An Assessment of the Internal Rating Based Approach in Basel II

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Abstract

The new bank capital regulation commonly known as Basel II includes a internal rating based approach (IRB) to measuring credit risk in bank portfolios. The IRB relies on the assumptions that the portfolio is fully diversified and that systematic risk is driven by one common factor. In this work we empirically investigate the impact of these assumptions by comparing the risk measures produced by the IRB with those of a more general credit risk model that allows for multiple systematic risk factors and portfolio concentration. Our tests conducted on a large sample of eurobonds over a ten year period reveal that deviations between the IRB and the general model can be substantial.

Keywords: Basel II, Internal Rating Based Approach, Credit Rating, Credit Risk.
JEL classification: G28, G32.
1. Introduction

With Basel II rapidly becoming the new standard for bank capital regulation around the world a lot of effort has been directed towards assessing the empirical validity of the risk models embedded in its Pillar 1. The first pillar prescribes that banks, depending on the degree of sophistication of their internal risk management function can qualify, subject to supervisory approval, for the adoption of the internal rating based approach (IRB) to credit risk capital calculation. The IRB, as its name suggests, allows banks to use internally derived credit ratings to measure the risk of loss stemming from individual loans. Losses are then aggregated across borrowers to produce an overall credit risk capital requirement. The aggregation is performed under two key assumptions: that idiosyncratic risk is fully diversified away and that systematic risk is dependent on one common factor across all exposures. In portfolios of loans to corporations, sovereigns and banks (and to specific retail customers) the IRB is built on the further assumption that the correlation of a borrower’s assets with the single systematic factor is negatively related, through a given functional form, to the borrower’s default probability.

Recent studies have shown that some of the IRB assumptions may not be consistent with empirical evidence. Perli and Nayda (2004) conclude that the negative relationship between asset correlation and default probability is not satisfied in the portfolio of retail exposure they analyse. Tarashev and Zhu (2008) observe that although model specification errors that follow from the restrictive assumptions of the IRB may produce relatively small biases in capital requirements in well-diversified portfolios, calibration errors can lead to more substantial inaccuracies. Calem and LaCour-Little (2004) find that geographical diversification, not explicitly accounted for in the IRB, has a sizeable impact on the risk of mortgage loan portfolios. Jacobson, Lindé and Roszbach (2005) when studying a large database of bank loans from two major Swedish banks find no evidence in support of the IRB assumption that SME loans and retail credits are systematically less risky than wholesale corporate loans. Jacobson et al (2006) also show, through a non-parametric technique based on resampling, that IRB capital can be substantially higher than economic capital.
Many other studies that test the accuracy of the IRB as applied to corporate loans adopt a parametric approach that relies on the Moody’s KMV model (Lopez 2004, Düllmann et al 2007, Tarashev and Zhu 2008). However, the parametric and non-parametric approaches so far employed in the literature, model credit risk as a binomial type of event whereby a borrower can be either in a default or in a non-default state and losses occur only in the default state. But, the value of an asset may decline and thus generate losses even in the non-default state. This happens when the borrower suffers a rating downgrade. In this paper we argue that downgrade losses may have a substantial impact on the value of the portfolio and find that downgrade risk may be one of the major causes of divergence between IRB capital and economic capital.

While taking downgrade risk into account we test the accuracy of the IRB on a portfolio of wholesale corporate exposures. We compare the regulatory capital from the IRB with the economic capital resulting from CreditMetrics, a popular credit risk model which provides a natural way to relax the assumptions underlying the IRB. The discrepancy between CreditMetrics and the regulatory model is measured and analysed over a ten year period on portfolios of eurobonds with different granularity and risk characteristics. This study follows closely Varotto (2008) but revises his restrictive asset correlation assumptions. Here, we compare the IRB and CreditMetrics by using a broad range of asset correlations that reflects empirical findings in recent literature.

Our main result is that the regulatory models and CreditMetrics are never consistently aligned over the whole sample period regardless of the correlation assumptions used to implement CreditMetrics. Over time we observe that the capital required by the regulatory models may be significantly in excess of or significantly short of the capital resulting from CreditMetrics, depending on economic conditions and portfolio characteristics. The implication is that in the former case the IRB can exacerbate credit rationing when excessive capital is required in a market downturn, while in the latter case it may leave banks over-exposed to credit risk.
The paper is organised as follows. Section 2 summarises the data used for our analysis. In Sections 3 and 4 we describe the IRB approach and the CreditMetrics model. In Section 5 we present our results and Section 6 concludes the paper.

2. Data

The data we use for this study were obtained through Reuters and include US dollar-denominated bonds issued by 502 firms\(^1\). Our criteria in selecting the bonds are (i) that they were neither callable nor convertible, (ii) that a rating history was available, (iii) that the coupons were constant with a fixed frequency, (iv) that repayment was at par, and (v) that the bond did not possess a sinking fund.

The composition of the total portfolio is shown in Table 1. 46.4% of the obligors are domiciled in the United States. A further 27.5% of the companies are headquartered in Japan, the Netherlands, Germany, France or the United Kingdom. 54% of the companies in our sample are in the financial services or banking sectors.

To implement CreditMetrics, we also needed: (i) transition matrices, (ii) default spreads and default-free yield curves over time, (iii) equity index data and (iv) a set of weights linking individual obligors to the equity indices. Transition matrices were sourced from Standard and Poor’s (see Vazza, Aurora and Schneck 2005). Default-free interest rates and spreads for different ratings categories were taken from Bloomberg.\(^2\) We also created an equity index dataset going back to 1983 and comprising 243 country and industry-specific MSCI indices. For each obligor, based on the domicile and industry classification provided by Reuters, we then chose one of these indices as the source of systematic risk.

\(^1\) Of these, 90% were eurobonds, the remainder are national bonds from several countries.

\(^2\) We used spreads for United States industrials since these had the longest series and the fewest missing observations.
3. The Internal Rating Based approach (IRB)

In this section we outline the main features of the IRB model. The exact specification of
the model depends on the type of borrower namely, large corporations, sovereigns, banks
and retail customers. We shall focus on large corporations and banks only, as the data we
employ for the empirical analysis are bonds issued by these types of obligors.

The IRB was derived by following the idea of Merton (1974) where default occurs when
the value of a firm’s assets falls below a given default trigger \(d\) (which depends on the
firm’s debt). By assuming that asset returns are normally distributed and driven by one
systematic factor \(X\), the standardised asset return of firm \(s\) can be written as,

\[
r_s = \sqrt{R_s} X + \varepsilon_s \sqrt{1 - R_s}
\]  

where \(X\) and \(\varepsilon_s\) are independent and distributed as a N(0,1). \(\varepsilon_s\) is an idiosyncratic risk
component and \(\sqrt{R_s}\) is the correlation between \(r_s\) and \(X\). If \(X\) is known, the default
probability of firm \(s\) conditional on \(X\) can be written as,

\[
PD_s(X) = P(r_s < d | X) = \Phi\left(\frac{d - X\sqrt{R_s}}{\sqrt{1 - R_s}}\right)
\]  

where \(\Phi\) denotes the cumulative standard normal distribution. Let \(PD_s\) be the
unconditional default probability of firm \(s\), then the conditional default probability can
be re-written as,

\[
PD_s(X) = \Phi\left(\frac{\Phi^{-1}(PD_s) - X\sqrt{R_s}}{\sqrt{1 - R_s}}\right)
\]  

\(\Phi\)

\footnote{For instance, if the firm’s credit rating is known, \(PD_s\) could be derived as the average default frequency
observed for that rating over a long time period.}
If, for the moment, we assume that the default loss of a loan of value 1 taken by company s will produce a loss of 1 (i.e. zero recovery) then the conditional probability in (3) also represents the expected loss the lender will suffer if firm s defaults, conditional on the specific state of the economy summarised by the systematic factor X. Regulators and banks are specifically interested in the maximum loss that a portfolio can generate over a particular time period and for a given confidence level. The IRB is based on a 1 year time horizon and a 99.9% confidence level. This, in our set-up, implies that we will be using one year default probabilities and consider losses in a severe downturn scenario. The severity of the downturn will be characterised by the factor X, and given the confidence level, will correspond to a value of X which will only be exceeded (on the negative side) with probability 0.1%. So, the expected loss on loan s under the specified scenario will be

\[ PD_s(X = X_{0.1%}) = \Phi \left( \frac{\Phi^{-1}(PD_s) - \Phi^{-1}(0.1%) \sqrt{R_s}}{\sqrt{1 - R_s}} \right) \] (4)

With the additional assumption that the portfolio is well diversified (and when all the loans have a value of 1) the 99.9% portfolio value-at-risk will be equal to the sum of all the expected losses of the loans in the portfolio, conditional on \( X = X_{0.1%} \). This is because by conditioning on X all losses are assumed to be independent of each other. Then, by the central limit theorem which can be invoked, by approximation, in a well diversified portfolio, the 99.9% quantile of the portfolio loss distribution tends to the distribution’s conditional expected value.\(^4\) Figure 1 shows that the difference between the VaR and the conditional mean falls to zero relatively quickly. For example, it takes only 40 (equally weighted) assets in a portfolio for such difference to fall below 5%. Then,

\[ VaR(99.9%) = \sum_s PD_s(X = X_{0.1%}) \] (5)

\(^4\) For more details on this see Gordy (2003) and Wilde (2003).
One can relax the assumption of zero recovery and unitary loan value. In the IRB, recovery rate and loan amount, conditional on the common factor $X$, are implicitly assumed to be independent of the default probability. This being the case (5) can be re-stated more generally as follows,

$$VaR(99.9\%) = \sum_s LGD_s (X = X_{0.1\%})PD_s (X = X_{0.1\%})EAD_s (X = X_{0.1\%})$$  \hspace{1cm} (6)$$

where $LGD_s$ is the loss given default of loan $s$ or one minus the loan’s recovery rate, and $EAD_s$ is the lender’s exposure in case loan $s$ defaults.

Regulatory capital in the IRB is designed to cover for unexpected portfolio loss only. This is because banks already set aside provisions against expected losses. Unexpected loss is defined as the difference between the portfolio value-at-risk and the portfolio expected loss,

$$UL = \sum_s (LGD_s (X = X_{0.1\%})PD_s (X = X_{0.1\%})EAD_s (X = X_{0.1\%}) - LGD_s PD_s EAD_s)$$ \hspace{1cm} (7)$$

Above we have presented the main ideas behind the IRB model. However, to arrive at the final expression for minimum capital requirement the Basel Committee has introduced some simplifications as well as a calibration factor. The final form of loan $s$ unexpected loss is then given by,

$$UL_s = CF \cdot MA_s \cdot LGD_s (X = X_{0.1\%}) \cdot EAD_s (X = X_{0.1\%}) \cdot [PD_s (X = X_{0.1\%}) - PD_s]$$ \hspace{1cm} (8)$$

This is achieved by imposing that the unconditional LGD is equal to the downturn, or conditional, LDG and the unconditional EAD is equal to the conditional one.\(^5\) CF is a

calibration factor that was introduced broadly to maintain the aggregate level of regulatory capital that was in place in the banking industry before the introduction of Basel II. The factor is currently equal to 1.06. MA_s is a maturity adjustment which grows with effective maturity, M, and falls as PD_s increases. The idea behind it, is that longer maturity bonds, which are riskier, should attract a higher capital charge. However, if PD_s goes up, the MA_s will fall because lower quality assets are exposed to downgrade risk to a lower extent than higher quality assets. In other words, the scope for loss in value due to a downgrade is larger for a AAA asset than for an asset with lower credit rating.\(^6\) The maturity adjustment is given by,

\[
MA_s = \frac{1 + (M_s - 2.5)b(PD_s)}{1 - 1.5b(PD_s)}
\]  \(9\)

where,

\[
b(PD_s) = (0.11852 - 0.05478 \ln(PD_s))^2
\]  \(10\)

b(PD_s) was calibrated on market data to produce the downgrade effect discussed above. M is obtained with a simplified duration formula and is defined as,

\[
M_s = \frac{\sum t \cdot c_t}{\sum c_t}
\]  \(11\)

where c_t denotes the cash flow of asset s at time t. By analysing a large database of US, Japanese and European firms Lopez (2004) found the square of the systematic factor loading \(\sqrt{R_s}\) to be a decreasing function of the probability of default and an increasing

\(^6\) This does not mean however that higher quality exposures will attract higher capital charges. Although they will have a higher MA, their PD, which has a dominant effect on the unexpected loss in (3), will drive down their risk weight. Therefore, the impact of MA as credit quality improves is to make the fall in capital requirement less sharp.
function of the size $V$ of borrower $s$. These findings are reflected in the IRB with the following relation,

$$R_s = 0.12 \cdot \eta + 0.24 \cdot (1 - \eta) - 0.04 \cdot \left(1 - \frac{V - 5}{45}\right)$$  \hspace{1cm} (12)$$

where the weight $\eta$ is given by,

$$\eta = \frac{1 - \exp(-50 \cdot PD_s)}{1 - \exp(-50)}$$  \hspace{1cm} (13)$$

and $V$ is measured in terms of the firm’s annual sales in million Euros. The size adjustment does not apply to companies that have a turnover of more than 50 million Euros, and is set at –0.04 for those with annual sales lower than 5 millions. Although we do not have turnover data for the companies in our sample, it is plausible to assume that their size is considerable, since most of them are eurobond issuers. Therefore, we do not apply the size adjustment. If we ignore the size adjustment, (12) indicates that $R_s$ is a weighted average of, i.e. it varies between, 0.12 and 0.24. The weights are a function of $PD_s$ and cause $R_s$ to decline as $PD_s$ increases.

Interestingly, regulatory capital under the IRB is additive - as is in Basel I - in the sense that to arrive at the total capital requirement one needs to sum the individual capital charge computed for each asset in the portfolio. However, while additivity in Basel I follows from the implicit assumption that assets in the portfolio are perfectly correlated, the IRB additivity does not rely on perfect correlation and is the result of the modelling assumptions described above.

We implement as closely as possible the IRB by using the information in our data sample. $M$ is estimated as in (6) and subject to a lower and upper boundary of 1 and 5 years respectively.\textsuperscript{7} For the probability of default, as we lack internal rating data, we

\textsuperscript{7} See BCBS 2006, para. 320.
assume that the bank’s internal rating system perfectly replicates that of a recognised rating agency, Standard and Poor’s. This is not implausible as in the IRB banks are allowed to map their internal ratings to agency ratings and employ the default probabilities of the latter.\(^8\) The PDs fed into the IRB model are, at each point in time and for each credit rating, the averages of point-in-time default rates for that rating over the previous five years.\(^9\) As prescribed in the IRB, we constrain default probabilities to be greater than or equal to 0.03\%.\(^10\) As we do not have information on downturn loss given default we use a flat LGD of 50\% across the whole sample.\(^11\) The same LGD will be employed when implementing CreditMetrics so that any specific choice of LGD should not affect the comparison we make between regulatory (IRB) and economic (CreditMetrics) capital. We construct portfolios that are equally weighted where each loan has an exposure at default of 1.

### 4. The benchmark model: CreditMetrics

The fundamental idea behind the IRB and CreditMetrics is the same in that in both models default risk is driven by the value of the borrower’s assets. However, in CreditMetrics, standardised asset returns are a function of several systematic factors \(X_1, X_2, \ldots, X_n\) instead of just one common factor as indicated in (1),

\[
r_s = \theta_{1,s}X_1 + \theta_{2,s}X_2 + \ldots + \theta_{n,s}X_n + \xi_s\epsilon_s
\]  (14)

where \(\theta_{v,s}\) is the loading of factor \(X_v\) and \(\xi_s\) is the weight of the idiosyncratic risk term.

In CreditMetrics the factors are represented by country and industry indices. As all our companies are classified by Reuters as belonging to one industry only, for each company

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\(^8\) See BCSB 2006, para. 462.

\(^9\) According to Basel II default probabilities should be estimated by taking average default rates over a minimum period of five years (See BCSB 2006, para. 447, 463).


\(^11\) More than 70\% of our bonds are unsecured, (57.34\% of unsecured proper and 14.14\% of senior unsecured) which, according to Carty and Lieberman (1996) have an average recovery rate of 51.13\%. Bonds with lower seniority (that is, subordinated), and hence with a lower recovery rate, account for only 1.55\% of the total sample.
we choose, as a sole systematic factor, the index of the relevant industry in the country where the company is domiciled. This simplifies the above return structure into one similar to (1) where, however, the only systematic factor is firm specific and not common across all firms in the portfolio,

\[ r_s = \theta_s X_s + \varepsilon_s \sqrt{1 - \theta_s^2} \]  

(15)

Beside multiple systematic factors, CreditMetrics also departs from the IRB in that it expands the concept of credit risk beyond default risk to include, explicitly, downgrade risk. Before reaching the default trigger \( \Phi^{-1}(PD_s) \) a change in asset value can cause a loss in the value of the firm’s debt. This happens when asset return crosses “transition” triggers which delimit the asset return range corresponding to specific credit ratings\(^\text{12}\),

\[
\begin{align*}
    r_s &\leq \Phi^{-1}(PD_s) & \text{Default} \\
    \Phi^{-1}(PD_s) &< r_s \leq \Phi^{-1}(PD_s + \pi_{g,CCC}) & \text{CCC rating} \\
    \ldots & \\
    r_s &> \Phi^{-1}(PD_s + \pi_{g,CCC} + \ldots + \pi_{g,AA}) & \text{AAA rating} 
\end{align*}
\]  

(16)

Here, \( \pi_{g,G} \) is the transition probability from initial rating \( g \) to end-of-year rating \( G \).\(^\text{13}\)

Under this basic set-up, the loss distribution of the portfolio, its value-at-risk and unexpected loss can simply be derived with Monte Carlo simulations. Given (15) the correlation between the asset return of borrowers \( s \) and \( v \) will be \( \theta_s \theta_v \rho_{s,v} \) where \( \rho_{s,v} \) is the correlation between indices \( X_s \) and \( X_v \). As in CreditMetrics asset returns are assumed to be normally distributed, one can generate correlated returns by using the

\(^{12}\) In this paper we use 8 coarse ratings D, CCC, B, BB, BBB, A, AA, AAA.

\(^{13}\) Transition probabilities can be found in rating transition matrices which are regularly published by all the major rating agencies. In this study, transition probabilities in a given year are estimated by taking the average of the previous 5 years’ point-in-time transition matrices. To take into account indirect migration and generate non-zero default probabilities for ratings at the top end of the rating scale we use the “generator” approach introduced by Jarrow, Lando and Turnbull (1997) and refined by Israel, Rosenthal and Wei (2001).
Cholesky decomposition of the correlation matrix made with the generic terms $\theta_s, \theta_v, \rho_{s,v}$. The resulting correlated returns can be used to identify joint rating scenarios and the distribution of the value of the loan portfolio under consideration.\(^{14}\) The portfolio loss distribution is then obtained by taking the difference between the 1 year forward value of the portfolio under the assumption that all exposures maintain their current rating and the generated portfolio values.\(^{15}\) Finally, the unexpected loss can be estimated as the difference between the 99.9% VaR on the loss distribution and the loss mean value. The comparison of the unexpected loss so derived with IRB unexpected loss will be the focus of our discussion in Section 5.

4.1 Systematic factor loading.

The CreditMetrics technical manual (Bhatia, Finger, and Gupton 1997) is silent about how the systematic factor loading $\theta_s$ should be estimated. Since asset return $r_s$, unlike stock returns (of listed companies), are unobservable,\(^{16}\) a simple regression of the systematic factor on asset returns is unfeasible. However, several studies have estimated implied asset correlations, $\theta_s \theta_v, \rho_{s,v}$, mostly by using Moody’s KMV\(^{17}\) or similar credit risk models (e.g. Hamerle et al 2003, Lopez 2004, Düllmann et al 2007, Rösch et al 2008 and Tarashev et al 2008).\(^{18}\) The findings of these studies in relation to international portfolios (like the ones considered in this work) are reported in Table 2.

Here, we do not estimate factor loadings directly but simply produce results under several correlation scenarios that reflect those reported in the literature. Our objective is threefold:

1. check the sensitivity of results to different correlation assumptions;

\(^{14}\) See Bhatia et al (1997), Chapters 10 and 11 for details.
\(^{15}\) See, for example, Crouhy et al (2000), Section 2.4.
\(^{16}\) It is doubtful whether asset value as reported in financial statements could be used in this context due to distortion introduced by accounting conventions and low frequency of observations.
\(^{17}\) Implied asset returns with the contingent claim approach exemplified by the Moody’s KMV model, can be obtained with a simple iterative procedure (see Vassalou and Xing 2004).
\(^{18}\) We explicitly set three target asset correlations 6%, 12% and 18% and then implement the target correlation in the IRB model which varies between 20% and 24% depending on the average credit quality of the portfolio considered.
2. account for factor correlation which is the main point of departure from the IRB where multiple factors are ruled out;
3. capture the empirical regularities found in Lopez (2004) and reflected in the IRB loadings $\sqrt{R_s}$.

To achieve the above we estimate asset correlation $\theta_s \theta_v \rho_{s,v}$ by imposing that $\theta_z$ equals $\sqrt{R_z}$. Then we re-scale correlations by a constant over the whole sample period so that average portfolio correlation across borrowers and over time equals, in turn, several target values between 6% and 24%. The 6-24% range was chosen because it covers most of the variation of average asset correlation found in the literature\(^{19}\)

Using asset correlations $\hat{\theta}_s \hat{\theta}_v \rho_{s,v}$ (where the hat above the factor loading denotes that the loading has been rescaled as indicated above) instead of the IRB specification $\theta_s \theta_v$ allows one to capture changes in the overall level of correlation in the market. Factor loadings as specified in the IRB only depend on the characteristics of the borrower alone (that is, its default probability and size). Indeed in periods of market turmoil when PD goes up, the IRB loading will fall as it is negatively related to PD through (12). On the other hand, empirical evidence suggests that in a crisis, market correlation tends to increase. This phenomenon, which escapes the IRB specification, can be detected by CreditMetrics via the parameter $\rho_{s,v}$ which varies over time and may better reflect the joint behaviour of assets in the economy.

5. Results

In this section we compare the capital charge of the IRB with that produced by the benchmark model on different portfolios. We consider a portfolio made of all the bonds in the sample, as well as a high-risk and a low-risk sub-portfolios with 40 exposures each,\(^{19}\)

\[^{19}\] We explicitly set three target asset correlations 6%, 12% and 18% and then implement the target correlation in the IRB model which, when calculated for the assets in our sample, varies between 20% and 24% depending on the average credit quality of the portfolio considered.
the ones with the lowest and highest ratings respectively among those available in our dataset at each point in time.

Figure 2 shows the IRB capital for the three portfolios as a percentage of portfolio value. As one would expect, given the sensitivity of IRB to credit ratings’ observed default probability, the ranking of the three portfolios’ capital requirement follows the risk implied from the portfolios’ average rating. Interestingly, while the capital for the whole portfolio and the low-risk one is downward sloping, the capital for the high-risk portfolio trends upwards for the first half of the sample, then stabilizes between July ‘94 and January ‘96 and finally slopes downward. The initial divergence in trend may partially be explained by a steady increase in sample size in the first half of the observation period. A large proportion of the new bonds are highly rated but a few have a lower rating than the existing ones. The new lowly rated bonds cause the average quality of the high-risk portfolio to deteriorate, while the larger set of high quality debt causes both the average and the high-quality portfolios to attract increasingly lower capital. Also the early nineties are characterised by larger default rates for low-rated companies. Instead, high investment grade firms remain broadly unaffected in that period.

Figure 3 shows the “excess” capital, that is the difference between IRB and benchmark capital, expressed as a percentage of portfolio value, under various asset correlation scenarios. The Figure reveals several interesting patterns. First, regardless of the level of asset correlation the IRB and the benchmark are never consistently aligned throughout the sample period. For instance, when average asset correlation is 12%, the IRB and the benchmark are remarkably close in the first three years of the sample (i.e. excess capital is close to zero) but then they start diverging, and increasingly so. Second, regardless of the level of correlation, in the last three years of the sample period excess capital is always positive, that is the IRB appears to be overly conservative. Third, when rescaling the correlation of the benchmark model to match the average IRB correlation, we observe the largest deviation between the two models, with the IRB undershooting the benchmark by 2.84% of portfolio value in April 1991.
Excess capital for the high-risk sub-sample reported in Figure 4 exhibits the same trend as for the whole sample. However, the magnitude of the excess across the whole spectrum of asset correlation scenarios is much larger now both in the positive and negative territory. Excess capital for the low risk sub-portfolio (Figure 5) deviates from zero in a less dramatic fashion but again it displays the familiar behaviour.

In summary, the difference between the IRB and the benchmark may vary substantially over time, an indication that the two models exhibit different sensitivity to mutating economic conditions (as proxied by factor correlations) and portfolio characteristics. This suggests that the inconsistency between the models can not simply be cured through, for example, a scaling factor. Moreover, the alternating sign of the excess capital shows that the IRB may lead to capital requirements that are not necessarily conservative, as often believed, but could in fact be too low at times.

5.1 Regression Analysis

In this section we explore the causes behind the time variation in the excess capital produced by the IRB. This will help us to identify the factors that may cause the regulatory model to deviate from the portfolio credit risk assessment of the benchmark. We carry out this analysis by regressing changes in the IRB excess capital on changes in the characteristics of the portfolios. These include, (1) average effective maturity, (2) average rating, (3) average default probability, (4) portfolio concentration as measured by the Herfindahl index, (5) systematic factor correlation, and (6) the level of downgrade risk. The rating variable has been constructed by assigning a numerical value to each rating as follows, AAA=27, AA+=25, AA=24, AA-=23 and so on. The wider gap between AAA and AA+ was present in conversion tables supplied by Reuters and denotes the absence of the AAA- rating. Variable (5) is measured indirectly as the impact of changes in factor correlation on the UL of the benchmark model. Downgrade risk is estimated as the difference in the unexpected loss of the benchmark when diagonal rating transition matrices (i.e. without downgrade risk) are replaced with full transition matrices. Regression results are reported in Table 3.
In the Table we show results under the four target correlation scenarios discussed in Section 4, 6%, 12%, 18% and IRB correlation. Results are broadly consistent across the correlation scenarios. Specifically, when significant, regression coefficients preserve the same sign across all correlation assumptions. Below, we shall comment in detail on the regression results in Panel A when the target correlation for the benchmark model is set to an average level of 12%. Coefficients in bold font in Panel A are also significant under all the other correlation specifications.

The coefficient of effective maturity has a positive sign and is highly significant across all portfolios. This means that banks can decrease the IRB regulatory capital both in absolute terms and relative to economic (i.e. CreditMetrics) capital - and thus engage in so called regulatory capital arbitrage - by decreasing the duration of the assets in their portfolios. This may be achieved for example with policies that favour short term lending or quick amortization of long term loans.

Downgrade risk has a negative and always significant sign (with 12% correlation) across all portfolios. In other words, when downgrade risk goes up, the benchmark model, which will be the only one to reflect such change, will produce higher unexpected loss. As a result, the IRB excess capital will fall. It is interesting to note that the maturity adjustment (see equation 9) was introduced in the IRB to account for downgrade risk.\textsuperscript{20} But, the facts that the two variables (in first difference) are lowly correlated (0.29 correlation in the whole sample) and that each can only explain a fraction of the volatility of the other,\textsuperscript{21} suggest that the maturity adjustment does not accurately capture downgrade risk.

The change in portfolio concentration has been used as an explanatory variable only for the whole sample as in the high-risk and low-risk sub-portfolios the number of assets (and hence concentration, given the equal weighting scheme adopted) is constant by

\textsuperscript{20} See, for example, Resti and Sironi (2007), p. 611.
\textsuperscript{21} When we regress one variable on the other, plus a constant, for the whole sample we find the adjusted R-squared to be 7.3%.
construction. Regression results show that the variable is consistently not significant across correlation scenarios. Most likely, this is because the number of assets in the whole portfolio is rather large (always greater than 68). As Figure 1 suggests, this implies that the error introduced by the IRB, which assumes that the portfolio is well diversified, is very small.

The reason for including both the credit rating and default probability variables in the regression is that default probability may change within the same rating category over time. So default probability and rating need not move in unison. The correlation between the two variables in first difference is negative, as expected. However, the level of correlation is not as high as to cause concern for multicollinearity (-41% in the whole portfolio, -79% in the high risk portfolio and –61% in the low risk portfolio, with IRB asset correlation). The rating variable influences both the IRB and the benchmark in a similar way since a change in rating brings about a change in the rating’s associated default probability. However, the negative sign of the variable’s coefficient suggests that the IRB capital increases relative to the benchmark as the rating deteriorates. This may follow because of the parameterisation of the IRB which appears to become more and more conservative as credit quality goes down.

Similarly, higher values of the default probability variable bring about higher IRB excess capital in the whole (average quality) portfolio. The result in the low risk portfolio is more curious as the default probability coefficient is negative and highly significant (it is negative although not significant also in the high risk portfolio). This implies that higher credit quality (i.e. lower PD) causes the gap between IRB and benchmark to grow, which appears to contradict our previous conclusions. In fact, the finding seems to be the result of the PD floor of 0.03% in the IRB. As the credit quality improves and the probability of default keeps falling below the 0.03% limit, CreditMetrics (i.e. economic) capital will fall while the IRB capital will not be affected. Hence, the gap between the two opens up. According to this line of reasoning, however, the rating’s coefficient in the low risk portfolio should be positive, while it is negative and highly significant. The reason for it, is that the rating variable for the low risk portfolio only changes in the first two years of
the sample period, when the usual negative relationship between credit quality and excess capital still applies. After that, the average rating of the portfolio is flat at the AAA level (as all assets in the high risk portfolio are AAA rated from January 1991). Therefore, the negative relationship between credit quality and excess capital does not carry into the remaining years (i.e. most of the sample period) when the PD floor effect kicks in.

The coefficient of the multiple factor variable is consistently negative and often significant only in the high risk portfolio. This may be because if factor correlation goes up (down) economic capital will go up relative to the IRB capital and excess capital will have to fall (increase) as the variable does not have any bearing on the regulatory model.

6. Conclusion

In this paper we study how the restrictive assumptions underlying the IRB approach may influence regulatory capital. We do so by comparing the capital requirement produced by the IRB with the economic capital resulting from a benchmark model in which the IRB assumptions are relaxed. Our comparison is performed over a 10 year period characterised by varying credit and market conditions. We find that the discrepancies between the IRB and the benchmark may be large. Our results may be summarised as follows: (1) The regulatory model exhibits large and protracted mis-alignment relative to the benchmark over the sample period. Specifically, in the last years of our observation period the IRB appears to be overly conservative. This should raise some concern – under the caveat illustrated below - as the finding combined with the higher risk sensitivity of the new regulation may well exacerbate credit rationing in an economic downturn. (2) The regulatory model may also produce too little capital relative to the benchmark. The implication is that banks adopting the IRB may at times be over-exposed to credit risk, despite the conservative approach taken in devising the new capital regulation. (3) The difference between the regulatory model and the benchmark depends on the combined effect of several variables. As a result, the discrepancy may not be corrected easily, for
example with the use of a constant scaling factor. (4) We find that the maturity adjustment appears to be a poor proxy for downgrade risk.

We should emphasise that our conclusions are based on the benchmark we employ in this study. Although the chosen benchmark provides a natural way to relax the assumptions in the IRB and is a widely popular model, its predictive ability has not been thoroughly investigated. Nickell et al (2007) who have conducted backtesting on the benchmark on a similar dataset to the one used in this study find it to be reliable on all the 11 portfolios they have investigated with the only exception of a portfolio of high risk assets where the model underestimated credit risk. This happened because of a cluster of losses that exceeded the model’s value-at-risk in 1995 and 1996. It should be noted that in that period, the IRB capital is above the one implied by the CreditMetrics model, for all but the lowest asset correlation scenario considered (6%). So it appears that with high-risk portfolios the IRB may lead to more adequate capital levels than the benchmark.

Furthermore, in this work we do not consider possible generalisations of the benchmark in which recovery risk and credit spread risk are accounted for as suggested for example in Kiesel et al (2001). These generalisations are likely to increase the inconsistency between the IRB and the benchmark. Moreover, the introduction of these risks leaves open the question of how to model the dependence between credit spreads, recovery rates and transition rates. We leave the investigation of these issues to future research.
References


### Table 1
**Portfolio Characteristics**

<table>
<thead>
<tr>
<th>Country</th>
<th>No. of firms</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>233</td>
<td>46.4</td>
</tr>
<tr>
<td>Japan</td>
<td>42</td>
<td>8.4</td>
</tr>
<tr>
<td>Netherlands</td>
<td>36</td>
<td>7.2</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>27</td>
<td>5.4</td>
</tr>
<tr>
<td>Germany</td>
<td>17</td>
<td>3.4</td>
</tr>
<tr>
<td>France</td>
<td>16</td>
<td>3.2</td>
</tr>
<tr>
<td>Other</td>
<td>131</td>
<td>26.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Industry</th>
<th>No. of firms</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Services</td>
<td>160</td>
<td>31.9</td>
</tr>
<tr>
<td>Banking</td>
<td>111</td>
<td>22.1</td>
</tr>
<tr>
<td>Utilities</td>
<td>27</td>
<td>5.4</td>
</tr>
<tr>
<td>Energy</td>
<td>20</td>
<td>4.0</td>
</tr>
<tr>
<td>Merchandising</td>
<td>15</td>
<td>3.0</td>
</tr>
<tr>
<td>Telecoms</td>
<td>15</td>
<td>3.0</td>
</tr>
<tr>
<td>Other</td>
<td>154</td>
<td>30.7</td>
</tr>
</tbody>
</table>

| Total            | 502          |     |
### Table 2
Asset correlation in internationally diversified portfolios

<table>
<thead>
<tr>
<th>Paper</th>
<th>Asset Correlation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lopez (2004)</td>
<td>10%-15%</td>
<td>Based on the KMV model, a single systematic factor, and global data. Correlation declines as average credit quality of the portfolio deteriorates.</td>
</tr>
<tr>
<td>Düllmann et al (2007)</td>
<td>4%-16%</td>
<td>Based on the KMV model, a single systematic factor, and a sample of EU firms.</td>
</tr>
<tr>
<td>Rösch et al (2008)</td>
<td>3.9%-5.4%</td>
<td>Moody’s historical default data. The first range refers to estimates in a portfolio with all credit ratings represented. The second range refers to investment grade ratings only.</td>
</tr>
<tr>
<td></td>
<td>16.9%-20.6%</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
Regression of changes in IRB excess capital on changes of portfolio characteristics

The table reports the estimation results of regressions that aim to explain changes in the internal rating based approach relative to CreditMetrics, the benchmark model. Excess capital is measured as the difference between the unexpected loss obtained from the IRB and that of the benchmark model under various asset correlation assumptions. The benchmark model is implemented with the inclusion of downgrade risk and multiple systematic factors. The sample includes monthly observations over the period from January 1989 to February 1998. Explanatory variables are in first differences and include: portfolio "Concentration" as measured by the Herfindahl index; average "rating"; average "default probability"; average "effective maturity"; " downgrade risk" which is estimated as the difference in unexpected loss of the benchmark model when diagonal rating transition matrices (i.e. without down-up grade risk) are replaced with full transition matrices; and the variable that measures systematic "factor correlation". The "high risk portfolio" ("low risk portfolio") columns denote a portion of the sample including, at each point in time, the 40 firms with the lowest (highest) rating. Parameters are estimated with ordinary least squares. ***, **, * indicate significance at the 1%, 5% and 10% confidence level respectively. Confidence intervals are estimated with standard errors adjusted for autocorrelation and heteroscedasticity.

<table>
<thead>
<tr>
<th></th>
<th>Whole Portfolio</th>
<th>High Risk Portfolio</th>
<th>Low Risk Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: 12% Average Asset Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.005</td>
<td>-0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Concentration</td>
<td>-54.88</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rating</td>
<td>-0.114</td>
<td>-0.377**</td>
<td>-1.114***</td>
</tr>
<tr>
<td>Default Prob.</td>
<td>22.33**</td>
<td>-3.373</td>
<td>-25155***</td>
</tr>
<tr>
<td>Effective Maturity</td>
<td>0.541***</td>
<td>0.549***</td>
<td>0.352***</td>
</tr>
<tr>
<td>Downgrade Risk</td>
<td>-0.728***</td>
<td>-0.523***</td>
<td>-0.400***</td>
</tr>
<tr>
<td>Multiple Factors</td>
<td>-0.033</td>
<td>-0.167***</td>
<td>-0.025</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.396</td>
<td>0.576</td>
<td>0.653</td>
</tr>
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</table>
### Table 3 – Continued

<table>
<thead>
<tr>
<th></th>
<th>Whole Portfolio</th>
<th>High Risk Portfolio</th>
<th>Low Risk Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: 6% Average Asset Correlation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.007</td>
<td>-0.015</td>
<td>0.001</td>
</tr>
<tr>
<td>Concentration</td>
<td>24.31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rating</td>
<td>-0.322***</td>
<td>-0.737***</td>
<td>-1.203***</td>
</tr>
<tr>
<td>Default Prob.</td>
<td>12.73</td>
<td>-1.915</td>
<td>-24184***</td>
</tr>
<tr>
<td>Effective Maturity</td>
<td>0.391***</td>
<td>0.319</td>
<td>0.335***</td>
</tr>
<tr>
<td>Downgrade Risk</td>
<td>-0.492***</td>
<td>-0.103</td>
<td>-0.385***</td>
</tr>
<tr>
<td>Multiple Factors</td>
<td>-0.091</td>
<td>-0.380***</td>
<td>-0.364***</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.458</td>
<td>0.689</td>
<td>0.686</td>
</tr>
<tr>
<td><strong>Panel C: 18% Average Asset Correlation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.010</td>
<td>0.007</td>
<td>-0.002</td>
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<tr>
<td>Concentration</td>
<td>29.64</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rating</td>
<td>-0.074</td>
<td>-0.165</td>
<td>-1.134**</td>
</tr>
<tr>
<td>Default Prob.</td>
<td>2.461</td>
<td>-0.781</td>
<td>-28283***</td>
</tr>
<tr>
<td>Effective Maturity</td>
<td>0.508***</td>
<td>0.463</td>
<td>0.340***</td>
</tr>
<tr>
<td>Downgrade Risk</td>
<td>-1.082***</td>
<td>-0.703***</td>
<td>-0.445***</td>
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<tr>
<td>Multiple Factors</td>
<td>0.008</td>
<td>-0.100</td>
<td>0.043</td>
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<tr>
<td>Adjusted R-squared</td>
<td>0.795</td>
<td>0.435</td>
<td>0.601</td>
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<tr>
<td><strong>Panel D: IRB Average Asset Correlation</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Constant</td>
<td>0.011</td>
<td>0.008</td>
<td>0.001</td>
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<tr>
<td>Concentration</td>
<td>39.81</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rating</td>
<td>-0.066</td>
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<td>-1.144*</td>
</tr>
<tr>
<td>Default Prob.</td>
<td>-12.77</td>
<td>-7.92</td>
<td>-25689***</td>
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<tr>
<td>Effective Maturity</td>
<td>0.449***</td>
<td>0.357</td>
<td>0.364***</td>
</tr>
<tr>
<td>Downgrade Risk</td>
<td>-1.129***</td>
<td>-1.014***</td>
<td>-0.706***</td>
</tr>
<tr>
<td>Multiple Factors</td>
<td>-0.051**</td>
<td>-0.108*</td>
<td>0.032</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.806</td>
<td>0.493</td>
<td>0.714</td>
</tr>
</tbody>
</table>
Figure 1

**Difference between Portfolio VaR and Conditional Mean**

Note. The figure shows the difference between the VaR and the conditional mean as a function of the number of assets in the portfolio for two levels of confidence. Each asset has a different probability of default and a different systematic factor loading both of which are generated from a uniform distribution in the (0,1) domain. The percentage difference in the y-axis is measured relative to the conditional mean.

Figure 2

**IRB Capital**

<table>
<thead>
<tr>
<th>Capital %</th>
<th>Whole sample</th>
<th>High risk sub-sample</th>
<th>Low risk sub-sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-89</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-90</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-91</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-92</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-93</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-94</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-95</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-96</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-97</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Jan-98</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Figure 3

IRB Excess Capital - Whole Sample

Excess %

Jan-89 Jan-90 Jan-91 Jan-92 Jan-93 Jan-94 Jan-95 Jan-96 Jan-97 Jan-98


Figure 4

IRB Excess Capital - High Risk Sample

Excess, %

Jan-89 Jan-90 Jan-91 Jan-92 Jan-93 Jan-94 Jan-95 Jan-96 Jan-97 Jan-98

Figure 5

IRB Excess Capital - Low Risk Sample

Excess, %

Jan-89  Jan-90  Jan-91  Jan-92  Jan-93  Jan-94  Jan-95  Jan-96  Jan-97  Jan-98