<td>M-GARCH-Crude Oil</td>
<td>0.7018</td>
<td>0.8154</td>
</tr>
<tr>
<td>M-GARCH-Composite</td>
<td>M-GARCH-S&amp;P500</td>
<td>0.7788</td>
<td>0.9199</td>
</tr>
</tbody>
</table>

Table 2: Significance Tests on the Variance of the Hedged Positions

8. Hedge Ratio Analysis

The rolling windows procedure allows the out-of-sample hedge ratios to change from month to month, as the data in the previous 60 months changes. Tables 3 and 4 present some statistics for the dynamic OLS and M-GARCH hedge ratios, respectively. For both single and composite hedges, and for OLS and M-GARCH estimation; the mean hedge ratios for crude oil futures are much lower than for S&P500 futures. There are two reasons for this difference. First, S&P500 futures have a larger size than crude oil futures. Second, the risk-minimizing single hedge ratio in equation (8) is the covariance between spot and futures returns divided by the variance of futures returns. Since the variance of crude oil futures returns is five times higher than for S&P500 futures (0.0085 versus 0.0017), while the covariance with returns on the Amex Oil Index is similar (0.0015 versus 0.0010), the crude oil hedge ratio is lower than for S&P500 futures due to the higher denominator. A similar argument applies to the composite hedge ratios.
Table 3: Hedge Ratio Analysis - OLS

<table>
<thead>
<tr>
<th></th>
<th>Single Hedging</th>
<th>Composite Hedging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crude Oil</td>
<td>S&amp;P500</td>
</tr>
<tr>
<td>Mean</td>
<td>0.1708</td>
<td>0.6522</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0036</td>
<td>0.0104</td>
</tr>
<tr>
<td>ADF test p-value</td>
<td>0.3911</td>
<td>0.3195</td>
</tr>
<tr>
<td>Sum of absolute changes</td>
<td>2.0505</td>
<td>3.6012</td>
</tr>
</tbody>
</table>

Table 4: Hedge Ratio Analysis - M-GARCH

<table>
<thead>
<tr>
<th></th>
<th>Single Hedging</th>
<th>Composite Hedging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crude Oil</td>
<td>S&amp;P500</td>
</tr>
<tr>
<td>Mean</td>
<td>0.2146</td>
<td>0.7123</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0378</td>
<td>0.0513</td>
</tr>
<tr>
<td>ADF test p-value</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sum of absolute changes</td>
<td>26.19</td>
<td>23.30</td>
</tr>
</tbody>
</table>

For each estimation method, the single and composite mean hedge ratios for each instrument were broadly similar. This is consistent with the risks hedged by the two futures being orthogonal. The average total hedge ratio (crude oil plus S&P500 futures) for the composite hedges was 0.9412 for OLS and 1.0096 for M-GARCH; and so the use of OLS leads to slightly smaller futures positions being taken than when M-GARCH was used.

An important question for hedgers is the frequency with which the hedge requires rebalancing. The variance of the hedge ratios for S&P500 futures is 2 or 3 times larger than for crude oil futures for both OLS and M-GARCH estimates. In addition, the variances of the M-GARCH hedge ratios are 2 to 10 times larger than for the OLS hedge ratios. These results indicate that the hedge ratios for crude oil futures are appreciably more stable than those for S&P500 futures; and that the OLS hedge ratios are much more stable than the M-GARCH hedge ratios (see figures 1 to 4).
To test for instability in the hedge ratios, the ADF test is applied. This tests for a stationary autoregressive process, against the null of a random walk. The $p$-values of the ADF tests in table 3 show that each of the series of dynamic OLS hedge ratios has a unit root. Therefore they follow a random walk process, allowing them to wander substantially from their average level, justifying the use of a dynamic hedging strategy. However, the hedge ratios estimated by M-GARCH are stationary; and so have a constant mean and variance, suggesting that dynamic hedging is unnecessary. This result is despite the M-GARCH hedge ratios having very much larger variances than the OLS hedge ratios.

It is possible that both hedging instruments address essentially the same risk, and the composite hedging regressions suffer from multicollinearity. In which case, due to sampling error, the individual hedge ratios will tend to be large and offsetting, i.e. negatively correlated. However, the correlation between the crude oil futures and S&P500 futures composite hedge ratios is 0.746 for OLS, and 0.383 for M-GARCH. These correlations indicate that such multicollinear effects are not a problem for composite hedging in this case.

Figures 3 and 4 show that occasionally the hedge ratios estimated using M-GARCH become negative (i.e. for a long position in the Amex Oil Index, the hedger buys crude oil futures). For composite hedges this may be because the vector of coefficients in multiple regressions takes into account the effect of the correlation between the Amex Oil Index and S&P500 futures.

9. Transactions Costs
The final rows of tables 3 and 4 show the sum of the absolute changes in the hedge ratios. These can be used to investigate how transaction costs (TC) might affect the different hedging strategies, and how they might increase due to composite hedging. Assuming the same linear relationship between transactions costs and the size of the rebalancing required for both hedging instruments, the transactions costs can be proxied by the sum of the absolute changes in the hedge ratio. The third and sixth columns of the first three rows of table 5 show the average transactions cost of a 1% improvement in effectiveness ($TC/e$), and reveal for OLS hedge ratios that this ratio is most favourable for composite hedges. Indeed, the transactions cost per 1% improvement in
effectiveness is 25% lower than for the best single hedge. For M-GARCH, the composite hedge
has a \( TC/e \) score that is 16% better than the best single hedge.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>M-GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TC</td>
<td>e</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>2.051</td>
<td>13.0</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>3.601</td>
<td>24.1</td>
</tr>
<tr>
<td>Composite</td>
<td>4.794</td>
<td>42.7</td>
</tr>
<tr>
<td>Crude Oil to Composite</td>
<td>2.743</td>
<td>29.7</td>
</tr>
<tr>
<td>S&amp;P500 to Composite</td>
<td>1.192</td>
<td>18.6</td>
</tr>
</tbody>
</table>

Table 5: Transaction Costs to Effectiveness Ratios

The last two rows of table 5 contain the marginal changes in transactions costs, effectiveness and
transactions costs for a 1% increase in effectiveness by moving from a single to a composite
hedge. For OLS, the cost per percentage point of the extra risk reduction from a composite hedge,
rather than the best single hedge (S&P500), is 0.064. This is less than half the cost per percentage
point of the single S&P500 hedge (0.149). For the M-GARCH hedges, the corresponding
improvement is 0.762 from a single hedge using S&P500 futures, with a \( TC/e \) score of 1.147.

The numbers in table 5 are not known to the hedger \textit{ex ante}, and so any decision on whether to
adopt a single or composite hedge must be based on forecasts. In addition, this decision depends
on the precise risk preferences of the hedger. However, if the forecasts are broadly similar to
those in table 5, the hedger will probably prefer the composite hedge to either of the single
hedges. The transactions cost of trading futures are very low, e.g. 10 or 40 basis points, and so
composite hedging may be a worthwhile expense to achieve a substantial risk reduction.

10. Dividends and Hedging Performance
The Amex Oil Index is a price index, and so does not include dividends. However, any investor
who owns the Amex Oil Index basket of shares receives dividends. Therefore, unlike the index,
the equity portfolio that is actually being hedged includes dividends. In contrast to many previous
studies, the effectiveness of single and composite hedging with dividends incorporated in the spot returns will be examined. Since dividend data on the Amex Oil Index is only available from February 1993, the comparison of hedging effectiveness with and without dividends in table 6 is based on a sample of 157 observations (February 1993 to February 2006).

For single and composite hedges estimated using OLS and M-GARCH, the inclusion of dividends leads to no change in the variance of the hedged portfolio; and small changes in effectiveness, both up and down. Therefore, the inclusion of dividends makes little substantive difference.

<table>
<thead>
<tr>
<th>Amex Oil Index Returns Including Dividends</th>
<th>OLS</th>
<th>M-GARCH (1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Instruments</td>
<td>Variance</td>
<td>e</td>
</tr>
<tr>
<td>Unheded</td>
<td>0.004</td>
<td>-</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>0.003</td>
<td>15.80%</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.003</td>
<td>22.86%</td>
</tr>
<tr>
<td>S&amp;P500 &amp; Crude Oil</td>
<td>0.002</td>
<td>38.43%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amex Oil Index Returns Excluding Dividends</th>
<th>OLS</th>
<th>M-GARCH (1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Instruments</td>
<td>Variance</td>
<td>e</td>
</tr>
<tr>
<td>Unheded</td>
<td>0.004</td>
<td>-</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>0.003</td>
<td>15.19%</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.003</td>
<td>22.72%</td>
</tr>
<tr>
<td>S&amp;P500 &amp; Crude Oil</td>
<td>0.002</td>
<td>38.08%</td>
</tr>
</tbody>
</table>

Table 6: Hedging Effectiveness With and Without Dividends

11. Maturity Effects
While the maturity of crude oil futures when the hedge is placed is always three weeks, maturity for S&P500 futures varies between 11 weeks and 3 weeks. As no empirical evidence is available on the effects of failing to allow for maturity effects when estimating S&P500 futures hedge ratios, this is investigated using a method proposed by Herbst and Marshall (1994) and Herbst,
Kare and Marshall (1993).

The no arbitrage condition for any futures contract is:—

\[ S_t = F_t^{-\gamma \tau} \]  \tag{13}

where \( S_t \) is the spot price of the underlying asset at time \( t \), \( F_t \) is the futures price at time \( t \), \( \gamma \) is the annual cost of carry until maturity, and \( \tau \) is the time to maturity of the futures contract at time \( t \), expressed as a proportion of one year. If the spot position to be hedged has a linear relationship with two futures hedging instruments, the regression equation to estimate the hedge ratios, expressed in terms of price changes, is:—

\[ \Delta S_t = \alpha + \beta_1 \Delta S_{1t} + \beta_2 \Delta S_{2t} + \epsilon_t \]  \tag{14}

Using equation (13) to replace \( F_1 \) and \( F_2 \) in equation (14) gives:—

\[ \Delta S_t = \alpha + \beta_1 \Delta S_{1t} e^{\gamma \tau} + \beta_2 \Delta S_{2t} e^{2\gamma \tau} + \epsilon_t \]

The maturity (\( \tau \)) of crude oil futures is constant and, assuming the cost of carry for crude oil (\( y_{2t} \)) does not change over time, equation (15) becomes:—

\[ \Delta S_t = \alpha + \beta_1 \Delta S_{1t} e^{\gamma \tau} + \beta_2 C \Delta S_{2t} + \epsilon_t \]

where \( C \) is a constant. Fitting equation (16) requires a knowledge of the cost of carry at time \( t \) for the S&P500 (\( y_{2t} \)), which is interest on the investment in the S&P500 basket and dividends received on the index basket. Since equation (13) can be rewritten as \( \ln(S/F) = -\gamma \tau \), each month the following regression is fitted to daily spot and futures prices for the S&P500 over the preceding 30 days:—

\[ \ln(S/F_t) = \alpha + \beta \tau + \epsilon_t \]  \tag{17}

The estimated values of \( \beta \) provide an estimate of the current S&P500 cost of carry (\( y_{2t} \)). Using these estimates, and setting \( C = 1 \), equation (16) is estimated 194 times with a 60 month window, as previously. Because equation (16) uses price changes, the OLS hedge ratios are given directly by the estimates of \( \beta_1 e^{\gamma \tau} \) and \( \beta_2 \). Table 7 shows the hedging effectiveness when changes in maturity are allowed for (equation 16), and when they are not, i.e. no maturity effects are included in equation (16). The effectiveness measures are very little changed by allowance for maturity effects, and so the earlier results based on equations (9) and (10) are robust to this omission.
12. Conclusions

Many hedges involve a cross hedge, and composite hedging may well be able to improve hedging effectiveness in such circumstances. However, composite hedging has been largely neglected by academics and practitioners. The reluctance of practitioners to use composite hedging may be due to an unwillingness to grapple with the technicalities of estimating two or more hedge ratios, or possible political issues within the company.

Using both S&P500 futures and Nymex crude oil futures to hedge the risk of the Amex Oil Index leads to a very substantial increase in out-of-sample effectiveness (more than the sum of the effectiveness of the single hedges), and this superiority of composite hedging over single hedging in variance reduction is statistically significant. This superiority is robust to using a variety of well-established techniques for estimating the hedge ratios. Allowance for transactions costs shows that composite hedging remains preferable to single hedging for OLS and M-GARCH hedge ratios. The incorporation of dividends and maturity effects does not change the main conclusion that, in this case, composite hedging is superior to single hedging.

References


Fig. 4: Out-of-Sample NYMEX Hedge Ratio in Composite Hedge