Regime-Dependent Smile-Adjusted Delta Hedging

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Most research on option hedging has compared the performance of delta hedges derived from different stochastic volatility models with Black-Scholes-Merton (BSM) deltas, and in particular with the ‘implied BSM’ model in which an option’s delta is based on its own market implied volatility. Various empirical studies of vanilla options on different equity indices have provided substantial evidence that minimum variance deltas outperform the partial derivative delta, but no clear evidence that they can consistently outperform the implied BSM delta, or other simple smile-adjusted deltas that are popular with option traders. This paper focuses exclusively on smile adjustments to BSM deltas with an emphasis on those which depend on the market regime. Using 16½ years of daily closing prices for FTSE 100 vanilla options, out-of-sample tests of their hedging performance clearly demonstrate that even the simplest of the regime-dependent smile adjustments will consistently and significantly improve on implied BSM delta hedging, for options of all moneyness and maturities and whether rebalancing is daily, weekly or fortnightly. For most options and over all hedging horizons the regime-dependent smile-adjusted delta-hedging errors are only 50% – 60% as large as the implied BSM hedging errors, on average. During volatile market periods the risk reduction is much greater than it is during tranquil periods.

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1. Introduction

The majority of option-pricing research focuses on the construction and calibration of models used to price over-the-counter (OTC) illiquid derivatives. Yet exchange trading on standard vanilla options is much more active than trading on OTC products. Market makers on exchanges can make huge profits from a high-volume business.\(^1\) The prices of liquid, exchange-traded vanilla options are determined by supply and demand, with market makers setting bid-ask spreads to make a small profit on the round trip after accounting for expected hedging costs. The more accurate their hedging the lower its cost, so the more competitive their spread and the greater their trading volume. Hence, the ‘daily bread’ of the vast majority of option traders depends on the accurate hedging of vanilla options.

Compared with the veritable explosion of research on pricing OTC options with path-dependent and/or exotic pay-offs, there has been relatively little research on hedging vanilla options. The market is incomplete when the price process has stochastic volatility, so there is no perfect hedge. Instead, traders should choose a hedge ratio to minimize the hedging costs over the life of the option. Such a delta is commonly termed ‘locally risk minimizing’ or ‘minimum variance’ because it is the delta which minimizes the instantaneous variance of the hedging error: see Schweizer (1991), Bakshi, Cao and Chen (1997, 2000), Frey (1997), Lee (2001) and others. Poulson, Schenk-Hoppe and Ewald (2009) demonstrate, using simulations and empirical data, that the particular stochastic volatility model used does not seem to matter that much – the important factor for accurate delta hedging is to use a minimum variance rather than a standard (partial derivative) delta.

Bakshi, Cao and Chen (1997) found no evidence that adding stochastic interest rates and/or jumps in prices improves the hedging performance of the Heston (1993) stochastic volatility model for S&P 500 options. Bakshi, Cao and Chen (2000) confirm these results even when hedging long term options. Kim and Kim (2004, 2005) find that the Heston model outperforms other stochastic volatility models for Kospi 200 options, confirming that jumps offer no significant improvement. Alexander, Kaeck and Nogueira (2009) explain how hedge ratios should be adjusted to account for the model risk stemming from time-varying calibrated parameters in a model that assumes the parameters are constant. The motivation for these investigations is that traders will use the same model for hedging vanilla options as they do for pricing complex options.

However, in practice many traders base vanilla option hedge ratios on simple adjustments to the standard lognormal model of Black and Scholes (1973) and Merton (1973) – referred to as BSM from henceforth. The simplest of these is the ‘implied BSM’ approach whereby each option’s price hedge ratios are evaluated at their own implied volatility. The empirical results in Bakshi, Cao and Chen (1997, 2000), Christoffersen and Jacobs (2004), Alexander, Kaeck and Nogueira (2009) and others has demonstrated how effective this hedging approach is compared with minimum variance hedging in various stochastic volatility models, especially for hedging at-the-money and high-strike options.

Smile adjustments to the implied BSM delta are derived from the slope of the implied volatility smile. An option pricing model’s calibration usually involves setting the model price equal to the BSM price based on the option’s implied volatility. Differentiating both sides of this equality yields the smile-adjusted delta as the implied BSM delta plus an adjustment term equal to the BSM vega times

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the implied volatility sensitivity to the underlying price. Derman (1999) introduced three different
ways of adjusting the BSM model to account for an equity index implied volatility smile, each being
relevant in a different market regime. They imply different types of ‘stickiness’ for the volatility in the
process for the underlying price $F$.\footnote{The vega is the option’s model price sensitivity to the option’s implied volatility. Although a smile-adjusted delta incorporates a vega effect there is no additional option in the hedge, i.e. the hedge is effected just by trading the futures in the amount specified by the smile-adjusted delta.} According to Derman:

- The sticky strike model applies during stable, trending equity markets. Here each option may
  be priced using a single, constant-volatility binomial tree for $F$, but each option uses a different
  tree in which the volatility is taken to be the implied volatility of the option. In this regime the
  implied BSM delta hedges are computed using the option’s own implied volatility.
- There are also multiple trees for $F$ in the sticky moneyness model, which applies when the mar-
  ket is in a range-bounded regime. But here the (constant) volatility of each tree depends on
  the option’s moneyness. Hence, to price the option one has to employ multiple trees, jumping
  between them as $F$ evolves over time to use the tree with the relevant moneyness of the option,
  given the price level at each node. For this reason sticky moneyness is also referred to as the
  floating smile model, and we need an additional adjustment to the implied BSM price hedge ratio,
  which depends on how the smile moves when $F$ changes.
- The only hedging model with a unique tree for $F$ that is used to price every option is the sticky tree
  model, which Derman’s advocates using during excessively volatile markets. Here the volatility
  is not constant over the tree, but is given by the forward volatility conditional on the price level
  at that node. This forward volatility is equivalent to the local volatility of Dupire (1994) and
  Derman and Kani (1994). Derman, Kani, and Zou (1996), Coleman, Kim and Verma (2001) and
  Crepey (2004) all show that the local volatility delta hedge ratio is a smile-adjusted delta where
  the implied volatility-price sensitivity is equal to the slope of the smile in the strike dimension.

There is conclusive evidence that deltas based on the sticky moneyness model provide worse hedges
than the implied BSM delta. The reason for this is that its price hedge ratios are theoretically equiva-
lent to the standard (partial derivative) price hedge ratios derived from any proper model for pricing
options on a tradable asset, which are well-known to perform poorly relative to the implied BSM. See
Alexander and Nogueira (2007) for further details. By contrast, local volatility (sticky tree) delta hedges
are usually more effective than implied BSM deltas, except possibly for hedging high-strike options
with frequent rebalancing. Several empirical papers support this: for instance, see Dumas, Pan and
Alexander and Nogueira (2007) and others. Vähämaa (2004) and Crepey (2004) find that the effective-
ness of the local volatility smile-adjustment has regime dependence; in fact, their results confirm
Derman’s intuition that the local volatility delta is superior only during excessively volatile periods.
Alexander, Kaeck and Nogueira (2009) demonstrate that local volatility deltas can outperform Hes-
ston minimum variance deltas for S&P 500 options, and for FTSE 100 options Alexander and Kaeck
(2010) show that this is true for options all moneyness and maturity, over both daily and weekly rebal-
ancing frequencies, and during both volatile and tranquil periods. They also confirm that the main
advantage of using the sticky tree/local volatility delta rather than the simple implied BSM delta is for hedging during volatile market conditions.

Given the evidence reviewed above this paper asks an obvious question: can we improve on the implied BSM and local volatility delta hedges by explicitly modelling regime-dependence in a smile-adjusted delta? We test the performance of several regime-dependent smile-adjusted deltas using 16 + 1/2 years of daily closing prices on FTSE 100 index options, thus covering several business cycles with both volatile and tranquil market conditions. 4 We consider two simple, ad hoc regime-dependent smile adjustments and then specify a formal model based on Markov switching of price-volatility sensitivities. In the following: Section 2 defines the various smile-adjustments and discusses their implicit assumptions about implied volatility dynamics; Section 3 describes the data; Section 4 presents the empirical results for the Markov switching model; Section 5 defines the hedging strategies and presents our results; and Section 6 summarizes and concludes.

2. SMILE-ADJUSTMENTS TO THE BLACK-SCHOLES-MERTON DELTA

We begin by presenting various smile adjustments that have already been used in the literature, making explicit their assumptions about the sensitivity of implied volatility to the futures price. Then we propose new smile adjustments that allow this sensitivity to depend on the current market regime, suggesting two ad hoc regime-dependent smile adjustments and one that is derived in the context of a Markov switching model.

Suppose a standard European option is written on one index futures contract, with price $F$. The purpose of delta-hedging is to build a portfolio which neutralizes the sensitivity of the option to changes in $F$, and the number of futures contracts that should be bought is the delta of the option. This may be calculated as the derivative of the option price $f(F, \sigma, r, t, K, T)$ with respect to $F$. Here $\sigma$ denotes the volatility of futures price, $r$ denotes the discount rate, $t$ denotes the time at which the option is priced, $K$ denotes the strike of the option and $T$ denotes the maturity date of the futures and the option. The option’s implied volatility $\sigma(K, T|F)$ is that volatility which equates the BSM price to the market price of the option. Its dependence on $K$ and $T$ arises because the implied volatility surface is not flat; nor is it invariant to changes in the underlying price, and hence its dependence on $F$. Indeed, the entire surface moves when the futures price changes. Hence, to calculate the first derivative of the option price with respect to the futures price we apply the chain rule:

$$\delta_{adj}(K, T|F) = \frac{\partial f}{\partial F} + \frac{\partial f}{\partial \sigma} \frac{\partial \sigma}{\partial F} = \delta_{BSM}(K, T|F) + v_{BSM}(K, T|F) \sigma_r(K, T|F),$$  

(1)

with the implicitly defined notation for $\delta_{BSM}$, $v_{BSM}$ and $\sigma_r$. Thus, to account for dynamics in implied volatility, the implied BSM delta $\delta_{BSM}$ should be adjusted by an amount which depends on the BSM option price-volatility sensitivity (vega) $v_{BSM}$ and on the implied volatility sensitivity to the futures price $\sigma_r$.

4We remark that all the empirical delta hedging studies reviewed above utilize no more than 4 years of daily data, with the exception of Dumas Fleming and Whaley (1998), who use 5 + 1/2 years. In fact, the majority of option hedging studies are based on only 1 or 2 years of data. The options studied are usually either S&P 500 or FTSE 100 index options, although Crepey (2004) also uses a 2001 sample of DAX 30, SM and DJIA index options data, and Poulson, Schenk-Hoppe and Ewald (2009) use a 2004-5 sample of Eurostoxx 50 and EUR/USM options, in addition to S&P 500 options.
(a) Sticky Models
A variety of different terminologies have been applied to these smile-adjusted deltas and our terminology is as follows: the ‘sticky strike’ (SS) delta denotes the implied BSM delta; the ‘sticky tree’ (ST) delta is the local volatility delta; and the ‘sticky moneyness’ (SM) delta denotes the partial derivative delta derived from any proper price process for a tradable asset.

In each of these hedging models the implied volatility-futures price sensitivity \( F(K_j; T) \) is related to the slope of the smile in the strike metric, \( F_m = F(K_j; T) \), as \( F(K_j; T) = k F_m \), with

\[
k = \begin{cases} 
0 & \text{(SS)}, \\
1 & \text{(ST)}, \\
-K/F & \text{(SM)}. 
\end{cases}
\]  

Now (2) defines the following very different skew dynamics when \( F \) changes:

- (SS): the skew does not move;
- (ST): the skew tilts by an amount equal to the slope of the skew at that point;
- (SM): the skew tilts in the opposite direction to the ST model, by an amount equal to \( K/F \) times the slope of the skew at that point.

If the skew has zero slope at some point, as it does if it has a typical hockey stick shape, then in the ST and SM models it pivots about this point. For an increase in \( F \), it tilts downward at lower strikes and upward at higher strikes in the ST model, and in the opposite direction in the SM model.

Our empirical study utilizes data over a 16\( \frac{1}{2} \) year period, with a great variety of option strikes being traded depending on the level of the futures price. So in order to construct continuous option price series we change from the strike to the moneyness metric, defining moneyness as \( m = F/K_j \).

The implied volatility as a function of moneyness is written \( \theta(m; T) = \theta(F; T) \), so the ATM volatility is \( \theta(1; T) = \theta(F; T) \). We write the smile-adjusted delta (1) as

\[
\delta_{\text{adj}}(m; T) = \delta_{\text{BSM}}(m; T) + \nu_{\text{BSM}}(m; T) \theta_t(m; T).
\]

Note that \( \sigma_t(K; T) = \theta_m(m; T) m = F^{-1}\theta_m(m; T) \), and so, since \( \sigma_t(K; T) = \theta_t(m; T) \), the latter being just a reparameterization of the former, the sticky models have:

\[
\theta_t(m; T) = \kappa \theta_m(m; T)
\]

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5Alternative terms include the ‘practitioner Black-scholes’ delta (e.g. Christoffersen and Jacobs, 2004), ‘the volatility-by-strike’ model (e.g. Rosenberg, 2000), and the ‘absolute smile’ method (e.g. Jackwerth and Rubinstein, 2001).
6The sticky moneyness model is also referred to as the ‘sticky delta’ model. Its delta is the scale-invariant delta defined by Alexander and Nougueira (2007), which is a partial price derivative hedge ratio that is model-free in the class of scale invariant models. So SM deltas include, for example, the partial derivative deltas of the Heston (1993) model and of any deterministic volatility function (e.g. a normal mixture diffusion) which has sticky moneyness implied volatility dynamics.
7Typically, the equity index smile slopes downward at low strikes and very often also slopes downward at and above the at-the-money (ATM) strike; it is usually only at very high strikes that the smile may be upward sloping, thus having a ‘hockey stick’ shape, and this is often termed the equity volatility skew, or ‘smirk’.
where

\[
\kappa = \begin{cases} 
0 & \text{(SS)}, \\
1/F & \text{(ST)}, \\
-m/F & \text{(SM)}. 
\end{cases}
\]  

Equations (3) – (5) define the smile-adjusted deltas \( \theta_m(m,T|F,t) \) that are derived from the slope of the smile in the moneyness metric. We shall estimate this slope by fitting a cubic polynomial of the form \( a_0(t) + a_1(t)m + a_2(t)m^2 + a_3(t)m^3 \) to the market implied volatility smile on each day \( t \), setting \( \hat{\theta}_m(m,T|F,t) = \hat{a}_1(t) + 2\hat{a}_2(t)m + 3\hat{a}_3(t)m^2 \).

(b) Approximate Minimum Variance Delta

Lee (2001), Section 7.2, derives an approximate locally risk minimizing or ‘minimum variance’ (MV) delta which, in the moneyness metric, is given by:

\[
\delta_{\text{MV}}(m,T|F) = \delta_{\text{BSM}}(m,T|F) + \frac{m}{F} \nu_{\text{BSM}}(m,T|F) \theta_m(m,T|F). 
\]  

As such it may be viewed as a smile-adjusted delta with \( \theta_r(m,T|F) = (m/F) \theta_m(m,T|F) \). We remark that the MV delta is based on exactly the opposite implied volatility dynamics to those of the SM delta.

(c) Ad-hoc Regime Dependent Smile Adjustments

So far we have considered four possible assumptions for implied volatility dynamics, each leading to a different value for \( \theta_r(m,T|F) \) in (3) which is derived entirely from the current smile \( \theta(m,T|F) \) and the current value of the underlying \( F \). However, Derman (1999) hypothesized that implied volatility dynamics depend on the market regime, as specified in the introduction, where a regime is defined by the characteristics of returns and volatility that prevails over a period of time. If this hypothesis is correct, then a recent period of historical data on the smile and the underlying will contain information that is relevant for determining the implied volatility dynamics.

In this sub-section we specify the dynamics by allowing \( \theta_r \) to depend on recent historical data, in addition to the current smile and underlying price. To this end we set \( \kappa = \kappa(m,T|F,t) \) in (4) to that multiple which minimizes the ex post standard deviation of the delta-hedging error over the past year. In other words, for each option, at each time \( t \) and fixing \( \kappa(m,T|F,t) \) we compute ex-post the delta-hedging error on every day during the past year and calculate its standard deviation. Then we choose that value for \( \kappa(m,T|F,t) \) which minimizes this standard deviation. This way, for every option, a time series of optimal multipliers \( \kappa \) in (4) is derived, whose values reflect the current regime by taking account of hedging error over the previous year. These deltas have an ad hoc regime dependence (RD) because the choice of a one year sample is arbitrary; a more refined approach might seek to optimize this period.

All the deltas defined so far are restricted by the limitation of the relationship (4) for the volatility-price sensitivity; when \( \theta_m(m,T|F) = 0 \) the implied volatility will not change with \( F \). That is, the smile can only pivot about the point where it has zero slope. To increase the possible smile dynamics we
now add a shift to the sticky-tree pivot movements by assuming:

\[ \theta_r (m, T|F) = \alpha(T|F, t) + \frac{1}{F} \theta_m (m, T|F). \]  

In section 5 we shall estimate \( \alpha(T|F, t) \) by considering the historical relationship between ATM volatility and the underlying price during the recent past. That is, we use the ATM volatility-price relationship to derive a shift in the smile that would apply even when \( \theta_m (m, T|F) = 0 \). As above we arbitrarily choose one year of historical data leading up to the time \( t \) that the hedge is taken, and perform a simple linear regression of daily changes in ATM volatility on the daily log return on the futures price. Then we set \( \hat{\alpha}(T|F, t) \) equal to the estimated slope coefficient, divided by \( F \) (since the regressor is the log return rather than changes in price). Since \( \hat{\alpha}(T|F, t) \) is derived using recent historical data, the delta derived from estimating (7) may be termed the ‘regime-dependent shift’ (RS). Below we shall specify a more rigorous regime-shifting volatility - price relationship. However, the advantage of the RD and RS deltas are that they may be implemented by option traders relatively easily.

\( \text{(d) Markov Switching Smile Adjustment} \)

To determine regime dependent values for \( \theta_r(m, T|F, t) \) in the context of a formal statistical model we introduce the ‘fixed moneyness spread’ (FMS) at time \( t \) as

\[ \theta^{\text{fms}}(m, T|F, t) = \theta(m, T|F, t) - \theta(1, T|F, t). \]

We do this because Derman (1999) notes that it is the relationship between the ATM volatility \( \theta(1, T|F, t) \) and the underlying price that appears to be regime dependent, and Alexander (1999) shows that the FMS has the same relationship with the futures price in all three of Derman’s sticky models, i.e.

\[ \theta^{\text{fms}}(m, T|F, t) = -b(t, T)F(t, T)(m(t, T) - 1), \]

where \( b(t, T) \) is relatively stable over time.

The model is constructed in two stages: (a) the estimation of a Markov switching model for the ATM volatility-price relationship, which determines a regime-dependent value for \( \theta_r(1, T|F, t) \); and (b) a principal component analysis (PCA) of the FMS which is used to extrapolate the regime-dependent \( \theta_r(1, T|F, t) \) to regime-dependent fixed-moneyness volatility-price sensitivities \( \theta_r(m, T|F, t) \).

Hamilton (1989) provided the first formal statistical representation of the idea that economic recessions and expansions influence the behaviour of economic variables. Since then, Hamilton’s Markov switching (MS) techniques have been further developed and widely applied to a variety of disciplines - see Frühwirth-Schnatter (2006) for a review. To specify a MS model, statistical tests are used to identify the optimal model structure and the number of distinct regimes. Non-standard distributions for test statistics require a computationally intensive bootstrapping framework, such as that recommended by Ryden et al. (1998). In addition to these we performed some very powerful inference tests based on the comparison of unconditional densities proposed by Ait-Sahalia (1996) and applied to Markov switching models by Breunig et al. (2003). A variety of different model struc-
tures were tested, including non-linear dependence and different numbers of states. The results of these tests, which are summarized in section 4, indicate that the optimal model of the ATM volatility-futures price relationship is, for all maturities, a two-state Markov switching model with the following specification:

$$y(t) = \alpha^s + \beta_1^s x(t) + \beta_2^s x(t-1) + \beta_3^s y(t-1) + \epsilon^s(t),$$

where $s = 1, 2$ denotes the state, $y(t)$ denotes the daily change in the $T$-maturity ATM volatility and $x(t)$ denotes the daily log return on the futures and $\epsilon^s \sim \text{NID}(0, \sigma^s_\epsilon)$. The state variable $s$ is assumed to follow a first-order Markov chain with constant state-transition probabilities $Pr(s_t = j|s_{t-1} = i) = \pi_{ij}$. Since $\sum_j \pi_{ij} = 1$, the transition matrix may be written:

$$\pi_{ij} = \begin{pmatrix} \pi_{11} & \pi_{21} \\ \pi_{12} & \pi_{22} \end{pmatrix} = \begin{pmatrix} \pi_{11} & 1 - \pi_{22} \\ 1 - \pi_{11} & \pi_{22} \end{pmatrix}. \tag{9}$$

Maximum likelihood estimation provides two sets of parameters, their standard errors, the transition matrix and a time series of conditional probabilities $\pi_t$ for state 1 to be the ruling state at time $t$. We shall label the states so that $\hat{\pi}_1 > \hat{\pi}_2$ and therefore identify state 1 with the ‘volatile’ regime and state 2 with the ‘tranquil’ regime.

Note that, for brevity of notation, we have supressed the dependence of (8) and (9) on the maturity $T$. However, it should be emphasized that there are six different MS models, one for each maturity $T = 30, 60, ..., 180$ days. Thus, based on (8) we have a regime-dependent and maturity-dependent value for the sensitivity of $T$-maturity ATM volatility to the daily change in the $T$ maturity futures price at time $t$.\(^9\)

Next we apply PCA to the covariance matrix $V(T)$ of the daily changes $\Delta \theta_t^{\text{fms}}(m, T|F, t)$ in $T$-maturity fixed moneyness spreads, and use three principal components in their representation:

$$\Delta \theta_t^{\text{fms}}(m, T|F, t) = \sum_{i=1}^3 \omega_i(m, T)p_i(t, T), \tag{10}$$

where $\omega_i(m, T)$ is the $m$th element of the $i$th eigenvector of $V(T)$ and $p_i(t, T)$ is the value at time $t$ of the $i$th principal component. Then, for each component we assume a time-varying but linear relationship with the log return on the futures:

$$p_i(t, T) = \gamma_i(t, T)\Delta \log F(t, T) + \epsilon_i(t, T) \tag{11}$$

where $\epsilon_i(t, T)$ are i.i.d. processes. Substituting (11) into (10) yields the FMS sensitivity to changes in the futures price:

$$\theta_t^{\text{fms}}(m, T|F, t) = \frac{\lambda(t, T)}{F(t, T)}, \tag{12}$$

where $\lambda(t, T) = \sum_{i=1}^3 \omega_i(m, T)\gamma_i(t, T)$. Writing $\theta_t(m, T|F, t) = \theta_t(1, T|F, t) + \theta_t^{\text{fms}}(m, T|F, t)$ and substituting

\(^9\)There are two possible values: the instantaneous sensitivity $\beta(T)/F(t, T)$ and the steady-state sensitivity $(\beta(T) + \beta(T)/(1 - \beta(T)))F(t, T))$. In the following we use the instantaneous sensitivity. In terms of their empirical hedging performance we find very little difference between the two.
(12) plus the regime-dependent sensitivity for the $T$-maturity ATM volatility in the above, yields:

$$\theta_t^r(m, T|F, t) = \frac{\theta_t^c(T) + \lambda(t, T)}{F(t, T)}.$$  (13)

Substituting this regime-dependent value for $\theta_t^r(m, T|F, t)$ into (3) gives the Markov switching (MS) deltas $\delta_{MS}^1(m, T|t)$ and $\delta_{MS}^2(m, T|t)$ which depend on whether the market is in state 1 or state 2 at the time the hedge is taken.

It remains only to decide on the signal to be used as an indication of the current market regime. To this end we employ the conditional regime probability $\pi(t, T)$ that is estimated from the MS model (8) and (9). For our hedging study we start by setting $\delta_{MS}^1(m, T|t)$ if $\pi(t, T) > \frac{3}{4}$ and $\delta_{MS}^2(m, T|t)$ otherwise. Then we investigate whether recent data has any useful information for the regime-switching signal, by setting $\delta_{MS}^1(m, T|t)$ if $\pi(t, T) > \tau(m, T|t)$ and $\delta_{MS}^2(m, T|t)$ otherwise, where $\tau(m, T|t)$ is chosen to minimize the ex post standard deviation of the hedging error at moneyness $m$ over the past year. Thus the selection of the threshold $\tau(m, T|t)$ for regime switching is based on a similar optimization to the selection of the multiplier in (c) above.

3. Data

The option data used for our empirical study, provided by NYSE Euronext, consist of closing prices and implied volatilities of European FTSE 100 index options from 2 October 1992 to 17 March 2009, a period of over 4000 days with over 2.3 million option prices in the raw dataset. At the start of the sample the price of the FTSE 100 futures was obtained using put-call parity because the options and futures markets closed at different times. Finally, time series of GBP LIBOR rates were downloaded from the British Banker's Association’s homepage and yield curve fitting was performed using Hermite splines.

The raw data were filtered for several reasons. In a few cases the put and call implied volatility for a given maturity and strike differed, and here we only used the implied volatility of the out-of-the-money (OTM) option, this typically being the more liquid of the two. Secondly, we deleted all ‘cabinet’ option prices and volatilities from the dataset. Cabinet options are those which are very deep OTM and hence are nearly worthless. They are rarely traded, mostly just to close open positions. The exchange is obliged to quote a price for these options whilst open interest is positive, so they quote the minimum price of £0.5, but the real value of these options is typically much lower. Hence to include these option prices in the database could distort our results. A third filter was based on the observation that, prior to 2008, the xx75 strike options were much less liquid than the xx25 strike options, and would therefore be liable to inaccurate stale prices. So we also deleted the xx75 strike options, prior to 2008, from the dataset.

A fourth filter was used to obtain consistent time series of prices and volatilities covering the entire sample period. For this we first constructed constant maturity option and futures prices with a fixed maturity of 30, 60, ..., 180 days. To this end, for every combination of strike and date we fitted

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10Following standard practice, to avoid excessive responses to occasional outliers the regime probability is smoothed and the regime probability trigger for entry to the high volatility regime is above $\frac{1}{2}$.

11We also constructed prices and volatilities at maturities one day, 5 days and 10 days less than these, because they are required for the hedging study.
arbitrage-free term structures of implied volatility, and evaluated these term structures at the fixed maturities.\textsuperscript{12} The best results were obtained using shape-preserving Hermite splines to interpolate and extrapolate the values of the implied volatility term structure, and we checked for no arbitrage in the result.\textsuperscript{13}

Just like cubic splines, Hermite splines consist of piecewise cubic polynomials. However, the second order condition at the knot points is replaced by a shape-preserving condition which is only on the slopes of the cubic polynomials at the knot points. Shape-preserving Hermite splines provide realistic extrapolated values, if these are necessary, and they are much less sensitive to data errors than cubic splines. Using Hermite splines we constructed roughly 100,000 term structures, so it is not possible to examine every term structure to verify that it is realistic. Instead we evaluated the result of Hermite spline interpolation and extrapolation across maturities by investigating the volatility skews for constant maturities of 30, 60, ..., 180 days.

There are no simple no-arbitrage constraints for the shape of the implied volatility skew except that the wings of the implied variance functions at deep in-the-money (ITM) and deep OTM strikes should be asymptotically linear in log-moneyness (see Lee, 2004). Hence, we transformed the constant maturity implied volatilities into the deltas of a constant maturity option, using constant maturity time series of discount rates and futures prices, and ensured that these deltas were monotonically decreasing with respect to strike; this way we ensured no-arbitrage with respect to strike. Then, to construct constant moneyness contracts, on each date in the sample and for each of the constant maturities considered, we fitted a cubic polynomial to the implied volatility skew. This approach, first used by Malz (1997), has also been applied by Weinberg (2001), Bliss and Panigirtzoglou (2002) and Lynch and Panigirtzoglou (2008).

Of course, constant maturity and moneyness contracts are artificial, non-traded instruments. Nevertheless, they allow one to examine hedging performance of a single contracts over the entire sample, which would not have been possible without this construction. In order to focus on liquid options we only examine the hedging of 30-day, 60-day, ..., 180-day options with moneyness 0.95, 0.96, ..., 1.04, 1.05. This way, our final arbitrage-free data set consists of 16\frac{1}{2} years of daily prices on 66 different options, which is much larger than any other dataset previously used to study option hedging.

4. Estimation of Regime-Dependent Smile Dynamics

Before performing the rolling window analysis it is necessary to fix the number of states and the functional form of the ATM volatility-price relationship in each state, and in particular to justify the specification (8). This lengthy analysis began with ordinary least squares estimation of single-regime models over the entire data period, testing out a variety of linear, log-linear and non-linear relationships between ATM volatility and futures price changes or returns. We found the best combination of

\textsuperscript{12}To avoid calendar arbitrage option prices of any given moneyness must be non-decreasing with respect to maturity. A necessary but not sufficient condition for this is that the total implied variance increases with term.

\textsuperscript{13}Other common approaches to fitting volatility term structures are cubic polynomials or cubic splines, and when these are used it possible to impose the no-arbitrage constraint on the optimization as explained by Fengler (2009). However, a cubic polynomial does not fit the raw data exactly and a cubic spline is too flexible to be of much use (especially for extrapolation) and it is very sensitive to any misquotes or stale prices that may have been left in the data after the filtering.
high modified $R^2$ and coefficient $t$-ratios to be based on a linear relationship between daily changes in ATM volatility and the daily log return on the futures, and for options up to 90 days maturity the fit is slightly improved by adding one lag of the dependent and independent variable. Then we applied the specification test of Breunig et al. (2003) to determine the number of states. This procedure compares the empirical density of daily changes in ATM volatility, $\hat{f}(y(i, T))$ (which we fit with a Gaussian kernel) and the unconditional density generated by the MS model, denoted $f_{MS}(y(i, T)|\hat{\theta})$ where $\hat{\theta}$ are the model parameters. The comparison of the two densities can be made through a distance metric, for instance the Kolmogorov-Smirnov (KS) statistic $\max_{j} f_{MS}(y(i, T)|\hat{\theta}) - \hat{f}(y(i, T))$ $\hat{f}(y(i, T))$ $\hat{f}(y(i, T))$ $\hat{f}(y(i, T))$, where $N$ is the number of observations. The smaller the distance between the two densities, the better specified the MS model.

To avoid unnecessary complexity in the rolling window analysis (which entails about 20,000 model estimations) we aim to use the same MS model for all maturities. Since short-term volatility is more variable than long-term volatility it is natural to suppose that state transition signals are most pronounced in short-term volatility-price relationships. For this reason we present model specification tests for the 30-day maturity. Table 1 reports the MSE and KS statistics for simple linear models with one, two and three states and for the two-state model (8) having one lag of both dependent and independent variables.

Table 1: Density Tests for Model Specification.

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE</th>
<th>KS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 State</td>
<td>0.481</td>
<td>0.726</td>
</tr>
<tr>
<td>2 States</td>
<td>0.362</td>
<td>0.607</td>
</tr>
<tr>
<td>2 States (lagged)</td>
<td>0.425</td>
<td>0.352</td>
</tr>
<tr>
<td>3 States</td>
<td>0.667</td>
<td>0.872</td>
</tr>
</tbody>
</table>

According to the KS statistic the best model is the two-state model with lagged variables, but according to the MSE this model is only second best, the best being the two-state model without lagged variables. Nevertheless, the hypothesis that (8) is the preferred specification is supported by other statistical tests. Starting with a linear model having no lagged variables we employ the likelihood ratio (LR) and Wald (W) tests specified by Ryden et al. (1998) as follows:14

- $H_0$: 1 State vs $H_1$: 2 States. Reject $H_0$ at 5%; $LR = -33.21$, $W = -4.1267$ (5% critical values: $LR = -40.08$, $W = -4.1277$);
- $H_0$: 3 States vs $H_1$: 2 States. Reject $H_0$ at 5%; $LR = 1.82$, $W = 18.52$ (5% critical values $LR = -6.05$, $W = -56.01$);
- $H_0$: 2 States vs $H_1$: 2 States (lagged). Reject $H_0$ at 5%; $LR = 24.76$, $W = 25.45$ (5% critical values $LR = -45.01$, $W = -42.55$).

Given these results we employ the specification (8) and (9) of a two-state MS model between daily changes in ATM volatility and the daily log returns on the futures of the same maturity.

---

14Since there are unidentified parameters under the null hypothesis of regime switching, standard tests do not converge to their usual distribution. Therefore, we obtain an empirical distribution of the test statistic under the null by bootstrapping and fitting the model and comparing this with the actual statitstic from our data set. Since this procedure requires the estimation of a switching model for every iteration, it is very computationally intensive. We therefore set a limit of 200 simulations and used 40 randomised starting values for each simulation.
Static Analysis
The MS-PCA model must be implemented in a dynamic framework in Section 5. First, we provide some intuition and discussion of the results when the model is estimated using the entire sample period, again focusing on the 30-day smile for illustration. Table 2 reports the results of estimating the model over the entire sample between Oct 1992 and Mar 2009, for the maturity $T = 30$ days.

### Table 2: Two-State MS Model Estimation.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Const.</th>
<th>x(t)</th>
<th>x(t-1)</th>
<th>y(t-1)</th>
<th>$\pi(i,j)$</th>
<th>Stdev</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Vol. Est.</td>
<td>-0.0007</td>
<td>-1.2076</td>
<td>-0.2906</td>
<td>-0.2096</td>
<td>0.94</td>
<td>0.0179</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0006</td>
<td>0.0303</td>
<td>0.0488</td>
<td>0.0322</td>
<td></td>
<td>0.0004</td>
</tr>
<tr>
<td>Low Vol. Est.</td>
<td>-0.0001</td>
<td>-0.6238</td>
<td>-0.0822</td>
<td>-0.1223</td>
<td>0.97</td>
<td>0.0052</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0001</td>
<td>0.0129</td>
<td>0.0199</td>
<td>0.0249</td>
<td></td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The two regimes are identified as a high volatility regime (regime 1) in which the standard error is almost four times higher than it is in the low volatility regime (regime 2). In both regimes the changes in FTSE futures log returns, and lagged log returns, and lagged changes in ATM volatilities are highly significant and have a negative impact on ATM volatility. The estimated parameters are significantly different in the two regimes. The size of all regression coefficients is much higher in regime 1, indicating that ATM volatility-price sensitivity is much greater when markets are volatile than when they are tranquil.\(^{15}\) The volatile regime is the least persistent of the two, as it has the lower transition probability. Figure 1 depicts the corresponding time series $\pi(t, 30)$ of the conditional regime probabilities. This identifies major periods of market turmoil such as the Asian property crash of 1997, the LTCM crisis in 1997, the bursting of the dot-com bubble in 2001-2002 and the global banking crisis of 2008-2009 which led to a global recession.

\(^{15}\)The ATM sensitivity to changes in the futures price, as defined in (13), is $\theta^1(30) = \hat{\beta}_1(30)/F(30) = -1.2076/3856.8 = -3.13 \times 10^{-4}$ in the volatile regime and $\theta^2(30) = -1.62 \times 10^{-4}$ in the tranquil regime. Clearly, the reaction of ATM volatility to futures returns is very different in volatile and tranquil periods. For instance, in the volatile regime a 5% fall in 30-day FTSE futures is associated with about a 6% rise in ATM volatility (i.e. $5 \times -1.2076$); but in the tranquil regime a similar fall in the FTSE futures precipitates little more than 3% rise in ATM volatility ($5 \times -0.06238 = 3.12\%$).

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Turning now to the principal component analysis part of the model, Figure 2 depicts the first three eigenvectors of the covariance matrix of the daily changes in the FMS at maturity 30 days, as a function of moneyness. This shows that the first eigenvector, which captures almost 70% of the variation, represents a tilt in FMS with moneyness ≤ 0.99, with the other FMS remaining virtually static; the second eigenvector, which captures about 23% of the variation, represents a tilt in FMS with moneyness ≥ 0.99, with the other FMS remaining almost static; and the third eigenvector, which captures about 7% of the variation, represents a change in convexity in FMS with moneyness < 1, at the same time as an almost equal and opposite change in convexity of FMS with moneyness > 1. A total of 99.75% of the variation over the entire period is captured using just three principal components. The eigenvectors have a similar shape at other maturities, although the tilt and convexity movements are less pronounced; but again virtually 100% of the variation may be captured using a 3-component representation.

**Figure 2: Eigenvectors at T=30 days, based on the entire sample.**

It remains to estimate the coefficients \( \gamma_i(t, 30) \) in (11) for the three principal components. In an advanced implementation of the model a Kalman filter or bivariate GARCH model could be applied, but to avoid this added complexity (and since we have to about 20,000 model estimations in the dynamic setting below) we shall employ a simple exponentially weighted moving average variance and covariance estimator, with a smoothing constant of 0.94.\(^{16}\) Thus, for \( i = 1, 2 \) and \( 3 \), we set

\[
\gamma_i(t, 30) = \frac{\text{Cov}(p_i(t, 30), \Delta \log F(t, 30))}{V(\Delta \log F(t, 30))}.
\]

Now, taking the value of \( \gamma_i(t, 30) \) and of the 30-day futures price at the end of the sample period, we apply (13) to obtain the range of volatility-price sensitivities depicted in Figure 3.

---

\(^{16}\)This value for the smoothing constant is commonly applied in the industry, being the value adopted by RiskMetrics for their daily Value-at-Risk calculations.
**Dynamic Analysis**

Having used the entire sample to determine which model is best specified, we now implement this model in a rolling window framework. This is to avoid ‘data snooping’. That is, in our hedging study we must determine the delta using data only up to the point in time when the hedge is implemented. A fairly large sample is required for specification of MS model parameters and for this reason we fixed a sample size of 1000 days (approximately 4 years) and estimated the models in a 4-year rolling window framework. That is, starting with the period Oct 1992 - Oct 1996, for each of six different maturities $T = 30, 60, ..., 180$ days, we estimated the MS-PCA model using 1000 observations on each of the variables (daily changes in ATM volatility, daily changes in FMS and daily log returns on the futures). Then each set of 1000 observations was rolled forward one day and the MS-PCA models were re-estimated. This was repeated until all $16rac{1}{2}$ years of data were exhausted. This way we estimated approximately 20,000 MS-PCA models. For brevity, we shall just summarize the 30-day results here. Full details on the rolling window results are available from the authors on request.

Figure 4 depicts the evolution of the percentage of variation explained by each of the first three principal components.\(^{17}\) This shows that the three components capture virtually all the movements in the FMS, with the total variation explained by the three components being in excess of over 99.75% for the vast majority of samples. As expected, $\theta_r(m, T|F, t)$ are very highly correlated across moneyness. They are also very highly correlated across maturity, and they become more negative and less dependent on moneyness during a highly volatile period. Time series of $\theta_r(m, 30|F, t)$ are depicted in Figure 5.\(^{18}\) To gain further insight to the behaviour of the sensitivities during different market regimes, we take 40 consecutive days during a tranquil period (July 2005) and plot the cross-section of $\theta_r(m, 30|F, t)$ as a function of moneyness, and compare this with the sensitivities taken from 40 days around one of the most volatile periods in the sample (October 2008). The result is depicted in Figure 6.

---

\(^{17}\)The percentage of variation explained by the $i$th component is the $i$th largest eigenvalue of the covariance matrix of the daily changes of the FMS, based on a rolling window of 1000 days, divided by the sum of all the eigenvalues.

\(^{18}\)There is very little difference between the steady-state and instantaneous sensitivities, so we only display the latter here.
Figure 4: Percentage of variation explained in rolling PCA (T=30 days)

Figure 5: Fixed-moneyness volatility sensitivity to changes in the futures price (T=30 days)

Figure 6 exhibits features that are common to other volatile and tranquil sub-samples: volatility sensitivities during crash periods are much greater and more variable from day to day than the sensitivities during normal market circumstances. Since these sensitivities are the source of difference between various smile-adjusted deltas, we expect that the MS-PCA model will produce hedging results that are very different from those based on deltas which are not regime-dependent, especially during a market crash.

5. Hedging Results

In our delta hedging strategies at each time $t$ a short position in a call option of moneyness $m$ and maturity $T$ is complemented by an investment in $\delta_{adj}(m, T|F, t)$ futures contracts of maturity $T$, where $\delta_{adj}(m, T|F, t)$ is one of the smile-adjusted delta hedges specified in Section 2. Defining $V(m, T|F, t)$ as the option value when the futures price is $F(t, T)$ at time $t$, the smile-adjusted delta is used to construct...
a self-financing portfolio by adding an investment of $B(m, T|F, t) = V(m, T|F, t) - \delta_{\text{adj}}(m, T|F, t)F(t, T)$ in a bond of maturity $T$ that returns the risk-free rate $r$. The initial value $\Pi(t)$ of this portfolio is zero, but if the hedge is held from time $t$ to time $t + \Delta t$ the value of the portfolio $\Pi(t + \Delta t)$ at the end of the hedging period, i.e. the hedging error, is:

$$\Pi(t + \Delta t) = rB(m, T|F, t)\Delta t + \delta_{\text{adj}}(m, T|F, t)\Delta F(t, T) - \Delta V,$$

(14)

where $\Delta F(t, T) = F(t + \Delta t, T) - F(t, T)$ and $\Delta V = (V(m, T|F, t + \Delta t) - V(m, T|F, t))$ are the changes in values of the futures and the option respectively, from time $t$ to time $t + \Delta t$.

We now measure the performance of each of the seven smile-adjusted hedges defined by (3),
which are summarized as follows:

\[
\theta_t(m, T|F, t) = \begin{cases} 
0 & (SS), \\
\frac{1}{T}\theta_m(m, T|F, t) & (ST), \\
-\frac{w}{T}\theta_m(m, T|F, t) & (SM), \\
\frac{w}{T}\theta_m(m, T|F, t) & (MV), \\
\kappa(m, T|F, t)\theta_m(m, T|F, t) & (RD), \\
\alpha(T|F, t) + \beta(m, T|F, t)\theta_m(m, T|F, t) & (RS), \\
F(t, T)^{-1}(\beta^*(T) + \lambda(t, T)) & (MS). 
\end{cases}
\]

In the MS delta the state indicator \( s = 1, 2 \) is determined by the conditional regime probability \( \pi(t, T) \) at time \( t \). We consider two variants of the MS delta: MS1, in which \( s = 1 \) if \( \pi(t, T) > \frac{3}{4} \) and \( s = 2 \) otherwise; and MS2, in which \( s = 1 \) if \( \pi(t, T) > \tau(m, T|t) \) and \( s = 2 \) otherwise. The parameters \( \kappa(m, T|F, t), \alpha(T|F, t), \beta(m, T|F, t) \) and \( \tau(m, T|t) \) are estimated using an in-sample period of 250 days, as described in Section 2.

For every \( (m, T) \in R \), with \( R = \{m, T|m = 0.95, 0.96, \ldots, 1.05; T = 30, 60, \ldots, 180 \ \text{days}\} \) and for \( \Delta t = 1, 5, 10 \ \text{days} \) we compute the standard deviation of the out-of-sample hedging error (14) over the entire out-of-sample period. Because 66 options are delta hedged and rebalanced over 3 different hedging horizons, and 8 different deltas are compared, a total of \( 66 \times 3 \times 8 = 1,584 \) standard deviations are computed. For brevity, only the results for hedging 30-day, 90-day and 180-day options with daily and 10-day rebalancing are summarized in Tables 3 and 4.19 The first row of each table shows the out-of-sample hedging error of the implied BSM delta hedge (labelled SS) and the remaining rows report the other delta hedging errors as a percentage of this.

Considering first the implied BSM delta hedge, we find that the performance is best for daily rebalancing of short term (30-day) options, with OTM calls being easier to hedge than ITM calls, and the hedging errors of ATM options giving larger standard deviations than both OTM and ITM calls. The hedging performance deteriorates as the maturity of the option increases and especially as the hedging horizon increases (compare the daily rebalancing results in Table 3 with the 10-day rebalancing results in Table 4). Over both daily and longer hedging horizons, OTM calls remain easier to hedge than other options, whatever their term. There is little or no improvement in ITM option’s hedging performance over that for ATM options for the longer term options.

Now consider the 7 other deltas, whose performance in Tables 3 and 4 is presented as a percentage of the implied BSM delta’s performance. In agreement with the previous literature the SM delta performs much worse than the implied BSM delta hedge for all options, irrespective of the rebalancing frequency. However, all other smile-adjusted hedges improve on the implied BSM hedge and the extent of this improvement increases with the maturity of the options. The ST and MV deltas perform very similarly, as expected, with the ST delta being marginally better for ITM options and the MV delta being marginally better for OTM options – for ATM options the deltas are the same. Much the best performance, for all options, is found with regime-dependent deltas, and with the RS and MS deltas in particular.

For options with maturity greater than 30 days the use of the full MS model with an optimized

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19Examination of the other results, which are available on request, would not change any of our conclusions.
### Table 3: Standard Deviation of Hedging Error, 1-day Rebalancing

<table>
<thead>
<tr>
<th>Maturity</th>
<th>30-day Moneyness</th>
<th>90-day Moneyness</th>
<th>180-day Moneyness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>RM</td>
<td>124%</td>
<td>127%</td>
<td>128%</td>
</tr>
<tr>
<td>ST</td>
<td>81%</td>
<td>79%</td>
<td>77%</td>
</tr>
<tr>
<td>MV</td>
<td>82%</td>
<td>79%</td>
<td>77%</td>
</tr>
<tr>
<td>RD</td>
<td>77%</td>
<td>72%</td>
<td>69%</td>
</tr>
<tr>
<td>RS</td>
<td>77%</td>
<td>71%</td>
<td>68%</td>
</tr>
<tr>
<td>MS1</td>
<td>73%</td>
<td>70%</td>
<td>68%</td>
</tr>
<tr>
<td>MS2</td>
<td>76%</td>
<td>68%</td>
<td>66%</td>
</tr>
<tr>
<td>SS</td>
<td>15.78</td>
<td>15.88</td>
<td>16.07</td>
</tr>
<tr>
<td>RM</td>
<td>131%</td>
<td>133%</td>
<td>134%</td>
</tr>
<tr>
<td>ST</td>
<td>74%</td>
<td>72%</td>
<td>71%</td>
</tr>
<tr>
<td>MV</td>
<td>75%</td>
<td>73%</td>
<td>72%</td>
</tr>
<tr>
<td>RD</td>
<td>62%</td>
<td>58%</td>
<td>56%</td>
</tr>
<tr>
<td>RS</td>
<td>65%</td>
<td>60%</td>
<td>56%</td>
</tr>
<tr>
<td>MS1</td>
<td>62%</td>
<td>59%</td>
<td>57%</td>
</tr>
<tr>
<td>MS2</td>
<td>59%</td>
<td>56%</td>
<td>53%</td>
</tr>
</tbody>
</table>

The threshold (the MS2 delta) has a clear advantage: for instance the standard deviation of the hedging errors is roughly half of that from the implied BSM delta hedge at both daily and 10-day hedging horizons, for ATM and OTM call options. This is a really substantial improvement on implied BSM hedging. The RS delta, which is much easier to implement than the MS deltas, is also very effective. In fact, with 30-day options and 10-day rebalancing the RS delta outperforms even the MS2 delta for near ATM and OTM options.

We now investigate how delta-hedging performance varies over time, as the market experiences volatile and tranquil periods. Is delta hedging more effective during tranquil periods? Does the choice of delta-hedging model matter more during volatile periods than during tranquil periods?

The answer to both these questions is most definitively “yes”. Figure 7 depicts the standard deviations of the daily-rebalanced hedging errors computed on an out-of-sample period of only the previous year (250 days), rolled over daily, for 30-day call options with moneyness 0.95, 1.00 and 1.05.20

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20The standard deviation of the MV delta-hedged portfolios were extremely close to those of the ST delta-hedged portfo-
Figure 7: Standard Deviation of Hedging Error Over Previous Year (Daily Rebalancing of Delta-Hedged 30-day Calls)
Table 4: Standard Deviation of Hedging Error, 10-day Rebalancing

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Moneyness</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-day</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>43.21</td>
</tr>
<tr>
<td>SM</td>
<td>117%</td>
</tr>
<tr>
<td>ST</td>
<td>87%</td>
</tr>
<tr>
<td>MV</td>
<td>88%</td>
</tr>
<tr>
<td>RD</td>
<td>83%</td>
</tr>
<tr>
<td>RS</td>
<td>77%</td>
</tr>
<tr>
<td>MS1</td>
<td>78%</td>
</tr>
<tr>
<td>MS2</td>
<td>76%</td>
</tr>
<tr>
<td>90-day</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>45.07</td>
</tr>
<tr>
<td>SM</td>
<td>127%</td>
</tr>
<tr>
<td>ST</td>
<td>77%</td>
</tr>
<tr>
<td>MV</td>
<td>78%</td>
</tr>
<tr>
<td>RD</td>
<td>67%</td>
</tr>
<tr>
<td>RS</td>
<td>68%</td>
</tr>
<tr>
<td>MS1</td>
<td>67%</td>
</tr>
<tr>
<td>MS2</td>
<td>61%</td>
</tr>
<tr>
<td>180-day</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>55.00</td>
</tr>
<tr>
<td>SM</td>
<td>124%</td>
</tr>
<tr>
<td>ST</td>
<td>79%</td>
</tr>
<tr>
<td>MV</td>
<td>80%</td>
</tr>
<tr>
<td>RD</td>
<td>57%</td>
</tr>
<tr>
<td>RS</td>
<td>66%</td>
</tr>
<tr>
<td>MS1</td>
<td>62%</td>
</tr>
<tr>
<td>MS2</td>
<td>57%</td>
</tr>
</tbody>
</table>

According to the criterion of minimizing the hedging error standard deviation over the past year, all delta hedges except SM improve on the BSM, with two exceptions: (1) hedging OTM calls (moneyness 1.05) during the very tranquil period between April 2004 and May 2006, where any delta hedging is particularly effective and it matters very little which model is used; and (2) the year from July 2006 to July 2007, where the simple regime dependent (RD) delta led to an unexpectedly large hedging error standard deviation, but only for the OTM calls. When hedging ATM options the RS and MS hedges perform almost identically, but as the option moves away from ATM the formal Markov switching model that we have introduced, especially with an optimized threshold delineating volatile and tranquil regimes, gives a clear advantage to traders who seek to reduce their spreads by using the most accurate delta hedges possible.

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6. Summary and Conclusions

We have employed a very much larger sample than any other hedging study to date (16\frac{1}{2} years of daily data on FTSE 100 index options) to calculate the efficiency gains, relative to the implied BSM delta, from utilizing several different smile-adjusted deltas. Considering the standard deviation of out-of-sample delta-hedging errors from 1-, 5- and 10-day rebalancing of delta-hedged vanilla call options, with moneyness 0.95, 0.96, \ldots, 1.05 and maturities 30, 60, \ldots, 180 days, we provide very strong evidence that tailoring the delta-hedge to the market regime makes a huge difference to hedging performance. On average over the entire sample period the regime-dependent delta hedges produce errors that are almost half the size of implied BSM delta-hedging errors, whatever the rebalancing period, with only slightly less of an efficiency gain even when hedging very short term (30-day) options. For all options and all rebalancing periods, regime-dependent deltas are especially effective during volatile market periods.

Considering seven different smile-adjustments to the implied BSM delta, four of which are regime-dependent, we show that the essential property to capture is that there is a shift of the smile accompanying a change in the underlying price. Furthermore, the size of this shift should depend on the current market regime. Three of our deltas have this property. One of these is very simple to compute, using the current smile and a short history of ATM volatility. Our results show that the most effective regime-dependent hedging of vanilla FTSE 100 index options is based on a Markov switching relationship between the ATM volatility and the underlying future price which has a straightforward extension to options of other moneyness via standard principal component analysis.

References


