Credit Analysis using EVT and Copula Functions

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Overview

- Current credit risk methods
- Credit risk and EVT
  - some results for default rates
- Credit risk and copula functions
  - preliminary results
- A credit rating model
  - how to calibrate the model to the market
- Valuation of credit derivatives
  - theory
  - results from the model

Current credit risk methods

It's not what it used to be...

- Old style credit risk analysis
  - 'How likely is it that a company will default?'
- New style
  - 'How likely is it that a company will default?'
  - 'How likely is it that a company's credit quality will deteriorate?'
  - 'How likely is it that two companies will default at the same time?'
  - 'Can I buy a credit derivative to reduce my capital allocation amount?'
- … it's a different game these days

Academic credit risk models

- In spite of a number of papers and advances, models remain relatively theoretical
  - mainly divide into 'reduced form' and 'structural' approaches
- There is little empirical testing of models and techniques
  - this is partly due to sparse or unavailable data
- Available models are difficult to apply
  - … and may give conflicting results

Internal credit risk models

- Often local banks have more local information
  - thus in theory local risk estimates may be more realistic than agency estimates
- Many companies are unrated
  - thus local risk estimates may be the only ones available
- Internal models differ widely
  - there are probably as many internal models as there are financial institutions
- … and can be very crude!

Data difficulties

- Default data is sparse
  - good because there are few defaults
  - bad because little data for estimations
  - particularly bad because default tails seem to be fat
- What there is is hard to get hold of
  - some is spread among many institutions
  - best data sets are probably held by rating agencies
- Data errors and inconsistencies are legion
  - for instance, the definition of default can vary widely

Risk of correlated defaults

- Portfolios can be very large
- Default correlations can become very important
- Do defaults tend to occur separately?
  - this would constitute something of a natural hedge
- … or does everything go bad at once?
  - thus reducing diversification benefits and increasing risk

External ratings and credit risk

- Rating agencies try to provide impartial, unbiased estimates of company/debt risk
- … this aim is thwarted by their popularity
- 'Quantum observation' problem
- eg, downgrades are known to cause more downgrades
- Some problems with agency ratings
  - they may change after it was appropriate
  - not all companies are rated
  - agencies may not have much local knowledge

Portfolio hedging with credit derivatives

- Use a basket product to hedge away credit risk
  - cheaper than individual contracts
- Starting to become popular
  - reduces credit risk
  - no actual portfolio adjustment necessary
  - may reduce necessary risk capital
- … but there are problems
  - Not much transparent pricing
  - Secondary market is small and illiquid
  - basket products are particularly difficult to value

Solutions?

- To correctly analyse fat tailed default distributions we need to use Extreme Value Theory
- To correctly analyse fat tailed dependencies we need to use copula functions
- We need a model which
  - can use EVT and copula results
  - takes into account rating band structures
  - acknowledges that external ratings influence credit quality
  - is easily adapted to use existing internal band-type credit ratings
- … and we need data!

EVT and credit risk

Distribution of maxima

- For credit risk we are concerned with the probability of large losses
- One way of modelling this is to look at distributions of maxima
- We do this because distributions of maxima are mathematically tractable
- For example, take 3 years of data, and divide it into months
- Take the greatest loss in each month (36 data points)
- What is the distribution of this series?

Distribution of maxima

- For any sample from any distribution, the maxima are asymptotically distributed as one of the following, where α is the tail parameter or tail index.
  - Heavy tails: the Fréchet distribution

    $x \leq 0$, $G(x) = 0$

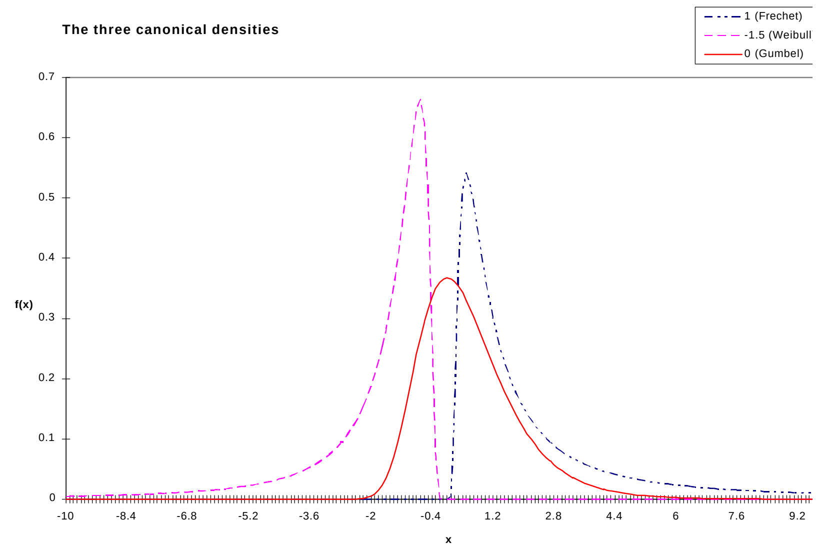
    $x > 0$, $G(x) = \exp(-x^{-\alpha})$, some $\alpha > 0$.
  - Short tails: the Weibull distribution

    $x > 0$, $G(x) = 1$

    $x \leq 0$, $G(x) = \exp(-(-x)^{\alpha})$, some $\alpha > 0$.
  - Light tails: the Gumbel distribution

    $-\infty < x < \infty$, $G(x) = \exp(-e^{-\alpha})$.

Distribution of maxima



Generalisation

- We can re-write the distributions, adding location μ and scale σ
  - Fréchet and Weibull distribution

    $G_{\tau;\mu,\sigma}(x) = \exp(-(1 + \tau.)_+^{-1/\tau})$, for $\tau \neq 0$.

    Fréchet if $\tau > 0$, $\mu \leq x < \infty$,

    Weibull if $\tau < 0$, $\mu \leq x \leq \mu - \sigma/\tau$.
  - Gumbel distribution

    $G_{0;\mu,\sigma}(x) = \exp(-e^{-x})$, $-\infty < x < \infty$.
- These are referred to as the 'Generalised Extreme Value Distribution', or GEV
  - $\tau = 1/\alpha$, and is called the shape parameter

Looking at tail behaviour only

- Use the Generalised Pareto Distribution

$$F_{\tau;\mu,\sigma}(x) = 1 - (1 + \tau.\frac{x-\mu}{\sigma})_+^{-1/\tau}.$$

  - if $\tau \geq 0$, then defined on $x \geq \mu$,
  - if $\tau < 0$, then defined on $[\mu, \mu - \beta/\tau]$

- If $\tau > 0$, $x \in [0,\infty)$, then tails are Fréchet as $x \to \infty$.
- If $\tau < 0$, $x \in [0,-\frac{1}{\tau})$, then tails are Weibull as $x \to \frac{1}{\tau}$
  - Weibull tails are zero for $x \geq \frac{1}{\tau}$
- If $\tau = 0$, then tails are Gumbel as $x \to \infty$.

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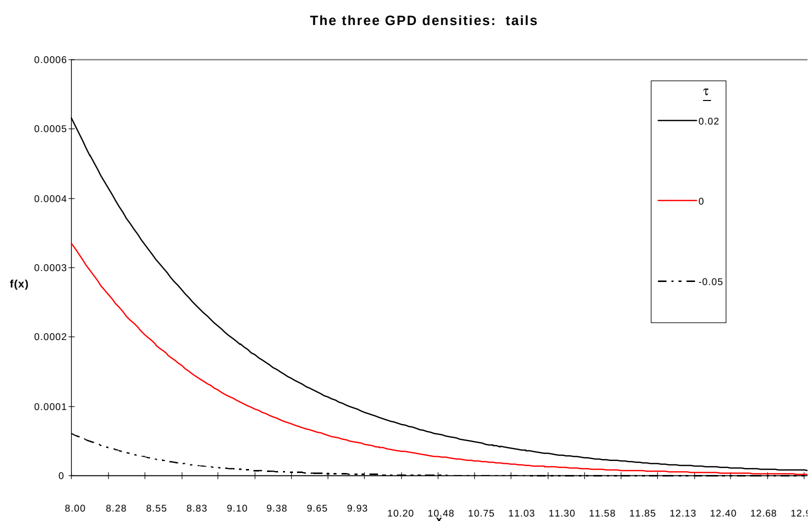
Comparison of tail distributions



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Generality of GPD

- For any large enough threshold, the conditional excess distribution is always a GPD, for some τ, μ, σ.
  - This applies to the whole tail, not just the tail of the distribution of maxima - ideal for data-sparse credit risk
- Thus the GPD is the best distribution to model any extreme tail
  - Can use all available data
- We need only find the three parameters
  - $\tau = 1/\alpha$ shape parameter
  - μ the location parameter
  - σ the scale parameter
- Of these, τ is the most important, as it describes the extent and fatness of the tail

Fitting the tail to the GPD

- Need to use maximum likelihood estimation
- Start by fitting just a few points in the very far tail
- Gradually include more points until estimate of τ stabilises
- Use various techniques
  - Pickand's estimator
  - Hill's estimator
  - MEL estimation

Pickand's estimator

- This is given by

$$\tau_p = \frac{\ln[Y_{(m)} - Y_{(2m)}] - \ln[Y_{(2m)} - Y_{(4m)}]}{\ln 2}$$

- where 0<m<N/4
- It is generally considered to be a poor estimator
  - needs a lot of tail data
  - susceptible to noise

Hill's estimator

- We can write a simplified version of the GPD, combining μ and σ into s.

  $F(x) = 1 - s^{\alpha} x^{-\alpha}$, for s > 0,

- with density

  $f(x) = \alpha s^{\alpha} x^{-\alpha-1}$

- Essentially, this is a general power law tail
- Maximum likelihood estimator of s and a:

  Given a set of observations, $X = \{X_i\}$ for i = 1,...,n,

  we choose s and α to maximise their joint density

  (normal maximum likelihood)

Hill's estimator

In practice maximise $L(X) = \ln \prod_{i=1}^{n} f(X_i)$, ie

$$L(X) = n.\ln \alpha + n\alpha.\ln s - (\alpha+1) \sum_{i=1}^{n} \ln X_i.$$

At a maximum of L(X) must have, eg for α,

$$0 = \frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + n.\ln s - \sum_{i=1}^{n} \ln X_i$$

so that the estimator for $1/\alpha$ is

$$\hat{\tau} = \frac{1}{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \ln X_i - \ln s = \frac{1}{n} \sum_{i=1}^{n} \ln (X_i/s).$$

Choose a threshold $u = X_{(m+1)}$.

Set s = u, and estimate

$$\hat{\tau} = \frac{1}{\alpha} = \frac{1}{m} \sum_{i=1}^{m} \ln X_{(i)} - X_{(m+1)}.$$

Hill's estimator

- This estimator is more robust than Pickand's
- It works for all heavy tailed distributions - it is more general than just the GPD
- Badly affected by autocorrelation
- Accuracy is improved by more data
  - estimate is unsettled at start (m<12), smoothing out later
- Need to judge where the tail begins - ie, what is m?
- In practice, need at least 10 points in tail - or more
- Choice of m can be judgmental or statistical

Uncertainty with Hill's estimator

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- We can place error bars on the value of t obtained with Hill's estimator

Suppose X_i are iid with

$$F(x) = 1 - ax^{-\alpha}[\,1 + bx^{-\beta} + o(x^{-\beta})\,],\ \ \alpha > 0.$$

then

$$E[\,\hat{\tau}\,] = \frac{1}{\alpha} - \frac{1}{\alpha}\left(\frac{1}{a}\right)^{\beta/\alpha}\frac{\beta b}{\alpha+\beta}\left(\frac{m}{n}\right)^{\beta/\alpha}$$

$$\mathrm{var}[\,\hat{\tau}\,] = \frac{1}{m}\frac{1}{\alpha^2}.$$

What is the MEL?

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- The mean excess loss function for a threshold parameter u is

  e(u) = E[Y - u | Y > u]

- For the GPD this has a limiting linear behaviour

$$e(u) \sim \frac{1}{\ }\sigma + \frac{\tau}{\ }u.$$

- If we choose a high value of u, u_c then e(u) is ~ linear for u > u_c
- If Y is standard normal then e(u) ~ 1/u

MEL function

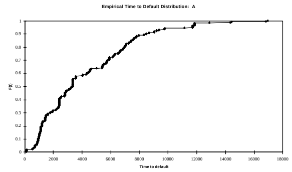
- If $Y_{(i)}$ is the tail data, then

$$e(u) = \frac{1}{N_u} \sum_{i=1}^{N_u} \left(Y_{(i)} - u\right)$$

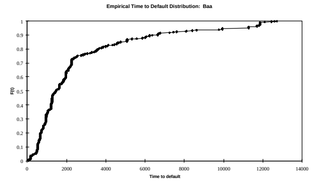
where $Y_{(N_u)} \geq u$ but $Y_{(N_u+1)} < u$.

- N_u is the number of observations greater than u
  - the exceedance number
- But what is u_c? Where does the tail start?

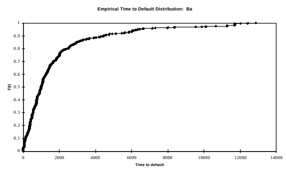
Empirical times to default, A to B



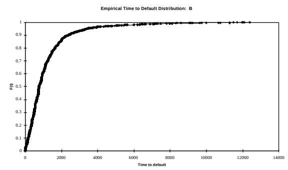
Rated at A



Rated at Baa



Rated at Ba



Rated at B

Empirical times to default, B to Ca

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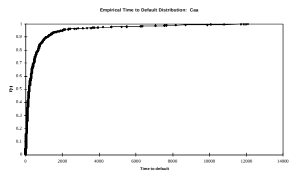
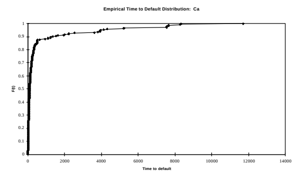


Rated at Caa



Rated at Ca

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Results for τ for different rating classes

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Results for shape parameter τ

Rating Band	*Upper Tail*	*Lower Tail*
A	4	-1 to –2
Baa	2	-0.5
Ba	3	-2
B	4	-1
Caa	1	-1
Ca	0.5	-1

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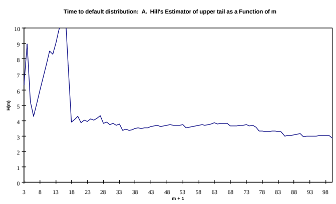
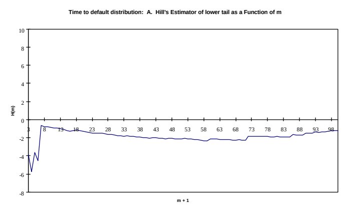
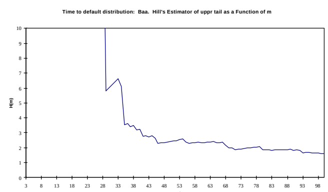
Times to default; tail behaviour



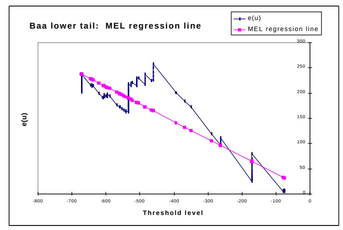
Hill's Estimator, A, upper tail



Hill's Estimator, A, lower tail

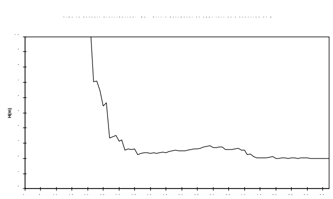


Hill's Estimator, Baa, upper tail

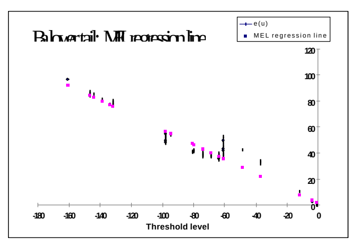


MEL Estimator, Baa, lower tail

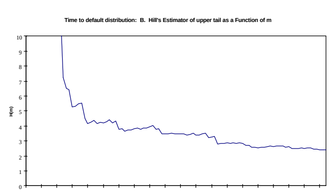
Times to default; tail behaviour



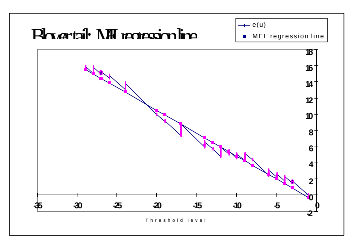
Hill's Estimator, Ba, upper tail



MEL Estimator, Ba, lower tail

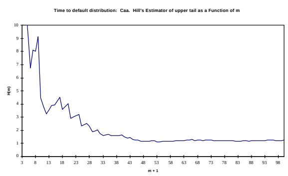


Hill's Estimator, B, upper tail

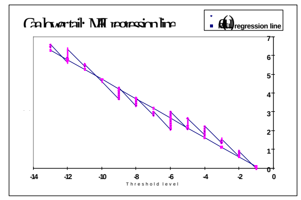


MEL Estimator, B, lower tail

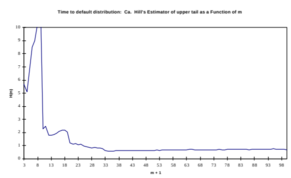
Times to default; tail behaviour



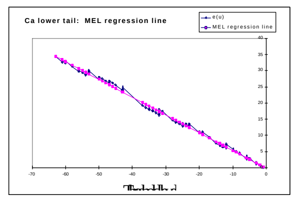
Hill's Estimator, Caa, upper tail



MEL Estimator, Caa, lower tail



Hill's Estimator, Ca, upper tail



Hill's Estimator, Ca, lower tail

Copula techniques and credit risk

Why correlation is inadequate

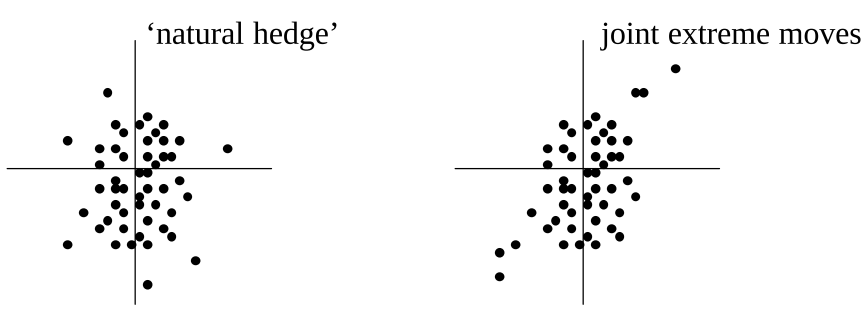
- Ordinary correlation between X and Y defined as

$$\rho(X,Y) = \frac{\operatorname{cov}[X,Y]}{\sqrt{s^2[X]s^2[Y]}}$$

- However, this will only fully capture the dependence between X and Y for a narrow class of X and Y
  - need elliptical relationship, i.e. $X=AY+\mu$, where Y is a spherical function
- Bivariate normal distribution IS elliptical
- ... but fat tailed distributions like credit risk are NOT
- Thus where EVT is used, correlation is useless

Same correlation, different tails

- Many joint distributions have the same correlation
- ...but they will not necessarily have the same copula
- Joint tail behaviour may not be distinguished except by the copula



The copula

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- The copula is a joint distribution function
- For the standard uniform distributions $U_1,\ldots,U_N$,

  $$C(u_1,\ldots,u_N) = \Pr[\, U_1 \le u_1,\ldots,U_N \le u_N \,]$$

  – the copula is therefore an N-dimensional array.

  – if N=2, the copula will be a surface

  – Can be empirical or analytical
- To use with any joint distribution $F(X_1,\ldots,X_N)$, first we need to find the 'marginal distributions' $F_i(X_i)$, which are uniform variates

  $$F_i(X_i)= U_i$$

  – the functions F_i transform the variables X to uniform variates
- Then F is defined by the copula of these marginal distributions

  $$F(X_1,\ldots,X_N) = C(F_1(X_1),\ldots,F_N(X_N))$$

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Examples of copulas

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- X, Y independent

  $$C(u,v) = uv \equiv \Pi(u,v)$$
- X, Y comonotonic, (ie Y = T(X), T increasing)

  $$C(u,v) = \min(u,v) \equiv M(u,v).$$
- X, Y countermonotonic, (Y = T(X), T decreasing)

  $$C(u,v) = \max(u+v-1,0) \equiv W(u,v).$$

- These are special copulas, useful in many applications
- For any copula C, must have $W \le C \le M$

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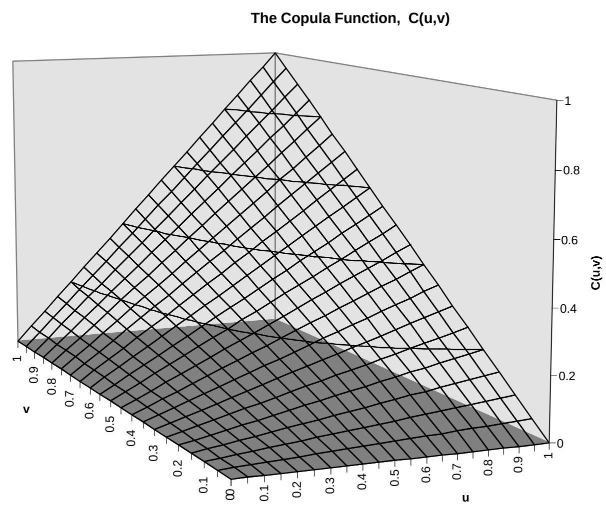
The copula Π (u,v)



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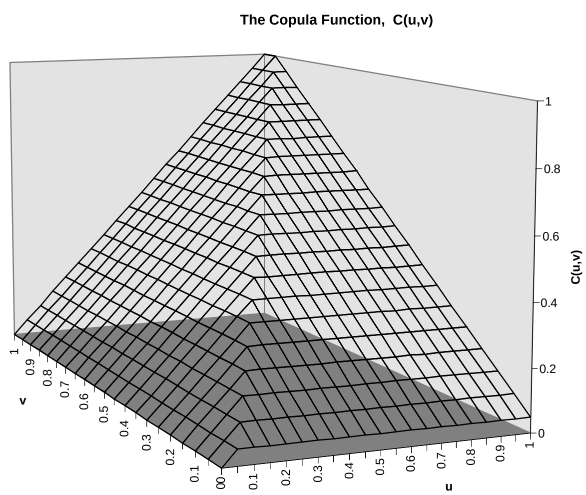
The copula M(u,v)



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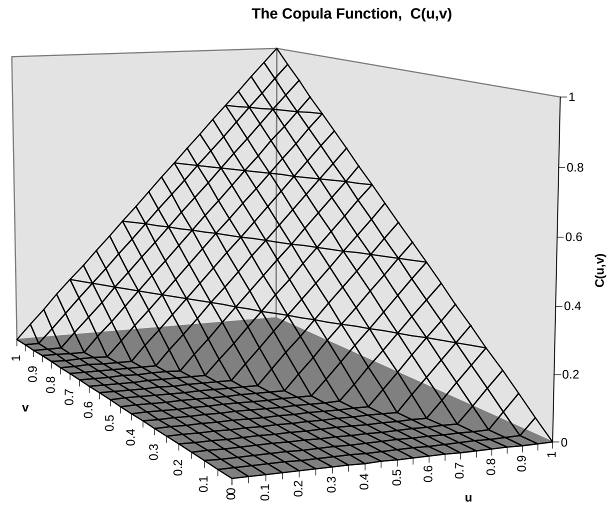
The copula W(u,v)



Archimedean copulas

- These are a particular branch of the copula family, where the copulas depend upon one single-parameter function
- The function is φ, where $\varphi : [0,1] \rightarrow [0,\infty)$ with $\varphi(1) = 0$,
- φ is convex, continuous, strictly decreasing
- Then C_φ is a copula where

$$C_\varphi(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v))$$

- Archimedean copulas are fitted to our data as they are the easiest to calibrate to the band structure imposed by credit ratings

Examples of Archimedean copulas

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- $\varphi(t)$ $= (1-t)^{\theta}$ $\theta \geq 1$

  $= (t^{-\theta} - 1)/\theta$ $0 \neq \theta \geq -1$ (Pareto)

  $= \ln((1-\theta(1-t))/t)$ $\theta \in [-1,1)$ (Ali-Mikhail-Haq)

  $= (-\ln t)^{\theta}$ $\theta \geq 1$ (Gumbel-Hougaard)

  $= \ln \frac{e^{-qt} - 1}{e^{-q} - 1}$ $\theta \neq 0$ (Frank)

  $= \ln(1 - \theta \ln t)$ $\theta \in (0,1]$ (Gumbel-Barnett)

  $= \frac{1-t}{1+(q-1)t}$ $\theta \geq 1$

  $= \exp(\theta/(t-1))$ $\theta \geq 2$

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Calibrating the Ait-Mikhail-Haq copula to the data

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- The two bond ‘cohorts’ whose dependency we analyse are the B and CAA
- We use data from 1981 - 2000, as there is a shift in behaviour in 1981
- We find the empirical B-Caa copula for monthly changes in this period
- We calibrate the Ait-Mikhail-Haq copula to this empirical copula
- The copula in fact resembles the independent copula, indicating that there is little dependence between B and Caa

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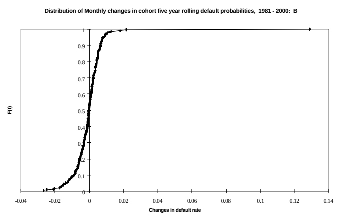
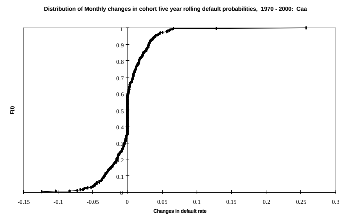
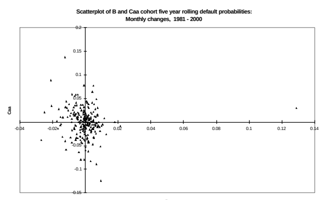
Default Dependency: B-Caa copula

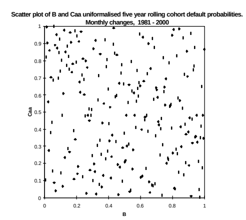
Monthly changes in five year default rates, 1981 - 2000





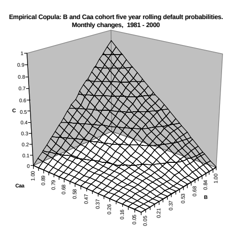
Marginal distribution of changes: B **Marginal distribution of changes: Caa**



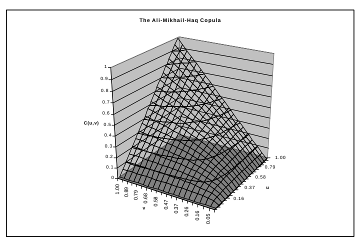


Scatterplot: B - Caa **Uniformalised scatter plot: B - Caa**

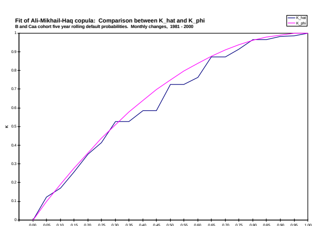
Fitting the copula



The empirical copula: B - Caa



The optimal Ait-Mikhail-Haq Copula



Empirical and best fit levels function

A Credit Model

Existing models

- Two main approaches:
- 'Reduced form models'
  - Exogenous default
  - Default is usually a stochastic process.
  - No attempt made to 'explain' default
  - examples are Duffie and Singleton, Jarrow, Lando and Turnbull, Duffie and Huang, Madan and Unal
- 'Structural' models
  - Default determined within the model
  - eg, default occurs when a state variable like 'firm value' hits a barrier
  - Try to 'explain' default
  - examples are the original Merton model, Zhou, Longstaff and Schwartz, Nielsen, Saa-Requejo and Santa-Clara

Necessary model attributes

- All credit models must include a survival function and a hazard rate
- Survival function s(t) is the probability of the bond surviving beyond time t
- Hazard rate h(t) is the instantaneous probability of default

$$s(t) = \exp\left(-\int_0^t h(t')dt' \right)$$

Hazard function

- It is necessary for the hazard function to be positive
- Different authors have made different assumptions about its form
  - Linear function of time — Li
  - Deterministic function of time obtained from credit spread curve — Li
  - Stochastic process — Several
  - Mean reverting stochastic process — Several
  - CIR process (has explicit solutions for survival times) — Finger
  - General affine process — Duffie & Singleton
  - Jump extension — Duffie & Garleanu
- There has as yet been little work on empirical determination of the hazard function

Desirable model attributes

- Ability to use agency ratings
  - … while recognising that they influence credit quality
  - … and also that they may lag events
- Ability to use internal ratings
  - which may be more accurate
- Incorporation of good tail estimation techniques
  - due to infrequent defaults and sparse data
- Good modelling of dependency
  - little work done in this area to date
  - copula techniques are the obvious candidate
- Possible extension to credit derivative valuation

Our model

- We combine the ratings approach of the Jarrow, Lando and Turnbull model with the flexibility of the Duffie and Singleton method
- Two state variables: credit quality q_t and interest rate risk r_t
- Ratings impose 'bands' on underlying credit quality
  - flexible enough to be easily implemented from internal systems
- Default probability λ_t is a deterministic function of q_t

$$\lambda_t(t=0) = 0$$

$$\lambda_t(t=\infty) = 1$$

Credit quality q_t

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- Unique to our model is the state variable credit quality q_t
  - q_t is what both agency ratings and internal ratings attempt to measure
- q_t goes from 0 (riskless) to 1 (default) but is only directly observable at default
  - otherwise only the bands (ratings) are observed
- q_t has a jump component to allow it to jump directly to 1 at any point

Recovery amount

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- This is important and very variable
- May depend on
  - Actual loss
  - seniority of debt
- Different models make different assumptions
  - Duffie and Singleton assume proportionate loss in market value
  - Jarrow, Lando, Turnbull assume a constant recovery rate as a proportion of face value
- We make the simple assumption that instruments pay off a constant bullet amount
  - this is because we are interested in dependency of defaults. Refinements may include more realistic recovery assumptions

Using Moodys ratings

- We define credit and default events as per the Moodys Corporate Bond Default database
  - Records dating from 1920 (most of our results are based on time series from 1970 onwards)
  - 15,000 issuers
  - 80,000 bonds
  - over 250,000 rating events
- We use the broad rating bands {Aaa, Aa, A, Baa, Ba, B, Caa, Ca, C} plus default
  - We ignore the narrower rating bands Baa1, Baa2 etc as changes between these are rarely considered to be credit or default events
- Most common 'default event' is a missed interest rate payment
- We use bond rating rather than issuer rating

Re-ratings in the model

- We assume that agency ratings accurately determine the credit band to which the bond currently belongs
- In the time between re-ratings, the credit quality is stochastic, but mean reverting about the centre of the last credit rating band
  - It may wander outside the band
- At a re-rating, the credit quality is reset to be at the centre of the band within which it is currently residing
  - Thus a re-rating may re-state the current rating, or announce a new one.
  - Satisfies 'lag' feature of ratings
  - Satisfies observation that ratings influence future credit quality

Using internal ratings with our model

- The variable q_t makes this model very flexible for use within existing frameworks
- In effect, we assume that internal systems measure the credit quality q_t, which may then be fed into the model
- External ratings both measure and influence q_t, and may be used for calibration

Calibrating the model to the market

- Two stages:
  - (1) Calibrate a single bond model for each risk class. Obtain the hazard rate function
  - (2) Calibrate the multi-bond model by obtaining the copula function which determines the degree of dependency between each risk class
- The calibration is based upon re-rating data, and therefore does not include a price of risk

Calibrating a single-bond model

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- Extract various data from Moody's database
  - Conditional distribution of re-rating times
  - Conditional time to default distributions
  - and conditional transition probabilities
- Use these to find parameters for process undergone by credit quality q
  - i.e. , for the CIR-type process

$$dq_t = a\,(m - q_t)dt + s\,\sqrt{q_t}\,dz$$

  - find parameters like α and σ.

Finding parameters for q_t process

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$$dq_t = a\,(m - q_t)dt + s\,\sqrt{q_t}\,dz$$

k	0	1	2	3	4	5	6	7	8
Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Ca	C
α_k	86.4	47.5	48.8	49.2	49.1	135.8	91.8	18.8	18.8
σ_k	1.94	1.16	1.04	0.98	1.00	1.00	1.00	1.00	1.00
RMSE x 10^5	25	366	102	115	385	489	705	1096	na

Multi-bond model calibration

- Choose bonds in same risk classes
  - or industry, geographic area, etc
- Obtain marginal distributions
  - We suppose that bonds within each risk class default independently but bonds in different risk classes have a default dependence structure which can be captured by and Archimedean copula
- Find empirical copula
- Estimate Archimedean copula
- Can then use results for various purposes like simulations or credit derivative valuation

Valuation of Credit Derivatives

Credit derivatives and dependent defaults

- What is a credit derivative?
  - An instrument whose payoff is contingent upon a credit history
  - Unique in that the credit quality of the seller may be an explicit factor in the valuation of the credit derivative
  - thus correlated default may be very important
  - Also many credit derivatives pay off conditional upon one or more defaults (basket products)
  - simultaneity of defaults becomes very important here
- First stage in process is to calibrate q_t process, finding parameters from default and re-rating data

Calculation of risky term structures

- First we calculate risky term structures for bonds in each rating band
- $D_t(T|r_t,q_t,k)$ is the price in the market at time t of a risky pure discount bond with maturity T in state (r_t,q_t,k).
- We assume that rates are constant over time, so $r_t = r = const$
- Then we have

$$D_t = e^{-r(T-t)}E[H_t]$$

$$r_t^k(T \mid q_t) = -\frac{1}{T-t}\ln D_t(T \mid q_t, k)$$

  - where $H_t = 1$ if the bond defaults prior to maturity, 1 otherwise. These rates define the risky term structure in rating band k. Since the riskless rate is constant, they represent a credit spread over the riskless rate

Risky term structures - results

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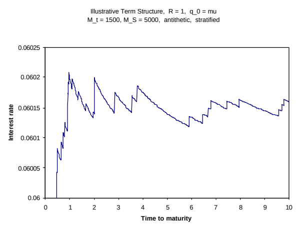
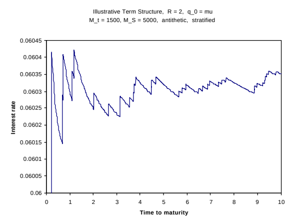
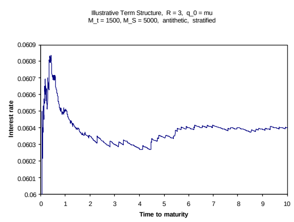


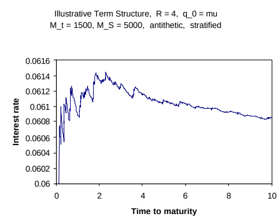
Aa term structure



A term structure



Baa term structure



Ba term structure

Risky term structures - results

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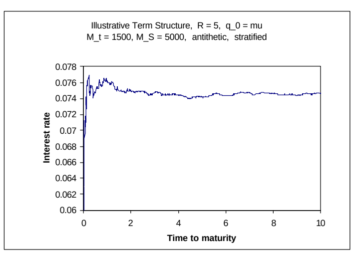
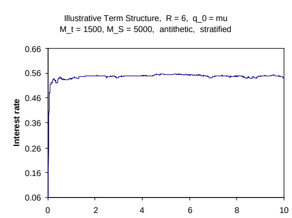
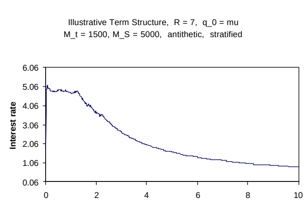


B term structure



Caa term structure



Ca term structure

Valuing a credit derivative within the model

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- We can value a credit derivative in a simplified version of the multi-bond model
- Select a second-to-default instrument
  - pays off 1 if each of 2 bonds defaults prior to a final maturity time
  - payoff received at time of second default
- Each individual bond follows a single bond model in a single rating band
  - Bond 2 is riskier,but with a better initial credit quality
  - Riskless rate = 6%
  - each bond matures at time 1
  - final maturity of the derivative is at time 1

Valuing a credit derivative within the model

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- We value the derivative when the default dependency is described by
  - a Gaussian copula
  - a Student copula with 4 degrees of freedom
  - A Student copula with 8 degrees of freedom
- We use a Monte Carlo method with antithetic variates
- Each of the copulas has a free parameter which allows the correlation ρ to vary
- For a Gaussian copula the value increases with ρ
- In the Student copulas the value of the derivative seems greatest when the correlation is close to zero

Credit derivative valuation results

	μ_0	α_0	σ_0	λ^D	q_0	Value	std err	YTM
Bond 1	0.6	10	1	0.02	0.5	0.9147	0.0002	8.91%
Bond 2	0.7	10	1	0.03	0.4	0.8843	0.0002	12.3

ρ		-1	-0.5	0	0.5	1
Gaussian	Value se	0.00176 0.0001	0.00161 0.0001	0.00178 0.0001	0.00171 0.0001	0.00183 0.0001
Student. 4 d.o.f.	Value se	0.00137 0.0002	0.00196 0.0003	0.00179 0.0002	0.00207 0.0002	0.00128 0.0002
Student, 8 d.o.f.	Value se	0.00179 0.0002	0.00189 0.0002	0.00176 0.0002	0.00188 0.0002	0.00175 0.0002

Conclusion

- Credit risk models are in general more theoretical than practical, with little empirical work
- Tail dependency in particular has historically been neglected, yet is particularly critical for assessment of portfolio default risk
- We present a model which utilises the Moodys rating database
- We introduce the useful parameter credit quality (q_t), to make for easy implementation with existing internal credit systems
- We have analysed the data using the most appropriate EVT and copula techniques
- We have valued a simple credit derivative with an easily extendable method