



The (De)merits of Minimum-Variance Hedging: Application to the Crack Spread

Carol Alexander

ICMA Centre, Henley Business School at Reading, UK

Marcel Prokopczuk

ICMA Centre, Henley Business School at Reading, UK

Anannit Sumawong

ICMA Centre, Henley Business School at Reading, UK

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ICMA Centre • The University of Reading
Whiteknights • PO Box 242 • Reading RG6 6BA • UK
Tel: +44 (0)1183 788239 • Fax: +44 (0)1189 314741
Web: www.icmacentre.rdg.ac.uk

Director: Professor John Board, Chair in Finance

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Abstract

We study the empirical performance of the classical minimum-variance hedging strategy, comparing several econometric models for estimating hedge ratios of crude oil, gasoline and heating oil crack spreads. Given the great variability and large jumps in both spot and futures prices, great care is required when processing the relevant data and accounting for the costs of maintaining and re-balancing the hedge position. We find that the variance reduction produced by all models are statistically and economically indistinguishable from the one-for-one “naïve” hedge. However, margin and transaction costs produced by GARCH-based models are excessive. Therefore we encourage hedgers to use a naïve hedging strategy on the crack spread bundles now offered by the exchange as it is the cheapest and easiest to implement. Our conclusion contradicts the majority of the existing literature, which favours the implementation of GARCH-based hedging strategies.

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Corresponding Author:

Anannit Sumawong,
ICMA Centre,
University of Reading,
Reading,
RG6 6BA, UK
Email: a.sumawong@icmacentre.ac.uk

1 Introduction

There exists a substantial literature on minimum-variance hedging of spot positions using futures contracts in which sophisticated econometric models are applied for estimating the hedge ratios. The majority of these studies conclude that advanced econometric tools improve the hedging performance over the naïve hedging strategy of shorting one futures contract per unit of spot exposure. However, most research ignores transactions and margin costs, and/or does not evaluate the improvement in a statistically meaningful way. Even, in some cases, insufficient care is taken to pre-filter the data for use in the analysis. Our contribution is to conduct an extensive out-of-sample study of minimum-variance hedging for a complex underlying position, with meticulous processing of the relevant data. We compare several popular hedging approaches and covariance estimation techniques with the simple naïve hedge, explicitly taking margin and transaction costs into account. In contrast to the majority of extant literature we find that none of the sophisticated methods are able to outperform the naïve hedge.

Minimum-variance hedging has been pioneered by Johnson (1960) and Stein (1961), and further refined by Ederington (1979), Hill and Schneeweis (1982), Figlewski (1984) and Herbst et al. (1989) amongst many others. Since Fama (1965) found that asset covariance structures are time-varying and Bollerslev (1982) introduced the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) method of estimating conditional variance the application of the GARCH family for estimating hedge ratios has been rapidly growing in popularity. Baillie and Myers (1991) first derived hedge ratios using the bivariate GARCH model. Kroner and Sultan (1993) utilise the CCC GARCH model in the foreign-exchange market and Gagnon et al. (1998) expand the study for multi-asset portfolios using the BEKK GARCH model. Haigh and Holt (2000) and Haigh and Holt (2002) analyse hedging in the freight and crack spread markets using a modified BEKK GARCH model. Further work on GARCH-based hedging includes Lee and Yorder (2007), Lien (2008), Lee (2009), Lee (2010), Chang et al. (2011), and Ji and Fan (2011). All these works conclude that a GARCH-based strategy is superior to other static hedges.

Supporters of GARCH hedge ratios argue in unison, that the implementation of GARCH is necessary in order to capture the time-varying asset covariance structure. Although it is true theoretically that GARCH-based minimum-variance hedging should provide greater variance reduction than naïve hedging, due to uncertainty in parameter estimates, and in the correct specification of the GARCH process, this may not be the case in practise. Moreover, typically, the hedge ratios derived from GARCH-type models

are extremely volatile, suggesting unrealistic re-balancing of the hedged portfolio. Doing so would amount to large transaction costs, a problem largely ignored by most of the literature. Moreover, often only in-sample hedging performance is considered. But in reality, the in-sample hedge ratios are irrelevant; only the out-of-sample hedge ratios are of any practical interest.

Besides, most previous papers utilise log returns in the analysis, but log returns are not realised and for assets with prices that can jump, log returns can be highly inaccurate even at the daily frequency. Additionally, since the hedged portfolio can have zero value, even its percentage return is undefined. Thus, the hedging analysis should be based on profit and loss (P&L) rather than returns. Another common feature among many of these papers is that the comparison between GARCH and OLS methods is performed on large estimation windows, which biases results towards the GARCH approach since the OLS model gives equal weight to outdated information at the beginning of the sample. Moreover, this only allows for a relatively small out-of-sample period, which consequently yields results with quite large standard errors. Another significant problem in many of the papers cited above is the use of the price data directly. Recently, Nguyen et al. (2011) have highlighted the pitfalls of using futures series with non-constant maturity, and in particular they highlight the bias in the minimum-variance hedge ratios thus obtained. To avoid this, our analysis is based on constant-maturity futures P&Ls.

We base our study on the problem of hedging crack spread positions. Although one might argue that most of the previous literature has studied the problem of hedging equity or pure commodity positions, we decided to use this underlying as it is a more complicated hedging problem, where prices are highly variable and subject to frequent jumps. As such, more advanced methods have a greater chance to improve the performance. Indeed, as mentioned above, Haigh and Holt (2002) conclude that multivariate GARCH models are superior for hedging the crack spread, although they use a mean-variance rather than a minimum-variance framework. The crack spread represents a simultaneous purchase and sale of crude oil and its refined products, mainly consisting of gasoline and heating oil. An $a : b : c$ crack spread is defined as going long a units of crude oil, and short b and c units of gasoline and heating oil, respectively. These ratios are set according to the refineries production technologies. In 1994 the New York Mercantile Exchange (NYMEX), which offers the highest trading volume on oil-related futures amongst all exchanges worldwide, introduced the possibility for refineries to put up a single margin for the 3:2:1 crack spread. Thus, if refineries hedge this position as a whole using futures contracts on crude oil, gasoline and heating oil in this fixed ratio margin costs are reduced and maintaining the account is simplified for both parties. The popularity of this product led NYMEX to

introduce single margins for any $a : b : c$ crack spread position.

The rest of this paper is structured as follows: section 2 presents the methodological framework; section 3 describes the data; section 4 presents the results; section 5 concludes.

2 Background and Methodology

When hedging spot exposure with futures, the problem of hedge ratio estimation typically arises due to a maturity mismatch. If the futures contract matures exactly on the due date of the spot positions, the hedge ratio will be one and the naïve hedge would always be optimal. We consider the typical case where the spot position is due weekly but the futures mature on a monthly cycle, so that a spot exposure typically arises before the futures contract matures. This is often referred to as the *delta hedging problem* and is discussed in seminal works, such as Cecchetti et al. (1988) and Baillie and Myers (1991).¹

There are two common hedging frameworks in the literature: minimum-variance, where the variance of the hedged portfolio is minimised; and mean-variance, where the investor's utility is maximised based on a mean-variance criterion. The origins of minimum-variance hedging stem from Johnson (1960) where, by minimising the variance of the hedged portfolio under first-order conditions, the hedge ratios can be expressed as a function of the variances and covariances of the spot and futures P&L. The hedged portfolio P&L, $\Delta\Pi_t$, can be described by vectors of m spot P&Ls, futures P&Ls and hedge ratios ($\Delta\mathbf{S}'_t$, $\Delta\mathbf{F}_t$ and $\boldsymbol{\beta}$) such that

$$\Delta\Pi_t = \mathbf{1}'_m \Delta\mathbf{S}_t + \boldsymbol{\beta}' \Delta\mathbf{F}_t ,$$

where $\mathbf{1}_m$ is a $m \times 1$ vector of ones. The variance, $V[\Delta\Pi_t]$, of this portfolio is given by

$$V[\Delta\Pi_t] = \mathbf{1}'_m V[\Delta\mathbf{S}_t] \mathbf{1}_m + \boldsymbol{\beta}' V[\Delta\mathbf{F}_t] \boldsymbol{\beta} + 2\boldsymbol{\beta}' Cov[\Delta\mathbf{F}_t, \mathbf{1}'_m \Delta\mathbf{S}_t] ,$$

where $V[\Delta\mathbf{S}_t]$ and $V[\Delta\mathbf{F}_t]$ are the covariance matrices of $\Delta\mathbf{S}_t$ and $\Delta\mathbf{F}_t$, respectively, and $Cov[\Delta\mathbf{F}_t, \mathbf{1}'_m \Delta\mathbf{S}_t]$ is a vector of covariances between $\mathbf{1}'_m \Delta\mathbf{S}_t$ and the individual elements of $\Delta\mathbf{F}_t$. Minimising the variance yields the optimal hedge ratio vector

$$\boldsymbol{\beta} = -V[\Delta\mathbf{F}_t]^{-1} Cov[\Delta\mathbf{F}_t, \mathbf{1}'_m \Delta\mathbf{S}_t].$$

On the other hand, for the mean-variance criterion, following Working (1953), the in-

¹It should be contrasted with the *roll-over* hedging problem, where the futures in the hedged portfolio mature before the spot positions.

vestor's expected utility value of the hedged portfolio is measured by the certainty equivalent income, $CEI[\Delta\Pi_t]$. Under certain restrictive conditions this can be expressed analytically as

$$CEI[\Delta\Pi_t] = E[\Delta\Pi_t] - \frac{1}{2\gamma}V[\Delta\Pi_t] ,$$

where γ is the coefficient of absolute risk tolerance. Maximising the CEI gives the optimal mean-variance hedge ratio vector as

$$\beta^* = V[\Delta\mathbf{F}_t]^{-1} (\gamma E[\Delta\mathbf{F}_t] - Cov[\Delta\mathbf{F}_t, \mathbf{1}'_m \Delta\mathbf{S}_t]) .$$

Although the latter framework is often employed, we have chosen the former for the following reasons: first, the analytic solution for mean-variance hedging imposes several restrictions (normally distributed P&Ls and an exponential or quadratic utility function) which are unrealistic for our hedging problem; Second, many studies that employ this framework, e.g. Lee and Yorder (2007), make the additional assumption that the speculative (futures) component of the mean-variance hedge follows a martingale process – but then the mean-variance framework ultimately collapses to the minimum-variance framework; Lastly, an out-of-sample analysis of the speculative component depends critically on the expected P&L from the futures series, which typically over-shadows the risk adjustment in the utility.² For volatile commodities such as oil, reducing the variance should be the main aim of hedging.

Estimation of hedge ratios for a multi-asset problem is carried out in two steps: First make a decision on the hedging model to be used (the models differ in the number of equations and variables considered); Second, estimate the relevant variance and covariance parameters for the chosen model. We consider each decision in turn.

2.1 Hedging Models

Let the $a : b : c$ crack spread spot and futures prices, S_t^z and F_t^z , be given by

$$S_t^z = -aS_t^c + bS_t^g + cS_t^h , \quad F_t^z = -aF_t^c + bF_t^g + cF_t^h ,$$

where S_t^c , S_t^g , S_t^h , F_t^c , F_t^g and F_t^h denote the spot and futures prices for crude oil, gasoline and heating oil, respectively. The realised hedged portfolio P&L, $\Delta\Pi_t = \Pi_t - \Pi_{t-1}$, is given by

$$\Delta\Pi_t = \Delta S_t^z + a\beta^c \Delta F_t^c - b\beta^g \Delta F_t^g - c\beta^h \Delta F_t^h , \quad (1)$$

²See Chopra and Ziemba (1993) amongst others.

where β^c , β^g , β^h are the hedge ratios. For the naïve hedge, $\beta^c = \beta^g = \beta^h = 1$.

The hedge ratios that minimise the variance of (1) can be obtained by solving the first-order conditions

$$\begin{bmatrix} a\beta^c \\ -b\beta^g \\ -c\beta^h \end{bmatrix} = \begin{bmatrix} a^2\sigma_{\Delta F_t^c \Delta F_t^c} & -ab\sigma_{\Delta F_t^c \Delta F_t^g} & -ac\sigma_{\Delta F_t^c \Delta F_t^h} \\ -ab\sigma_{\Delta F_t^g \Delta F_t^c} & b^2\sigma_{\Delta F_t^g \Delta F_t^g} & bc\sigma_{\Delta F_t^g \Delta F_t^h} \\ -ac\sigma_{\Delta F_t^h \Delta F_t^c} & bc\sigma_{\Delta F_t^h \Delta F_t^g} & c^2\sigma_{\Delta F_t^h \Delta F_t^h} \end{bmatrix} \begin{bmatrix} a\sigma_{\Delta S_t^z \Delta F_t^c} \\ -b\sigma_{\Delta S_t^z \Delta F_t^g} \\ -c\sigma_{\Delta S_t^z \Delta F_t^h} \end{bmatrix}, \quad (2)$$

where σ_{ij} denotes the covariance between i and j . This method is analogous to Ordinary Least Squares (OLS) regressions of the spot P&L on the hedging instrument(s). In other words, finding the hedge ratios in (2) is analogous to performing the single-equation, multiple-variable regression

$$\Delta S_t^z = \alpha - a\beta^c \Delta F_t^c + b\beta^g \Delta F_t^g + c\beta^h \Delta F_t^h + \epsilon_t, \quad (3)$$

where ϵ_t denotes the regression residuals. We refer to this hedging model as the *single-equation, multiple-variable model*. Since each commodity is exposed to closely related risk factors, it is expected that multicollinearity is present in this setting. Consequently, hedge ratios derived from (3) may have biased standard errors yielding to imprecise hedge ratio estimates.

Alternatively, one might employ a *multiple-equation, single-variable model* in which the hedge ratios are estimated via three asset-by-asset regressions, as follows:

$$\begin{bmatrix} a\Delta S_t^c \\ b\Delta S_t^g \\ c\Delta S_t^h \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} a\beta^c \Delta F_t^c \\ b\beta^g \Delta F_t^g \\ c\beta^h \Delta F_t^h \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{bmatrix}. \quad (4)$$

This model does not account for the covariances between the futures P&Ls at all. In other words, the hedge ratio calculation is the same as in (2), but the off-diagonal elements in the square matrix are assumed to be zero.³

The first model requires estimates of several covariances and variances and each are prone to estimation errors. In contrast, the second model assumes all futures covariances to be zero. A third possibility is to simply impose the restriction $\beta^c = \beta^g = \beta^h = \beta^z$, whereby one obtains the parsimonious *single-equation, single-variable model* for which the estimation errors might be significantly reduced. This model, which is nested in (3),

³We also estimated hedge ratios using generalised least squares (GLS) in a seemingly unrelated regression equations (SURE) system for this model. As the hedging effectiveness results were indistinguishable we do not report them in the following.

is given by

$$\Delta S_t^z = \alpha + \beta^z \Delta F_t^z + \epsilon_t , \quad (5)$$

with optimal hedge ratio given by

$$\beta^z = \frac{\sigma_{\Delta S_t^z, \Delta F_t^z}}{\sigma_{\Delta F_t^z}^2} . \quad (6)$$

The price to pay for a parsimonious model is the implicit assumption of constant correlations between the futures P&Ls. This model has not previously been considered in the literature, but when the correlations between the components of a multiple hedge portfolio are high, then so are the estimation errors in the covariances of the futures P&Ls in (3). Hence, one might expect a superior performance from the *single-equation, single-variable model* despite its restrictive assumptions on correlation.

2.2 Estimation Methods

We now turn to the econometric methods used to estimate the variances and covariances in the hedging models. We employ four different popular estimation methods: OLS; exponentially weighted moving averages (EWMA); the standard symmetric GARCH; and an asymmetric GARCH model. To conduct an out-of-sample study we re-estimate all parameters of the OLS and GARCH models using a rolling window of length n . The parameter of the EWMA model is fixed, a priori.

With OLS, variances and covariances of two assets Y_1 and Y_2 are simply estimated by their sample counterparts

$$\hat{\sigma}_{\Delta Y_1, t}^2 = \frac{1}{n-1} \sum_{i=0}^n (\Delta Y_{1, t-i} - \Delta \bar{Y}_{1, t})^2 , \quad (7)$$

and

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_2, t} = \frac{1}{n-1} \sum_{i=0}^n (\Delta Y_{1, t-i} - \Delta \bar{Y}_{1, t})(\Delta Y_{2, t-i} - \Delta \bar{Y}_{2, t}) , \quad (8)$$

respectively. EWMA variances and covariances are estimated via the recursions

$$\hat{\sigma}_{\Delta Y_1, t}^2 = (1 - \lambda) \Delta Y_{1, t-1}^2 + \lambda \hat{\sigma}_{\Delta Y_1, t-1}^2 , \quad (9)$$

and

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_2, t} = (1 - \lambda) \Delta Y_{1, t-1} \Delta Y_{2, t-1} + \lambda \hat{\sigma}_{\Delta Y_1 \Delta Y_2, t-1} , \quad (10)$$

where λ is the EWMA decay coefficient which takes a value between 0 and 1. With a

lower λ more emphasis is placed on the most recent observations and the model hence becomes more reactive to changing market conditions.

GARCH variances and covariances are obtained using the BEKK model specification of Engle and Kroner (1995). For a vector of zero mean P&Ls $\Delta\mathbf{Y}_t$, the multivariate GARCH covariance matrix estimate \mathbf{H}_t is based on the dynamics

$$\mathbf{H}_t = \mathbf{A}'\mathbf{A} + (\mathbf{B}'\Delta\mathbf{Y}_{t-1})(\mathbf{B}'\Delta\mathbf{Y}_{t-1})' + \mathbf{C}'\mathbf{H}_{t-1}\mathbf{C}, \quad (11)$$

where \mathbf{A} , \mathbf{B} , \mathbf{C} are $m \times m$ matrices of the BEKK parameters for m assets. The parameter estimates are obtained by maximising the log-likelihood function⁴

$$\ln L(\theta) = -\frac{1}{2} \sum_{t=1}^n (\ln(|\mathbf{H}_t|) + \Delta\mathbf{Y}_t' \mathbf{H}_t^{-1} \Delta\mathbf{Y}_t). \quad (12)$$

As it is well known that the symmetric GARCH specification can be improved by allowing for an asymmetric variance response to shocks we also employ the asymmetric GARCH BEKK specification (AGARCH) of Grier et al. (2004). Here the variances dynamics are specified as

$$\hat{\mathbf{H}}_t = \mathbf{A}'\mathbf{A} + (\mathbf{B}'\Delta\mathbf{Y}_{t-1})(\mathbf{B}'\Delta\mathbf{Y}_{t-1})' + \mathbf{C}'\hat{\mathbf{H}}_{t-1}\mathbf{C} + (\mathbf{D}'\Delta\mathbf{Y}_{t-1}^*)(\mathbf{D}'\Delta\mathbf{Y}_{t-1}^*)', \quad (13)$$

where \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} are $m \times m$ matrices of the asymmetric BEKK parameters for m assets and \mathbf{Y}_t^* is a vector of $\max\{Y_t, 0\}$ for a positively skewed sample or $\min\{Y_t, 0\}$ for a negatively skewed sample.

For ease of presentation, we abbreviate the hedging models and estimation techniques by $model_{ij}$ where $model$ denotes the estimation method, i.e. $model = \{OLS, EWMA, GARCH, AGARCH\}$, and $i = 1, 3$ and $j = 1, 3$ denote the number of equations and variables in the regression system respectively. For example, $EWMA_{13}$ refers to the single-equation, multiple-variable model as specified in (3) where the variances and the covariances are estimated using the EWMA method, etc.

In total, seven hedging models are analysed: naïve, OLS_{31} , OLS_{13} , OLS_{11} , $EWMA_{11}$, $GARCH_{11}$ and $AGARCH_{11}$. For the EWMA, GARCH and AGARCH estimation methods we omit results for multiple-equation or multiple-variable models because preliminary results, based only on the OLS models, show that the three regression configurations are more or less equally effective. Moreover, the proliferation of parameters when GARCH and AGARCH models are applied to multiple-equation or multiple-variable models ex-

⁴We use Kevin Sheppard's UCSD GARCH toolbox for the estimation, available at http://www.kevinshppard.com/wiki/UCSD_GARCH.

acerbates the problem of parameter estimate instability, which is discussed later on with reference to Figure 3.

2.3 Margin and Transaction Costs

When trading futures on the NYMEX, transaction costs arise from the round trip commission charged by the exchange and from the bid-ask spread. Since the early 2000's, the NYMEX has reduced the round trip commission costs from \$15.00 to \$1.45 per futures contract bought and sold. Although the NYMEX is an open-outcry market (which allows limit orders) the hedger is assumed to place market orders, to prioritise the variance reduction over possible gains from trading. The bid-ask spreads of the three considered commodity futures, x , y and z are defined as the average spread between the bid and the ask price divided by the average mid price of each commodity futures. The dollar value of the bid-ask spreads, TC_t , arises from re-balancing the portfolio and is given by:

$$TC_t = aF_t^c|\Delta\beta_t^c|x + bF_t^g|\Delta\beta_t^g|y + cF_t^h|\Delta\beta_t^h|z . \quad (14)$$

The modulus signs are placed to indicate how the hedger loses the spread regardless of the direction of trade. We follow Dunis et al. (2008) and set x , y and z to be 1 bps, 10 bps and 12 bps, respectively. Although these are bid-ask spreads of first-to-mature rollover series, we assume that these are constant through the first two months of the term structure and hence use the same for our constant-maturity futures.

Margin costs arise from raising the initial margin and from marking-to-market the maintenance margins. In the past decade there have been several changes to the NYMEX margin requirement rules. When trading an $a : b : c$ crack spread, NYMEX calculates the initial margin based on the portfolio Value-at-Risk at the 5% or 1% level for commercial (hedgers) and non-commercial (speculators) traders respectively. For a hedger who shorts a crack spread expiring in 1 month, the initial margin is approximately \$10, \$18 and \$7 per 3:2:1, 5:3:2 and 2:1:1 crack spread bundles respectively (as opposed to \$15, \$20 and \$10 for a speculator).⁵ We shall focus on the costs incurred by refineries, which are generally treated as hedgers by the clearing house. The total cost m_t^i from raising the initial margin is

$$m_t^i = \beta_{z,t}N(r_t^d - r_t^f) , \quad (15)$$

where r_t^d is the cost of raising the initial margin, r_t^f is the risk-free rate of return gained

⁵For an $a : b : c$ crack spread, a “bundle” indicates simultaneously going long a barrels of crude oil, short b barrels of gasoline and short c barrels of heating oil. These approximations were obtained from NYMEX on 06/06/2011.

from depositing in the margin account, N is the initial margin required per crack spread bundle and $\beta_{z,t}$ is the number of crack spread bundles purchased. In the cases where the hedge ratios do not allow for exact transaction of the bundles (i.e. $\beta^c \neq \beta^g \neq \beta^h$), the approximation $\beta^z \approx \frac{a\beta^c + b\beta^g + c\beta^h}{a+b+c}$ is taken instead. The refinery is assumed to raise debt for the initial margin, r_t^d is set as the average cost of debt in the industry. The top ten US refineries are currently, on average, rated AA by Moody's. Hence Moody's AA bond index was chosen as a proxy for the cost of debt r_t^d . Three-months US T-bill rates are used as a proxy for r_t^f .

The gains and losses from the maintenance margin arise from the movement in the futures prices every day. These are marked-to-market daily but as we work with weekly data we employ a linear approximation of the daily changes in the margin account. The weekly interest on the margin account m_t^m , assuming no margin calls, is therefore approximated as

$$m_t^m = \frac{1}{2} (a\beta_t^c \Delta F_t^c - b\beta_t^g \Delta F_t^g - c\beta_t^h \Delta F_t^h) r_t^f . \quad (16)$$

The total hedged portfolio P&L including margin and transaction costs $\Delta \Pi_t^*$ may now be expressed as

$$\Delta \Pi_t^* = \Delta \Pi_t + m_t^m - m_t^i - TC_t . \quad (17)$$

2.4 Performance Measurement

Hedging effectiveness is measured by the Ederington Effectiveness (EE) calculated as

$$EE = \frac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2} , \quad (18)$$

where σ_u^2 and σ_h^2 are the variances of the hedged and the unhedged portfolios, respectively. We compute the EE for each model in two ways: (i) in-sample using unconditional variances over the whole sample period and (ii) out-of-sample using a rolling window of EWMA variances with $\lambda = 0.99$. The EWMA method is preferred to a rolling window of unconditional variances because the latter produces ghost features where the variances are augmented as long as a spike in the P&L remains inside the window. This is also to avoid any possible bias the unconditional EE may have over the conditional EE as highlighted by Lien (2009), as well as to examine the robustness of the models' performance over the analysis period. To test whether the variance reductions from each model are significantly different from the naïve hedge we apply the standard F-test for equality of variances.

3 Data

3.1 Spot Prices

Wednesday spot prices from 30/12/1992 to 23/02/2011 of Cushing WTI light-sweet crude oil, New York Harbour heating oil No.2, unleaded gasoline and RBOB gasoline barges are taken from Platts. In the rare cases where Wednesday is not a trading day, the price on Tuesday is taken instead.⁶ The delivery location of the spot prices is the same as their corresponding NYMEX futures. We use Platts prices as these are collected from a window of physical commodity buyers which truly reflect the spot of the physical commodity trades. Platts prices are determined at 4:30pm GMT as opposed to the NYMEX futures prices which are determined at 5:00pm GMT-5/6 (depending on summer/winter time zones) which poses a non-synchronicity problem between the two sources. This may invoke a downward bias on the daily correlation between the spot and futures prices, but our analysis is on weekly data with weekly hedging horizons. As such, this relatively minor time difference will have negligible effect on the empirical results.

3.2 Futures Prices

Wednesday NYMEX futures prices of crude oil, heating oil, and gasoline from 30/12/1992 to 23/02/2011 are based on the NYMEX closing price. Among these three commodities, gasoline production has undergone some changes over time and therefore, since 2006, the NYMEX has no longer offered the original unleaded gasoline futures, replacing them by Reformulated Blendstock for Oxygen Blending (RBOB) gasoline futures. Due to data availability and low liquidity in the early years of the RBOB futures market, we switch from unleaded to RBOB gasoline in different years in the spot (2003) and futures markets (2006). This problem is of limited importance as both types of gasoline face the same demand and supply trends so that the prices are extremely highly correlated.

There are two ways to create a continuous series of futures P&Ls: the rollover method and the constant-maturity method. A standard rollover series is constructed by taking a futures price series up to a rollover date, the price series then jumps to the prompt futures series which is taken up to the next rollover date and so on. Often, the rollover dates are roughly a week before maturity to avoid thin market trading but for the commodities we study there is no need for this adjustment since trading continues in high volumes right up to the maturity date.

⁶And in the case where Tuesday is not a trading day as well as Wednesday, Monday's price is taken instead. In the circumstance where none of those days are trading days, the week is omitted entirely.

However, there are two problems associated with the rollover futures series. First, as explained by Nguyen et al. (2011) where unlike constant maturity series, any regression relating spot data to futures data will be contaminated by the “saw-tooth” pattern in the basis. Second, gasoline and heating oil futures contain breaks in the term structure due to strong seasonality effects: prices at year-end periods are expected to be lower/higher than the rest of the year, and start of the year prices are expected to be higher/lower than the rest of the year. Hence, futures contracts which mature at these periods are often priced at a significant step above or below the rest of the year. When rolling from a mid-year contract to a year-end contract (say, October to November) an artificial jump is created through this seasonality effect. In fact, we obtain up to four jumps in the rollover series per year, which produces outliers in the P&L data that are merely an artefact of the rollover methodology and are difficult to deal with statistically.

Such artefactual jumps are not present in the constant maturity futures P&L series that are constructed by linearly interpolating between two futures with maturities above and below the constant maturity target.⁷ Galai (1979) compares two methods for creating constant maturity futures which he terms the value index (interpolation between prices) and the return index (interpolation between returns). Galai shows that the return index method is the only one that provides realisable investments. As we require realisable investments to implement the optimal hedge ratios in practise, but our analysis must also be based on P&L rather than returns, we adapt Galai’s return index method to the P&L as follows:⁸

$$\Delta F_{t,T} = \eta_t \Delta F_{t,T_1} + (1 - \eta_t) \Delta F_{t,T_2}, \quad 0 \leq \eta_t \leq 1, \quad (19)$$

where $\Delta F_{t,T}$ is the constant-maturity futures P&L expiring in T days, $\Delta F_{t,T_1}$ and $\Delta F_{t,T_2}$ are the futures P&Ls expiring at T_1 and T_2 respectively, and

$$\eta_t = \frac{T_2 - (t + T)}{T_2 - T_1}, \quad T_1 < T < T_2.$$

A reasonable choice for T is 44 calendar days, i.e. approximately 1.5 months. With this choice there will always be two maturities straddling the constant maturity. Of course, to maintain a constant maturity series of ΔF_t^z for the regression (5) the constant maturity must be the same for all futures.

⁷With constant maturity futures the hedger is required to re-balance at the frequency of the data (in our case every week) but the transaction costs incurred are negligible.

⁸Note that several existing hedging studies adopt the value index method, which has the clear disadvantage of being non-investable.

3.3 Summary Statistics

Tables 1 and 2 report summary statistics and correlations of the weekly spot and constant maturity futures P&L distributions based on the entire sample period. Crude oil spot and futures are less volatile than gasoline and heating oil spot and futures, and in each case the spot is more volatile than the futures. Each P&L except spot heating oil is slightly negatively skewed and all series are highly leptokurtic.

	ΔF^c	ΔF^g	ΔF^h	ΔS^c	ΔS^g	ΔS^h
μ	0.0294	0.0988	0.0469	0.0718	0.0814	0.0868
σ	2.2428	2.6110	2.6191	2.3394	3.0467	2.7847
τ	-0.2923	-0.3527	-0.0619	-0.1063	-0.3127	0.0597
\varkappa	7.2883	3.8765	7.9457	6.9776	3.6993	6.6830

Table 1: Summary statistics for weekly constant-maturity futures and spot P&Ls for the sample period 30/12/1992 to 23/02/2011. The total number of observations is 938 for each series. μ, σ, τ and \varkappa denote the mean, standard deviation, skewness and excess kurtosis, respectively.

	ΔF^c	ΔF^g	ΔF^h	ΔS^c	ΔS^g	ΔS^h
ΔF^c	1	-	-	-	-	-
ΔF^g	0.8553	1	-	-	-	-
ΔF^h	0.9003	0.8384	1	-	-	-
ΔS^c	0.9704	0.8248	0.8666	1	-	-
ΔS^g	0.7334	0.9305	0.7368	0.7073	1	-
ΔS^h	0.8375	0.7839	0.9505	0.8104	0.6913	1

Table 2: Correlation matrix between spot and futures P&Ls for the sample period 30/12/1992 to 23/02/2011. The total number of observations is 938 for each series.

Figure 1 displays the P&L time series for all six variables. We observe that all series show rising volatility from the year 2000 onwards. Surges in prices produced by unexpected supply shortages result in frequent jumps in all the series. In many cases a decoupling of spot and futures prices results in a jump in the basis which is difficult to hedge effectively with the one-for-one ratio, and possibly also with a minimum variance hedge ratio. Only one, very extreme spike in the data was removed. This was during the week of Hurricane Katrina, during which we assume no trades were made.

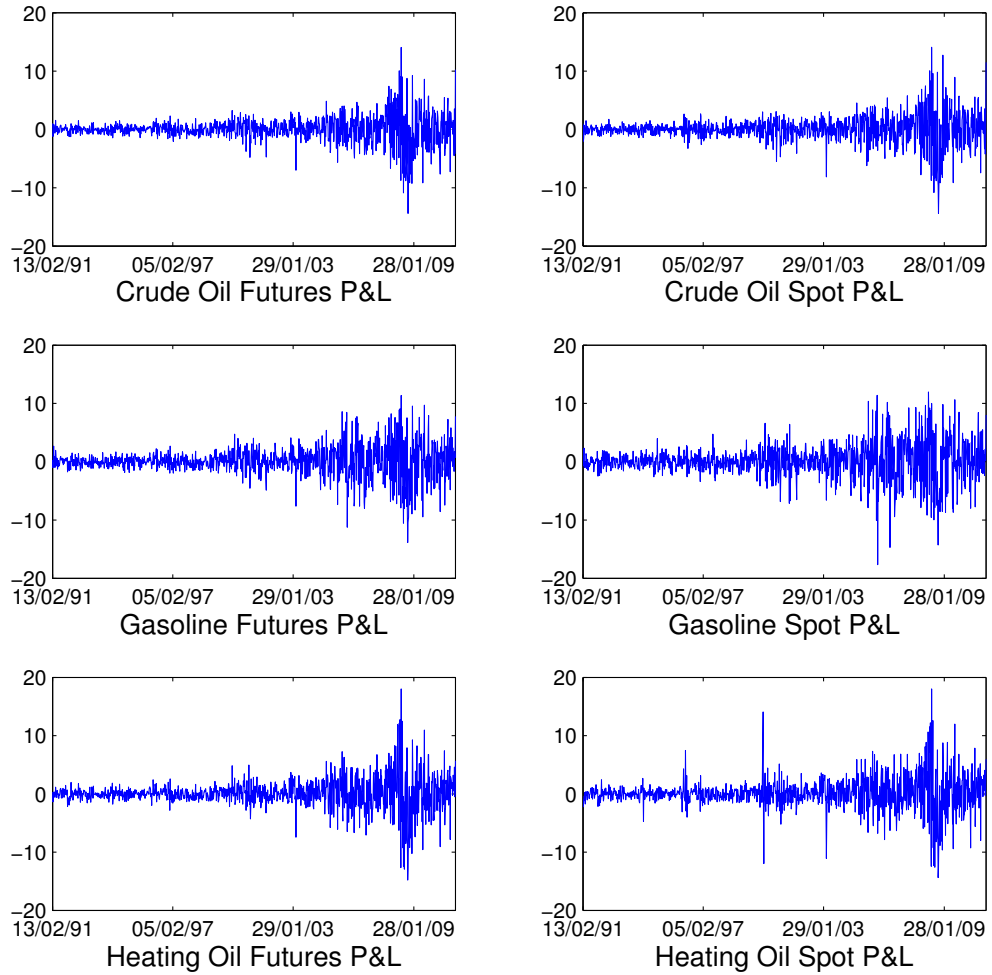


Figure 1: Spot and constant-maturity futures P&L series for each commodity. Period: 30/12/1992 - 23/02/2011. Prices for the week of 28/08/2005 - 02/09/2005 have been removed due to abnormal market conditions caused by hurricane Katrina. The investor is assumed to make no trades on this week.

4 Empirical Results

We study the hedging performance of seven different models: naïve, OLS₃₁, OLS₁₃, OLS₁₁, EWMA₁₁, GARCH₁₁ and AGARCH₁₁ both, in-sample and out-of-sample. For the in-sample analysis, parameters are estimated using the entire data set, i.e. 938 weekly observations. Hedge ratios are then calculated based on these parameters and held constant for computing the hedge performance. But clearly, the in-sample analysis is just a data-fitting exercise – it is the out-of-sample analysis that matters for practical purposes. Here, the parameters are estimated using a rolling window of 260 weeks.⁹ The hedge ratios estimated at time t are then applied to the one step ahead P&L. The hedger is assumed to re-estimate the parameters every week.¹⁰ Since the EWMA parameter λ is always constant, EWMA results are the same both in-sample and out-of-sample.

All empirical results presented are for the 3:2:1 crack spread bundle, as many refineries have this approximate crack spread and the original NYMEX margin bundles were also based on this spread. Robustness checks were carried out by repeating our analysis for two other crack spread ratios, namely 5:3:2 and 2:1:1. Since our qualitative conclusions remain the same for these other crack spread bundles detailed results are not reported here. However, they are available from the authors on request.

4.1 Hedge Ratios

Table 3 reports the average hedge ratios for each model and their standard deviations. In-sample hedge ratios are reported for completeness, we focus the following discussion on the out-of-sample hedge ratios. The multiple-equation model OLS₃₁ yields hedge ratios closer to 1.0, yet the single-equation models produce hedge ratios nearer to 1.3. It is tempting to conclude that the OLS₁₃ model produced these higher hedge ratios because of multicollinearity. However, both OLS₁₃ and OLS₁₁ produce hedge ratios of roughly the same magnitude, which brings into question any such conclusion.

The fact that hedge ratios deviate from 1 is simply a reflection of the maturity mismatch between the spot and the futures: the maturity of the futures are constant at 44

⁹For the OLS methods we have also employed windows of length 104, 156, and 208 to ensure that this choice is not the driver of our results. No significant differences were found. For the GARCH models, shorter windows are not feasible due to the number of parameters to be estimated. In some few instances, the optimisation of the GARCH parameters failed to converge. We then used the estimates from the previous week.

¹⁰A separate analysis for annual recalibration of parameters was also carried out, attempting to tame the volatility of the GARCH parameter estimates. Hedging performance was found to be inferior to the alternative weekly re-estimation method. Parameters also jump on a year-to-year basis. Hence the results are not presented here but are available on request.

in-sample	OLS ₃₁	OLS ₁₃	OLS ₁₁	EWMA ₁₁	GARCH ₁₁	AGARCH ₁₁
β^c	1.021	1.274	1.276	1.381	1.360	1.262
	-	-	-	(0.169)	(0.281)	(0.211)
β^g	1.085	1.305	1.276	1.381	1.360	1.262
	-	-	-	(0.169)	(0.281)	(0.211)
β^h	1.011	1.663	1.276	1.381	1.360	1.262
	-	-	-	(0.169)	(0.281)	(0.211)
out-of-sample						
β^c	1.035	1.373	1.238	1.381	1.364	1.353
	(0.021)	(0.131)	(0.121)	(0.169)	(0.300)	(0.298)
β^g	1.171	1.449	1.238	1.381	1.364	1.353
	(0.055)	(0.120)	(0.121)	(0.169)	(0.300)	(0.298)
β^h	1.085	1.238	1.238	1.381	1.364	1.353
	(0.061)	(0.301)	(0.121)	(0.169)	(0.300)	(0.298)

Table 3: Average in-sample and out-of-sample hedge ratios with standard deviations in parentheses. In-sample ratios are estimated using the entire sample period. Out-of-sample hedge ratios are estimated using a moving window of 260 weeks. Period: 30/12/1992 - 23/02/2011.

days, whereas the spot is due weekly, so hedge ratios are expected to be larger than 1 since at this maturity the futures variance is lower. In fact, the OLS₃₁ model produces smaller hedge ratios, closer to 1, because all cross-market correlations are assumed to be zero. As they are certainly not (see Table 2) this produces a substantial bias. On the other hand, the OLS₁₃ assumes a equal cross-market correlation across all commodities – an assumption that seems reasonable in light of Table 2.

Figure 2 displays the evolution of the OLS models' out-of-sample hedge ratios over time. One can observe that OLS₃₁ and OLS₁₁ are relatively stable. In contrast, the hedge ratio for the heating oil contract of OLS₁₃ in particular exhibits some substantial transitions over time. This is also reflected by the relatively high standard deviation in Table 3. The AGARCH₁₁ estimation method produced the most volatile out-of-sample hedge ratios. Although this characteristic is expected given that GARCH parameters are generally more sensitive with respect to innovations in the data, the volatility of the hedge ratios should roughly be of the same magnitude as the EWMA₁₁ hedge ratios.¹¹ According to Table 3 however, the out-of-sample GARCH₁₁ hedge ratios are roughly 33% more volatile than the EWMA₁₁ hedge ratios.

¹¹For some periods, the GARCH hedge ratios are unmanageably large for the investor (e.g. up to ± 5 times the spot investment). To control these, when the absolute value of the GARCH hedge ratios exceed twice the absolute value of the EWMA hedge ratio, the GARCH hedge ratio from the previous time step is used instead.

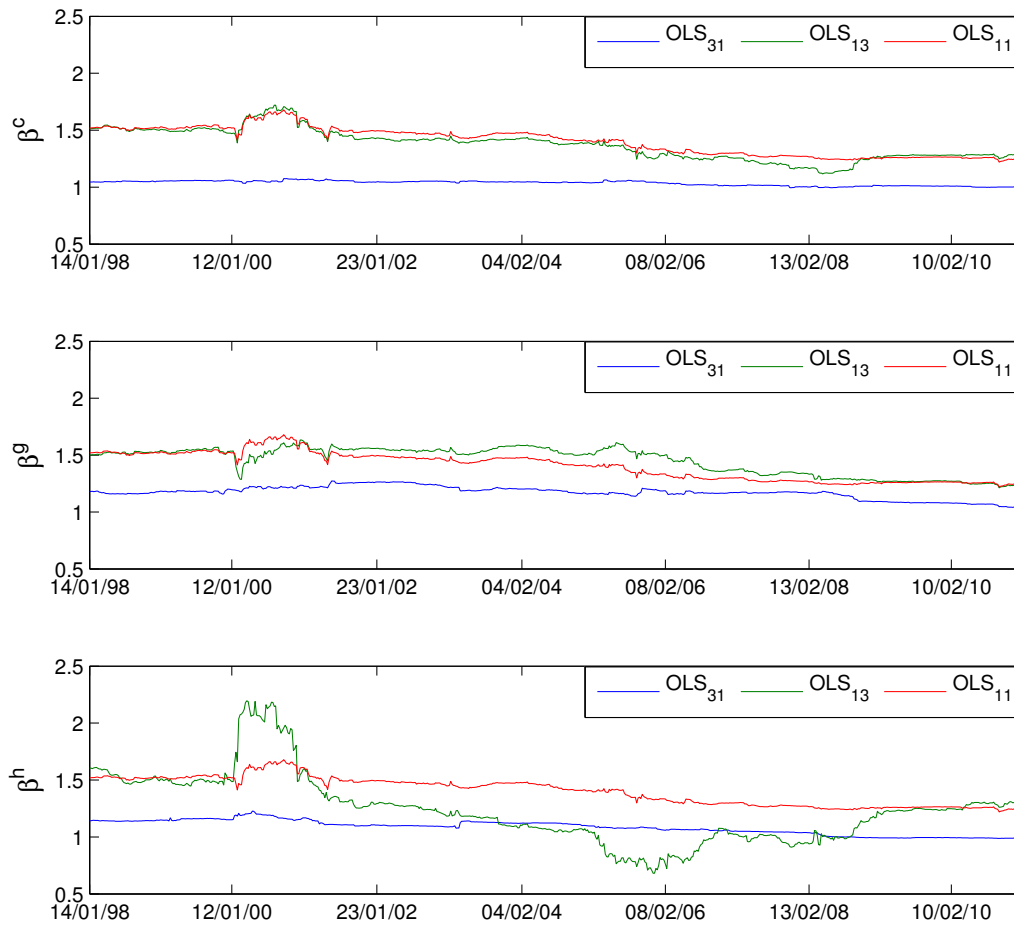


Figure 2: OLS hedge ratios calculated using a moving window of 260 weeks. Period: 14/01/1998 - 23/02/2011.

This is also shown in Figure 3, which compares the behaviour of the hedge ratios derived for the single-equation, single-variable models over time. Note how volatile the GARCH hedge ratios are over time. Would a serious risk manager implement a hedging strategy that involved re-balancing more than 100% of the hedging portfolio from week to week? This casts serious doubts on the merits of GARCH-based hedge ratios.

The GARCH model parameter estimates are highly volatile over time. Table 4 displays the means of the estimated GARCH parameters and their standard deviations measured over the entire out-of-sample period. Clearly, the estimates are far from being stable. Extending the length of the estimation windows up to 8 years did not produce substantially more stable estimates. Hence, the problem is not one of convergence to

local optima instead of a global optimum, but rather an intrinsic problem with applying GARCH models for hedging when there are frequent jumps in a highly volatile basis. In this situation, large changes in the conditional variance parameter estimates are only to be expected. Indeed, the finding of highly unstable GARCH hedge ratios is nothing peculiar for our data set. Previous studies, e.g. Lee and Yorder (2007) and Lee (2010), have found similar results.

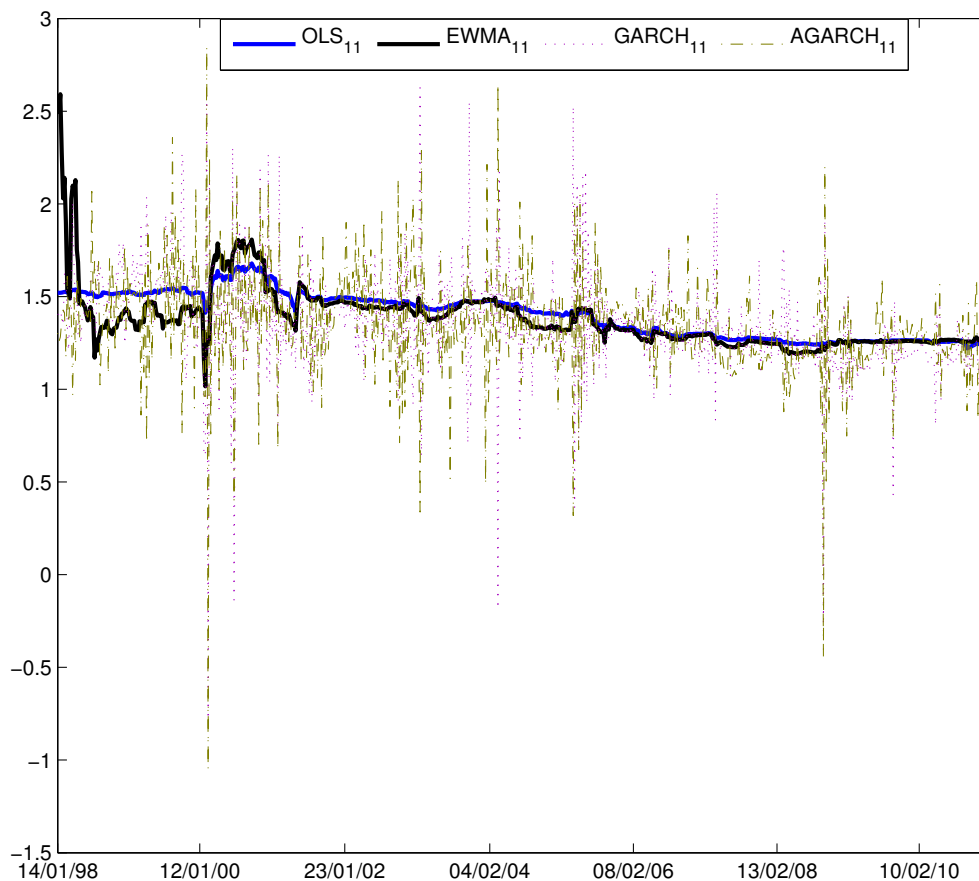


Figure 3: Comparisons between out-of-sample hedge ratio estimates of the OLS₁₁, EWMA₁₁, GARCH₁₁ and AGARCH₁₁ models. EWMA₁₁ hedge ratios estimated with $\lambda = 0.99$. Period: 24/12/1997 - 23/02/2011.

Another problem concerns transaction costs. Re-balancing a hedge with such extreme swings will amount to much higher transaction costs in comparison to the other methods having more stable hedge ratios. Table 5 presents the average transaction costs (including margin costs) of the seven hedging strategies. One can see that the GARCH₁₁ and AGARCH₁₁ models produce average transaction costs of \$0.06 per bundle. A refinery

Parameter	GARCH		AGARCH	
	Mean	St. Dev.	Mean	St. Dev.
A_{11}	0.544	(0.675)	0.363	(0.878)
A_{21}	1.203	(0.995)	0.989	(1.987)
A_{22}	0.141	(0.788)	0.038	(0.213)
B_{11}	0.002	(0.323)	0.014	(0.378)
B_{12}	-0.883	(0.672)	0.713	(0.866)
B_{21}	-0.011	(0.563)	0.058	(0.156)
B_{22}	0.672	(0.151)	-0.466	(0.476)
C_{11}	1.049	(0.310)	0.810	(0.399)
C_{12}	1.861	(0.283)	1.772	(0.426)
C_{21}	-0.147	(0.678)	0.006	(0.226)
C_{22}	-0.546	(0.197)	-0.434	(0.293)
D_{11}	-	-	0.078	(0.187)
D_{12}	-	-	0.263	(0.555)
D_{21}	-	-	-0.085	(0.175)
D_{22}	-	-	-0.223	(0.406)
log-likelihood	-1177.400	(231.649)	-1168.800	(232.563)

Table 4: Out-of-sample mean and standard deviation of GARCH₁₁ and AGARCH₁₁ parameter estimates using a 260 weeks rolling window. Period: 24/12/1997 - 23/02/2011.

	naïve	OLS ₃₁	OLS ₁₃	OLS ₁₁	EWMA ₁₁	GARCH ₁₁	AGARCH ₁₁
In-sample	0.007 (0.004)	0.008 (0.005)	0.010 (0.019)	0.009 (0.005)	0.012 (0.009)	0.024 (0.027)	0.036 (0.040)
Out-of-sample	0.007 (0.004)	0.018 (0.010)	0.022 (0.011)	0.011 (0.006)	0.012 (0.009)	0.062 (0.073)	0.061 (0.070)

Table 5: Average transaction and margins costs in \$ per spot bundle, numbers in parentheses represent standard deviations. Period 30/12/1993 - 23/02/2011.

that purchases 50,000 3:2:1 crack spread bundles per week for example, would be paying \$156,000 per year only to implement their hedging strategy. This is very large in comparison to the other models, especially the naïve strategy, where hedging does not require re-balancing and the associated transaction and margin costs are near zero.

4.2 Hedging Effectiveness

We now consider the hedging effectiveness of each model. The main question is whether the effort to implement more advanced models and the associated transaction costs pay

off in a superior hedging performance? Table 6 shows the overall hedging performance measured by the unconditional EE of each model both, in- and out-of-sample. In the more relevant out-of-sample test, all models produce variance reductions in the range of 65-70% with the OLS₁₁ as the most effective model. The GARCH₁₁ and AGARCH₁₁ models perform worst, with an EE of 66.33% and 66.46%, respectively.

	naïve	OLS ₃₁	OLS ₁₃	OLS ₁₁	EWMA ₁₁	GARCH ₁₁	AGARCH ₁₁
In-sample	66.71%	67.50%	69.35%	69.88%	69.51%	66.87%	65.63%
Out-of-sample	66.71%	67.33%	69.16%	69.73%	69.51%	66.33%	66.45%

Table 6: Whole-sample EE of each model in percentage points. Hedge ratios calculated using 260 weeks. Whole-sample data points: 678. Period 30/12/1993 - 23/02/2011. Whole-sample unhedged portfolio variance: 44.34 \$². Hedged portfolio includes margin and transaction costs.

Figure 4 displays the out-of-sample conditional EE for each model over time. It is variable throughout the sample period and occasionally reacts to the jumps in the basis. For instance, during the first quarter of 2000 the hedging effectiveness of all models drops to almost 0% but then rises to about 40% after about 3 months. This is due to the surge in heating oil prices (note the spike at this time in the bottom, right-hand graph in Figure 1). We have not excluded data from this event because the price shift occurred over a period of two months, and hedging would have been necessary over such a long period. Although the GARCH models are expected to perform better under these conditions since they are more capable to react to changing market conditions, here they produce roughly the same hedging effectiveness as all the other models.

From Figure 4 we can conclude that all models have similar effectiveness throughout the entire sample period. To test this more formally, we perform a standard F-test, for equality of variances: between the variance of P&L resulting from the naïve hedge and the P&L variance from each of the models. We use the out-of-sample P&L and evaluate the F-statistic using a rolling window to calculate the individual variances.¹² Figure 5 depicts these F-statistics together with lines showing the critical values at the 90% and 95% confidence level. We fail to reject the null hypothesis that the hedge portfolio variance produced by more advanced models is significantly smaller than the naïve strategy in every instance. Thus we conclude that no model is able to improve upon the naïve hedge, utilising the 3:2:1 bundle offered by NYMEX. The same conclusion was reached for all the $a : b : c$ crack spreads considered, although detailed results have not

¹²We have used a one year window (52 weeks) in order to obtain a long out-of-sample period. Results for longer windows (260 weeks) yield identical conclusions.

been reported for brevity.

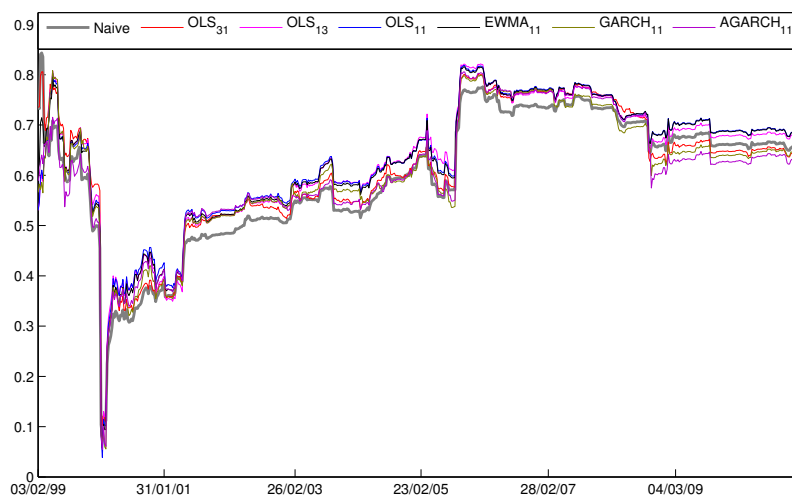


Figure 4: Out-of-sample analysis: EWMA EE of each model. Period: 03/02/1999 - 23/02/2011. Hedge ratios calculated using 260 weeks.

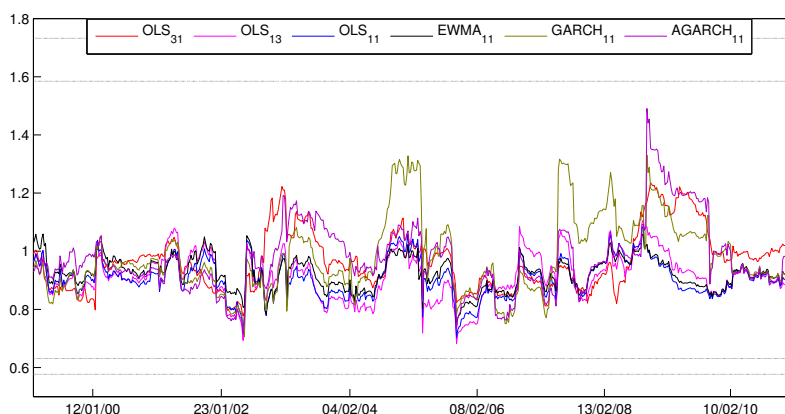


Figure 5: Rolling-moving F-statistic for testing the equality of variances between each hedging model and the naïve hedged portfolio. Rolling-moving variances are calculated using a 52 weeks window. Hedge ratios calculated using 260 weeks. Period: 03/02/1999 - 23/02/2011. Horizontal lines indicate two-sided critical values at 5% and 10% significance levels, respectively.

Lastly, to ensure that our results are not driven by our assumption regarding the size of transaction costs, we repeat the analysis ignoring transaction costs. The change in EE after including transaction and margin costs for all models are found to be very small and mostly negative, the largest being -0.18% from the AGARCH model. This may be because the transaction costs are, on average, positively correlated with the hedged portfolio and

hence add to the portfolio variance. To improve the hedging models, one could account for the correlation between the margin/transaction costs and the spot and futures P&L when minimising the portfolio variance, i.e. minimising the variance of $\Delta\Pi_t^*$ as opposed to the variance of $\Delta\Pi_t$. However, this requires estimates of additional covariance terms and unless done accurately, could undermine the variance reduction process. The resulting effects would also be minimal given that the changes in EE incurred by the costs are so small.

5 Conclusions

We have compared seven different models for estimating hedge ratios for crack spread delta hedging. Although all models are found to produce a healthy amount of variance reduction (roughly 67% on average) none of them is able to outperform the naïve hedging strategy. In fact, the most complex models delivered the worst hedging results. These strategies are not only more complicated to implement but also generate the highest transaction costs. Therefore we conclude that, due to instability in parameter estimates and excessive transaction costs, it is pointless to follow any other strategy than the naïve.

Our findings contradict a fair body of existing literature which concludes that model-based minimum-variance hedging is superior. We have shown that the hedging effectiveness was statistically indistinguishable between all the models considered. This finding was based on a very long out-of-sample period, but we would have reached the same conclusion had we used much shorter sub-periods or, indeed, had we based conclusions on in-sample analysis alone as some previous studies have done. Moreover, we have taken much more care with the data than most other studies, using the best (Platts) spot prices, constructing realisable constant-maturity futures (rather than unrealisable ones, or using rollover series that are tainted by artificial jumps in P&L) and taking meticulous care to account for all the costs involved in hedging.

We have also shown that it is not the cost of minimum-variance hedging that matters – not even for the excessively variable hedge ratios prescribed by GARCH models. The main point for end-users to take away from our study is that, even for complex underlyings such as spreads on oil-related commodities which produce a basis that is extremely variable and jumpy, the maturity mismatch justification for minimum-variance hedging is simply not viable. It may be that minimum-variance hedging can improve on the so-called “naïve” hedge when a proxy futures contract must be used – but even this remains an open question waiting for a thorough empirical analysis.

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