Dependent jump processes with coupled Lévy measures

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ABSTRACT

I present a simple method for the modeling and simulation of dependent positive jump processes through a series representation. Each constituent process is represented by a series whose terms are equal to a transformation of the jump times of a standard Poisson process. The transformations are given by the inverses of the respective marginal Lévy tail mass integral functions. The dependence between the various constituent processes is given by a probabilistic copula for the inter-arrival times of the various standard Poisson processes.

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1 INTRODUCTION

If Y is a random variable with distribution function F then $F(Y)$ is uniformly distributed on $[0,1]$. Reciprocally, if V is uniformly distributed on $[0,1]$ then $F^{-1}(V)$ has distribution function F. The counterpart result for Lévy measures states: if $(X_t)_{(t\geq 0)}$ is a jump process with the tail mass of its Lévy measure given by the function U then $\{U(\Delta X_k)\}_{(k\geq 1)}$ are distributed as jump times of a standard Poisson process, where $\{\Delta X_k\}$ denotes a sequence of jumps of the process ordered by decreasing magnitude. Reciprocally, if $\{\Gamma_k\}$ is a sequence of jump times of a standard Poisson process then $\{U^{-1}(\Gamma_k)\}$ is equal in distribution to a sequence of ordered jumps of the process (X_t). Since jump times of a standard Poisson process are uniformly distributed across time, it means that Lévy measure integrals are "uniformly distributed on $[0,\infty)$" as probability-integrals are uniformly distributed on $[0,1]$.

A multidimensional Lévy measure can be constructed by linking marginal Lévy measures through a Lévy copula. A Lévy measure $\nu(B)$, for $B \in \mathfrak{B}(\mathbb{R}^d)$ is the expected number per unit time of joint jumps whose sizes belong to B. Any probabilistic (ordinary) copula C is a joint distribution function of standard uniform random variables: $C(v_1,\dots,v_n) = \mathbb{P}[V_1 \leq v_1,\dots,V_n \leq v_n]$. Similarly, any Lévy copula $C_L(x_1,\dots,x_d)$ is the expected number of jumps by a vector of standard Poisson processes whose times of arrival occur jointly before $x_1,\dots,x_d$:

$$C_L(x_1,\dots,x_d) = \mathbb{E}[\#\{k : \Gamma_k^1 \leq x_1,\dots,\Gamma_k^d \leq x_d\}] \tag{1.1}$$

Consequently, simulating paths of a multidimensional jump process when the dependence is specified through a Lévy copula is fundamentally related to the simulation of standard Poisson processes. The jump times of the Poisson processes are dependent in such a way as to satisfy the property in equation (1.1) for all $(x_1,\dots,x_d)$. There are two equivalent methods to achieve this requirement: either through the joint distribution of the jump times or through the joint distribution of successive inter-arrival times. We can start with a Lévy copula and derive the implied distributions as in Tankov (2003a). El-Bachir (2008) discusses a conditional sampling technique suitable for this approach. Alternatively, this paper shows that we can construct new Lévy copulas by directly specifying these distributions.

A *positive pure jump* Lévy process $(X_t)_{(t>0)}$ has stationary and independent positive jumps and is of finite variation, i.e. $\lim_{\Delta t_k\to 0} \sum_{t_k\leq t} |X_{t_{k+1}} - X_{t_k}| < \infty$ where $0 = t_1 < t_2 < \cdots < t_n = t$ and $\Delta t_k = t_{k+1} - t_k$. The distribution of X_t for any time $t > 0$ is infinitely divisible and its characteristic function satisfies the Lévy-Khintchine formula:

$$\mathbb{E}\left[e^{i\langle z,X_t\rangle}\right] = e^{-t\Psi(z)},\ z \in \mathbb{R}^d \tag{1.2}$$

$$\Psi(z) = \int_{\mathbb{R}^d} \left(1 - e^{i\langle z,x\rangle}\right) \nu(dx) \tag{1.3}$$

where ν is a measure on $\mathbb{R}^d\backslash\{0\}$ such that $\int(1 \wedge |x|)\nu(dx) < \infty$.

The tail mass of the Lévy measure is defined as $U(x_1,\dots,x_d) = \nu([x_1,\infty) \times \cdots \times [x_d,\infty))$ for $(x_1,\dots,x_d) \in \mathbb{R}^d\backslash\{0\}$ and such that $U(x_1,\dots,x_d) = 0$ if $x_j = \infty$ for some $j \in 1,\dots,d$ and U is finite everywhere except at zero, $U(0,\dots,0) = \infty$. The marginal Lévy measures have as their tail masses the margins $U_k(x_k) = U(0,\dots,0,x_k,0,\dots,0)$ of the multidimensional Lévy measure's tail mass. In this paper, I only consider continuous tail mass integrals.

Tankov (2003a) defines a $d-$dimensional Lévy copula as a $d-$increasing grounded function $F : [0,\infty]^d \to [0,\infty]$ with margins $F_k, k = 1\dots d$, which satisfy $F_k(u) = u, \forall u \in [0,\infty]$. He also extends Sklar (1959)'s theorem showing that any $d-$dimensional tail mass U can be constructed as a Lévy copula taking the margins of U as arguments. Conversely, if F is a Lévy copula and $U_1, \dots, U_d$ are one-dimensional tail masses, then $U(x_1, \dots, x_d) = F(U_1(x_1), \dots, U_d(x_d))$ defines a $d-$dimensional tail mass.

While Tankov (2003a) focuses the analysis on Lévy processes with positive jumps only, Kallsen and Tankov (2006) extends it to general Lévy processes with both positive and negative jumps. Lévy copulas for processes with both positive and negative jumps can be constructed from positive Lévy copulas but are less tractable. Fortunately, processes with jumps of the same sign are already a rich enough class for various applications. For the sake of clarity I focus on the case of processes with positive jumps only, a.k.a. subordinators.

2 Standard Poisson representation for Lévy processes

Ferguson and Klass (1972) shows that the ordered jump magnitudes $\Delta X_1, \Delta X_2, \dots$ of a Lévy jump process (X_t) have the same distribution as $U^{-1}(\Gamma_1), U^{-1}(\Gamma_2), \dots$, where $\{\Gamma_k\}$ denotes the jump times of a standard Poisson process. This justifies the series representation of the process (X_t) on the unit time interval $t \in [0,1]$:

$$X_t = \sum_{i=1}^{\infty} U^{-1}(\Gamma_i) \mathbf{1}_{\{V_i \in [0,t]\}} \tag{2.1}$$

where $\{V_i\}$ is a sequence of i.i.d. random variables uniformly distributed on $[0,1]$[1]. The proof consists in showing that the series converges for each t and the resulting process is a Lévy process with the same characteristic exponent as X_t. The same result follows as the converse of the direct statement in theorem 2.1, where it is shown that the distribution of $U(\Delta X_1), U(\Delta X_2), \dots$ is the same as the distribution of $\Gamma_1, \Gamma_2, \dots$. The alternative proof[2] presented here has the advantage of making explicit the link with the corresponding theorem for distribution functions of random variables.

Theorem 2.1. *Let $(X_t)_{(t\in[0,1])}$ be a one-dimensional Lévy process on the unit time interval with Lévy measure density ν. Denote the tail mass function of ν by $U(x) := \int_x^\infty \nu(y)dy$ with the inverse function $U^{-1}(x) = \inf\{u > 0 : U(u) \le x\}$.*

If $\{\Delta X_i\}_{(i\ge1)}$ is a sequence of ordered jumps (magnitudes) of (X_t) along a sample path, then $\{U(\Delta X_i)\}_{(i\ge1)}$ are distributed as the jump times of a standard Poisson process.

Conversely, if $\{\Gamma_i\}_{(i\ge1)}$ is a sequence of jump times of a standard Poisson process, then $\{U^{-1}(\Gamma_i)\}_{(i\ge1)}$ have the same distribution as the ordered jumps $\{\Delta X_i\}$ of the process (X_t).

[1] We restrict the discussion to processes on the unit time interval for simplicity but the case of an arbitrary finite time interval $[0,T]$ can be handled by simply replacing the sequence of $\{\Gamma_i\}$ by $\{\frac{\Gamma_i}{T}\}$ and the sequence $\{V_i\}$ of i.i.d. uniforms on $[0,1]$ by a sequence $\{TV_i\}$ of i.i.d. uniforms on $[0,T]$.

[2] The proof in Fergusson and Klass (1972) is more involved but more general as it includes the case of infinite variation and processes with fixed points of discontinuity.

Proof. Let $(X_t^\varepsilon)_{(t\geq 0)}$ denote the compound Poisson approximation of the process (X_t) obtained by truncating jumps that are smaller than ε:

$$X_t^\varepsilon = \sum_{i=1}^{N_t} \xi_i$$

where $(N_t)_{(t\geq 0)}$ is a Poisson process with intensity $\theta = U(\varepsilon)$, and $(\xi_i)_{(i\geq 1)}$ an i.i.d. sequence with distribution $\frac{\nu(x)}{\theta}\mathbf{1}_{\{x>\varepsilon\}}$.

The process (X_t^ε) is a Lévy process with characteristic exponent $\Psi_\varepsilon(z) = \int_\varepsilon^\infty (1-e^{izx})\nu(dx)$. Furthermore, $\lim_{\varepsilon\downarrow 0}\Psi_\varepsilon(z) = \Psi(z)$, where $\Psi(z)$ represents the characteristic exponent of the Lévy process (X_t). Hence, as $\varepsilon \to 0$ we have the convergence in law: $X_t^\varepsilon \to X_t$ by Lévy's continuity theorem.

Since $\frac{U(x)}{\theta}$ represents the cumulative distribution function of the jump magnitudes of X_t^ε, then $\frac{U(\Delta X_i^\varepsilon)}{\theta}$ is distributed as a uniform random variable on $[0,1]$. Hence, $\{U(\Delta X_i^\varepsilon)\}_{(i\geq 1)}$ is a sequence of i.i.d. uniforms on $[0,\theta]$. An ordered version of this sequence has the same distribution as a sequence of jump times of a standard Poisson process on the interval $[0,\theta]$.

When $\varepsilon \to 0$, we have $\theta \to \infty$ and the convergence in law: $X_t^\varepsilon \to X_t$. By continuity of U, we can deduce the convergence in law: $U(\Delta X_i^\varepsilon) \to U(\Delta X_i)$ and conclude that $\{U(\Delta X_i)\}$ are distributed as the jump times of a standard Poisson process.

Using the fact that $U^{-1}(U(x)) = x$, the converse statement is deduced from:

$$\mathbb{P}[U^{-1}(\Gamma_i) \leq x] = \mathbb{P}[U^{-1}(U(\Delta X_i)) \leq x] = \mathbb{P}[\Delta X_i \leq x]$$

□

For any probabilistic (ordinary) copula C, $C(v_1,\dots,v_n)$ is the probability that the components of a vector of standard uniform random variables are jointly smaller that $v_1,\dots,v_n$: $C(v_1,\dots,v_n) = \mathbb{P}[V_1 \leq v_1,\dots,V_n \leq v_n]$. Theorem 2.2 states a similar result for Lévy copulas where the probability is replaced by the expected number of vectors, and the standard uniforms are replaced by jump times of standard Poisson processes, i.e. "uniforms on $[0,\infty]$". For any Lévy copula F, $F(x_1,\dots,x_d)$ is the expected number of vectors of jump times of standard Poisson processes whose components are jointly smaller than $x_1,\dots,x_d$: $F(x_1,\dots,x_d) = \mathbb{E}[\#\{k : \Gamma_k^1 \leq x_1,\dots,\Gamma_k^d \leq x_d\}]$.

Theorem 2.2. *If F is a d-dimensional Lévy copula, then for all $x_1,\dots,x_d \in [0,\infty)$, $F(x_1,\dots,x_d)$ is the expected number k of jump times $\Gamma_k^1,\dots,\Gamma_k^d$ of a d-dimensional vector of standard Poisson processes occurring before $x_1,\dots,x_d$ respectively:*

$$F(x_1,\dots,x_d) = \mathbb{E}[\#\{k : \Gamma_k^1 \leq x_1,\dots,\Gamma_k^d \leq x_d\}] \tag{2.2}$$

Furthermore, for all $k \geq 1$ the joint distribution of the vector $(\Gamma_k^1,\dots,\Gamma_k^d)$ is:

$$\mathbb{P}\left[\Gamma_k^1 \leq y_1,\dots,\Gamma_k^d \leq y_d\right] = \int_0^{y_1} e^{-y}\frac{y^{k-1}}{(k-1)!}\partial_x F(x,y_2,\dots,y_d)|_{x=y}dy \tag{2.3}$$

Equivalently, for all $k \geq 1$ the conditional joint distribution of the vector of inter-arrival times $(\tau_k^1, \ldots, \tau_k^d) := (\Gamma_k^1 - \Gamma_{k-1}^1, \ldots, \Gamma_k^d - \Gamma_{k-1}^d)$ given the previous jump times $\mathbf{\Gamma_{k-1}} := (\Gamma_{k-1}^1, \ldots, \Gamma_{k-1}^d)$ is given by:

$$\mathbb{P}\left[\tau_k^1 \leq z_1, \ldots, \tau_k^d \leq z_d / \mathbf{\Gamma_{k-1}}\right] = \int_0^{z_1} e^{-y} \partial_x F(x + \Gamma_{k-1}^1, z_2 + \Gamma_{k-1}^2, \ldots, z_d + \Gamma_{k-1}^d)|_{x=y} dy \qquad (2.4)$$

Proof. For some one-dimensional Lévy tail mass function U_i, we have that $U_i\left(U_i^{-1}(x)\right) = x$. Thus, $F(x_1, \ldots, x_d) = F\left(U_1\left(U_1^{-1}(x_1)\right), \ldots, U_d\left(U_d^{-1}(x_d)\right)\right)$. Since $U_1, \ldots, U_d$ are marginal tail mass functions and F is a Lévy copula, Tankov's extension to Sklar's theorem implies the existence of a process (Y_t) with d-dimensional Lévy tail mass U such that $U(x_1, \ldots, x_d) = F(U_1(x_1), \ldots, U_d(x_d))$. Hence, $F(x_1, \ldots, x_d) = U\left(U_1^{-1}(x_1), \ldots, U_d^{-1}(x_d)\right)$. Using the definition of a Lévy measure, we can then deduce:

$$F(x_1, \ldots, x_d) = \mathbb{E}\left[\#\{t \in [0,1] : \Delta Y_t^1 \in [U_1^{-1}(x_1), \infty), \ldots, \Delta Y_t^d \in [U_d^{-1}(x_d), \infty)\}\right]$$

Note the equivalence between the events $\{\Delta Y_t^i \in [U_i^{-1}(x_i), \infty)\}$ and $\{U_i\left(\Delta Y_t^i\right) \in (0, x_i]\}$. We can now use the result of theorem 2.1 to conclude that:

$$F(x_1, \ldots, x_d) = \mathbb{E}[\#\{k : \Gamma_k^1 \leq x_1, \ldots, \Gamma_k^d \leq x_d\}]$$

where $\{\Gamma_k^1\}, \ldots, \{\Gamma_k^d\}$ are sequences of standard Poisson jump times.

Denote the conditional joint distribution of $\Gamma_k^2, \ldots, \Gamma_k^d$ conditional on $\Gamma_k^1 = x_1$ by P_{x_1}:

$$P_{x_1}(x_2, \ldots, x_d) := \mathbb{P}\left[\Gamma_k^2 \leq x_2, \ldots, \Gamma_k^d \leq x_d / \Gamma_k^1 = x_1\right]$$

Since $\Gamma_k^1, \ldots, \Gamma_k^d$ have the same distribution as $U_1(\Delta Y_k^1), \ldots, U_d(\Delta Y_k^d)$ where $\{\Delta Y_k^i\}$ denotes the sequence of jumps of Y^i ordered by decreasing magnitudes, we can write:

$$P_{x_1}(x_2, \ldots, x_d) = \mathbb{P}\left[U_2(\Delta Y_k^2) \leq x_2, \ldots, U_d(\Delta Y_k^d) \leq x_d / U_1(\Delta Y_k^1) = x_1\right]$$

This last conditional distribution function has been shown in Tankov (2003b) to satisfy:

$$\mathbb{P}\left[U_1(\Delta Y_k^2) \leq x_2, \ldots, U_d(\Delta Y_k^d) \leq x_d / U_1(\Delta Y_k^1) = x_1\right] = \partial_x F(x, x_2 \ldots, x_d)|_{x=x_1}$$

We can now deduce:

$$\mathbb{P}\left[\Gamma_k^1 \leq y_1, \ldots, \Gamma_k^d \leq y_d\right] = \int_0^{y_1} P_x(y_2, \ldots, y_d) f_{\Gamma_k^1}(x) dx$$

and replace the probability density $f_{\Gamma_k^1}$ of Γ_k^1 by its expression to obtain equation (2.3).

To deduce equation (2.4), note that:

$$\mathbb{P}\left[\tau_k^1 \leq z_1, \ldots, \tau_k^d \leq z_d / \mathbf{\Gamma_{k-1}}\right] = \mathbb{P}\left[\Gamma_k^1 \leq z_1 + \Gamma_{k-1}^1, \ldots, \Gamma_k^d \leq z_d + \Gamma_{k-1}^d\right]$$

and $\tau_k^1 = \Gamma_k^1 - \Gamma_{k-1}^1$ is a standard exponential random variable. □

3 EQUIVALENCE BETWEEN THE LÉVY COPULA AND AN ORDINARY COPULA

Simulating paths of a multidimensional jump process is therefore related to the simulation of standard Poisson processes. The dependence between the components is given by the joint distribution of their jump times in equation (2.3) or equivalently by the conditional joint distribution of their inter-arrival times in equation (2.4).

We can start with a Lévy copula and derive the implied distributions. This is the procedure followed by Tankov (2003a) generalizing the series representation for multidimensional Lévy processes:

$$X_t^k = \sum_{i=1}^{\infty} U_k^{-1}(\Gamma_i^k)\mathbf{1}_{\{V_i \in [0,t]\}},\ k = 1,\dots,d,\ t \in [0,1] \tag{3.1}$$

where $\{\Gamma_i^1\},\dots,\{\Gamma_i^d\}$ are d random sequences independent from $\{V_i\}$. The sequence $\{\Gamma_i^1\}$ represent the jump times of a standard Poisson process, while the vector $(\Gamma_i^2,\dots,\Gamma_i^d)$ conditionally on Γ_i^1 has distribution function $\partial_{x_1}F(x_1,\dots,x_d)|_{x_1=\Gamma_i^1}$.

Alternatively, we can construct new Lévy copulas by directly specifying the distributions in equation (2.3) or equation (2.4). Since the distribution function of exponential random variables and its inverse are more tractable than those of jump times of Poisson processes, working with the distribution of inter-arrival times is preferred and allows the use of probabilistic copulas to model the dependence between the inter-arrival times:

$$\begin{aligned}\mathbb{P}\left[\Gamma_k^1 \le x_1,\dots,\Gamma_k^d \le x_d / \mathbf{\Gamma_{k-1}}\right] &= \mathbb{P}\left[\tau_k^1 \le x_1 - \Gamma_{k-1}^1,\dots,\tau_k^d \le x_d - \Gamma_{k-1}^d\right] \\ &= C\left(\mathbb{P}[\tau_k^1 \le x_1 - \Gamma_{k-1}^1],\dots,\mathbb{P}[\tau_k^d \le x_d - \Gamma_{k-1}^d]\right)\end{aligned} \tag{3.2}$$

where C denotes some suitable probabilistic copula whose existence is guaranteed by Sklar's theorem.

The sequences $\{\Gamma_k^1\},\dots,\{\Gamma_k^d\}$ are jump times of standard Poisson processes, thus the inter-arrival times of a given sequence are i.i.d. standard exponential random variables. Intuitively, dependence between the various sequences necessarily translates into dependence between the inter-arrival exponential random variables of the different Poisson processes. This is formalized in theorem 3.1, where the equivalence between a Lévy copula and a probabilistic copula for inter-arrival times of the standard Poisson processes is proved.

Theorem 3.1. *Let $(X_t) := (X_t^1,\dots,X_t^d)$ be a $d-$dimensional Lévy jump process with marginal Lévy tail mass functions $U_1,\dots,U_d$. The process (X_t) has a unique Lévy copula $F:\ [0,\infty]^d \to [0,\infty]$ if and only if there exists a unique probabilistic copula $C:\ [0,1]^d \to [0,1]$ such that for all i:*

$$\mathbb{P}\left[U_1(\Delta X_i^1) \le x_1,\dots,U_d(\Delta X_i^d) \le x_d / \Delta X_{i-1}^1,\dots,\Delta X_{i-1}^d\right] = C\left(1 - e^{-(x_1 - U_1(\Delta X_{i-1}^1))},\dots,1 - e^{-(x_d - U_d(\Delta X_{i-1}^d))}\right) \tag{3.3}$$

where $\Delta X_1^j,\dots,\Delta X_i^j,\dots$ denote the ordered jumps of the process (X_t^j).

Proof. Let $\mathbb{P}_{i-1}$ denote the probability conditional on the information $\Delta X_{i-1}^1,\dots,\Delta X_{i-1}^d$. If F is a

Lévy copula, then theorem 2.2 states that

$$\begin{aligned}\mathbb{P}_{i-1}\left[U_1(\Delta X_i^1)\le x_1,\dots,U_d(\Delta X_i^d)\le x_d\right] &= \mathbb{P}\left[\Gamma_i^1\le x_1,\dots,\Gamma_i^d\le x_d/\mathbf{\Gamma_{i-1}}\right]\\ &= \mathbb{P}\left[\tau_i^1\le x_1-\Gamma_{i-1}^1,\dots,\tau_i^d\le x_d-\Gamma_{i-1}^d\right]\end{aligned}$$

where $\{\Gamma_k^1\},\dots,\{\Gamma_k^d\}$ are sequences of jumps times of standard Poisson processes whose dependence is uniquely determined by the Lévy copula such that equation (2.4) is satisfied. Since $\mathbb{P}[\tau_i^j \le x_j-\Gamma_{i-1}^j] = 1-e^{-(x_j-\Gamma_{i-1}^j)}$, Sklar's theorem implies the existence of a unique copula C such that equation (3.3) holds.

Conversely, if $(X_t^1,\dots,X_t^d)$ is a d-dimensional jump process such that equation (3.3) is satisfied, then theorem 2.1 guarantees that for all $j=1,\dots,d$, the distribution of $U_j(\Delta X_1^j),U_j(\Delta X_2^j),\dots$ is the same as that of the jump times $\Gamma_1^j,\Gamma_2^j,\dots$ of a standard Poisson process. Let the joint transition probability from $\Gamma_{k-1}^1,\dots,\Gamma_{k-1}^d$ to $\Gamma_k^1,\dots,\Gamma_k^d$ be given by:

$$\mathbb{P}\left[\Gamma_k^1\le x_1,\dots,\Gamma_k^d\le x_d/\mathbf{\Gamma_{k-1}}\right] = C\left(1-e^{-(x_1-\Gamma_{k-1}^1)},\dots,1-e^{-(x_d-\Gamma_{k-1}^d)}\right)$$

We can deduce by convolution and using Sklar's theorem that for all $k\ge 1$ there exists a unique copula C_k such that

$$\mathbb{P}\left[\Gamma_k^1\le x_1,\dots,\Gamma_k^d\le x_d\right] = C_k\left(\mathbb{P}[\Gamma_k^1\le x_1],\dots,\mathbb{P}[\Gamma_k^d\le x_d]\right)$$

Let $F:\ [0,\infty]^d\to[0,\infty]$ be the function:

$$F(x_1,\dots,x_d)=\sum_{k=1}^{\infty} kC_k\left(\mathbb{P}[\Gamma_k^1\le x_1],\dots,\mathbb{P}[\Gamma_k^d\le x_d]\right)$$

We can deduce from the properties of the copulas $C_k,\ k=1,2,\dots$ that F is a grounded, $d-$increasing function with uniform margins. Thus, we can conclude that F is a Lévy copula. □

4 Concluding remarks

The result of theorem 3.1 justifies the use of probabilistic (ordinary) copulas to model the dependence between Lévy jump processes by specifying joint distributions for the increments of marginal tail mass integrals Γ_k^is. This has some advantages in terms of tractability over using Lévy copulas. Indeed, some tractable probabilistic copulas have known efficient algorithms for simulation that avoid conditional sampling. Furthermore, statistical fitting of Lévy copulas requires either computationally expensive simulation-based estimation or deriving a complicated likelihood function for $\Gamma_k^1,\dots,\Gamma_k^d$ by successive conditioning. On the other hand, estimation procedures for probabilistic copulas are readily available.

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